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PRINCIPLES OF VALUATION

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OF
VALUATION

BY

JOHN ALDEN GRIMES

AND

WILLIAM HORACE CRAIGUE

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ELJIVA SHERMAN CRAIGUE

PREFACE

The origin of the compound interest deferment method for valuing a single payment due at a future date is lost in antiquity, and the extension of this method to the valuation of equal annual amounts, or annuities, dates back at least three centuries. The first record of a sinking fund is that established by Sir Robert Walpole in England during the year 1716, and it may be assumed that the development of the compound interest sinking fund annuity formula was practically contemporaneous. The premises of valuation, which are called the compound interest actuarial method by the authors, have been accorded mention in several publications, among others the textbook on interest sponsored by the Institute of Actuaries, of London; but, in so far as the authors are aware, the complete mathematics of that method of valuation are developed in this book for the first time. The Hoskold type of sinking fund valuation, employing two rates of interest for valuing speculative income, was expounded by H. D. Hoskold[1] in a book published in 1877. Hoskold developed a formula for the valuation of equal annual income beginning at date of valuation, but reverted to compound interest discount for periods of deferment. George King[2] applied different principles of deferment, which are somewhat similar to the mathematical principles followed in this book. Based upon this theory, as explained by T. A. O'Donohue,[3] "the investor must increase his holding to the extent of the natural increase in capital at the accumulative [safe] rate, but it is not logical to assume that the extra interest, for which the speculation is made, can be invested in the transaction to acquire interest at the remunerative [speculative] rate." This reasoning has been attacked as discordant with financial practice, since the investment of unpaid interest is not optional with the investor but is required of him. However,

[1] "Engineers Valuing Assistant," Longmans, Green & Co., London.

[2] "The Theory of Finance," third edition in 1898.

[3] "Transactions of the Institute of Mining Engineers," volume 32, page 388, London, 1907.

the mathematical principle that all contributions to the sinking fund must be accorded the same treatment (whether individual contributions are positive, negative or zero in amount) can hardly be questioned, and gives the same result as that developing from the theory of investment just quoted. The Morkill type of sinking fund valuation was developed by D. B. Morkill.[1] The three-interest-rate sinking fund method of valuation was developed by the authors, who desire to acknowledge mathematical assistance received from Harold Alden Wheeler in the derivation of the formula for valuing equal annual income in accordance with these premises. The previously mentioned mathematical developments apply only to the simple case of equal annual income. Henry Louis[2] recently published formulas suited to the valuation of income increasing by equal annual amounts or percentages in accordance both with the compound interest annuity premises and the Hoskold-O'Donohue sinking fund valuation premises.

The initial work on the mathematics of valuation, contained herein, was commenced by the authors with the approval of Orr Ross Hamilton, then chief of the section of the Bureau of Internal Revenue engaged in the valuation of metal mines for various tax purposes, but the principal work has been done outside of office hours over a period of several years, the results of that work now being published with the permission of the Commissioner of Internal Revenue, with the understanding that the book shall be confined to the theory of valuation and shall exclude any reference to methods or practices of any office of the Bureau of Internal Revenue. The tables of valuation factors were computed as a regular part of the office work of the Bureau of Internal Revenue as time became available. Engineers to whom credit is due for assistance in computing and checking the tables are Raymond E. Adams, Everett Wilcox, H. D. Lambert, Granville S. Borden, Everett Hurt, and William Huff Wagner.

The valuation of equal annual income is the simplest form of valuation, to which the authors have added but one new method, the three-interest rate, sinking fund method. The development of mathematical methods for valuing any other type of income, regular or irregular, increasing or decreasing,

[1] "Mining and Scientific Press," volume 116, page 691; volume 117, page 276.

[2] "Mineral Valuation," Charles Griffin & Co., Ltd., London, 1923.

subject to tax or tax-free, and according to any mathematically sound premises of valuation, comprises the principal claim of this book for consideration. The various valuation methods are correlated and compared, and it is finally demonstrated that the valuation of any character of income, under any of the mathematical premises, may be accomplished through the use of one simple general formula, which will be found on page 52, as applied to tax-free income, and with modification on page 101 for income subject to tax.

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PRINCIPLES OF VALUATION

Chapter I

INTRODUCTION

The objective of this book is the development and comparison of the several mathematical principles which are adapted to the valuation of future income, and a discussion of these principles with respect to their applicability or inapplicability to commercial usage. A threefold treatment has been chosen: the statement of premises, the conversion of these premises into algebraic formulas, and the arithmetical illustration of the use of such formulas. The only individual factors of valuation which will be discussed are those common to all types of valuation, such as the future rate of interest and the treatment of future capital expenditures necessary to the realization of the anticipated income. It is assumed that the evaluator is competent to make accurate estimates of both the amount of the future income and the time distribution of its receipt, otherwise there is no basis for an appraisal by analytical methods. The authors' purpose is to describe as many as possible of the mathematical tools which may be employed in valuation, leaving the procurement of material with which to work to the evaluator, whose skill and judgment in the selection of material and in the use of the tools will determine the quality of his product.

The accuracy of any valuation will depend partly upon the capability, experience, and judgment of the evaluator and partly upon the tools and material with which he must work. Errors in judgment will inevitably occur in valuation work; but compounding such errors by the use of inapplicable valuation formulas cannot be justified by the excuse so often heard, that, since there is probability of some unavoidable error of uncertain amount, it does not matter if the evaluator knowingly introduces other and avoidable errors.

Valuation may be defined as the act of estimating the worth of, or setting a price upon, anything. Since this is a very comprehensive definition, it is well to state at once that the authors propose to cover only a particular phase of the general subject, that is, the valuation of properties which may be expected to yield future income and which possess worth solely by reason of potential earning power. Such income producing properties are of various kinds: tangible and intangible, lands, natural resources, capital goods which are the products of past industry and useful as aids to future production, franchises, patents, good will, in fact anything which has either a known or a potential future earning capacity. Another element that must be eliminated from a general discussion is that of barter, which necessarily enters into all determinations of price, but which is foreign to estimations of intrinsic value. The price at which an income producing property passes from one ownership to another is determined, not only by its value, but by such intangible elements as the comparative knowledge, bargaining ability, foresight, and need of the buyer and the seller. A careful analysis of these elements is required in the study of comparative sales prices which have obtained for similar properties. The accuracy of a valuation depends largely upon the ability of the evaluator to reach sound conclusions with respect to these intangible elements based upon analysis of the prospective transaction as well as from such studies of completed transactions; but consideration of these important factors finds no place in an exposition of the mathematical principles of valuation.

It must be recognized that many sales and purchases of income producing properties will be consummated without resort to a mathematical analysis of the elements of value. Numerous transactions occur on the basis of comparisons between the property under consideration and other properties of essentially similar nature which have recently been sold at various prices or which may be acquired at known prices. Other transactions involve the necessity of the owner of the property to sell or the extraordinary need of a particular buyer for a specific property. Still others are purchased on speculation at prices known to be exorbitant, with the expectation that buyers can be found at still more exorbitant prices. But the basis underlying all other types of valuation is the

fundamental method requiring a mathematical analysis of the present worth of future expected income, and generally known as the capitalization method. Briefly, this method consists in the determination of a value which will be returned, together with a suitable rate of interest, by the future income which may be expected from the property. Such a method is of general application, while others are of limited local or temporary utility, and it is essential that any sound valuation should consider this basic process as well as derived modes of appraisal.

The universal recognition which has been accorded the capitalization method of appraisal will be apparent from the divers sources of the following quotations from authorities on various phases of the subject of valuation.

"As an investment, land is valued, as is any other form of income producing property, by capitalizing its annual return at the current rate of interest."[1] "The price of land thus varies directly with the amount of its rent and inversely with the rate of interest."[2]

The principles here enunciated so clearly and concisely have long been regarded as axiomatic by leading economists and it would be difficult to find a standard textbook on economics which does not include similar statements.

"The basis of agricultural land values has been established since the time of Ricardo, and throws light on the fundamentals of our problem. Value in urban land, as in agricultural land, is the resultant of economic or ground rent capitalized."[3] This statement by the president of the Lawyers Mortgage Company is based upon a lifetime of experience in the valuation of real estate for a company engaged in the business of placing conservative mortgage loans on such property. Few, if any, authorities on real estate valuation harbor divergent opinions.

Turning from land to natural resources the following expressions of opinion may be cited as representative.

"Various methods of valuation for mining properties have been suggested, but all authorities seem to agree in accepting as the fairest and most practicable and logical, a capitalization

[1] "Principles of Economics," by Henry Rogers Seager, p. 239.
[2] *Ibid.*
[3] "Principles of City Land Values," by Richard M. Hurd, p. 1.

of the estimated earnings, when such estimate can reasonably be made. Hoover and Finlay both emphasize this."[4] "A proper value of a mining property is the present value of the prospective net earnings taking into account probable variations in output and value."[5] These statements are the opinions of experts in the valuation of natural resources of all descriptions—mines, quarries, oil and gas wells, timber, water power, and other natural resources. They are quoted as the well considered and authoritative judgments both of a governmental commission and of a national professional organization which speaks for thousands of the leading mining engineers of the world.

"The main thesis adopted is that the exchange value of all capital goods arises out of earning capacity, and that the value of securities, representing in various ways property rights to capital goods, likewise reflects earning power. In other words, security values arise from capitalizing earnings."[6] This thesis is well supported by the subsequent treatment of the subject as related to different types of the securities of corporations engaged in numerous classes of business with varying degrees of success. From the same book the following additional statements are quoted for emphasis: "It is universally accepted that the value of a piece of land is nothing more nor less than its rental income capitalized at the current rate of interest." "There are . . . strong economic grounds for considering the exchange values of all capital goods as derived from the income such goods are capable of producing . . . " ". . . when the earnings which a given unit of capital is capable of producing fall, the value of the capital also falls." ". . . a going concern . . . is always considered by investors in the light of its capacity to yield income." ". . . the closer we approach the equity represented by common stock, the greater the importance which must be attached to earnings in appraising security values."

The following quotations are from a well-known handbook of the accounting profession.

[4] "Report of the United States Coal Commission," Vol. II, p. 1028.

[5] "Report of the Committee of the American Institute of Mining and Metallurgical Engineers," appointed in 1919 to formulate recommendations with respect to the valuation of mines incident to the application of Federal taxes.

[6] "Valuation of Industrial Securities," by Ralph Eastman Badger, p. 8.

"Rules governing tax valuations depend upon tax laws. Fundamentally, such a valuation must be a valuation of real estate, a capitalization of net earnings which is equivalent to a going concern valuation, or else a franchise valuation made to determine the amount of a franchise tax."[7]

"Franchises are usually valued by the net earnings rule." "Where franchise or going concern value is determined on the basis of earnings, actual cost is necessarily abandoned."[8]

"Sometimes market value is secured by capitalization of net earnings,"[9] in the valuation of public utilities.

It should be noted at this time, however, that the valuation of public utilities is usually for the specific purpose of establishing the rate of earning to be allowed, or the income to which the public utility is fairly entitled, and that such values cannot be determined by appraising income. This type of valuation is practically the only one which is an exception to the fundamental rule that value depends upon earnings, and this exception arises from legislative limitations upon economic law. The market value of a public utility is generally recognized as different from its value for rate making purposes, and is customarily determined by capitalizing estimated future earnings.

An almost unlimited number of opinions might be cited to show the practically universal acceptance of the concept that the power to produce income creates and determines capital value, while opinions to the contrary are notable for their comparative rarity. This theorem has been generally accepted by economists, financiers, engineers, accountants, and other persons engaged in the commercial valuation of divers types of income producing property, and it is likewise recognized as a sound basis of valuation in numerous court decisions and by many executive agencies of the State and Federal Governments.

Accepting the hypothesis that the intrinsic value of any property for investment purchases is dependent upon its capacity to yield income and upon the rate of interest to be expected during the period of investment, it becomes desirable

[7] "The Accountant's Handbook," by E. A. Saliers, pp. 440–442.
[8] *Ibid.*
[9] *Ibid.*

to consider the extent to which mathematical principles may be utilized in commercial practice. They are evidently worthless when applied to properties from which no income may be expected, while maximum application will be found in the appraisal of future income which is assured both as to amounts and times of receipt, a type of income which has been defined as a safe investment. When the amounts and times of receipt of the future income are more or less uncertain or speculative, the capitalization of expected income still finds wide application in commercial practice; but when the future receipt of income is so speculative that its purchase becomes a gamble this method of valuation is no longer applicable. If the right to receive future income is purchased as a gamble, the same principles are applicable as in any other form of gambling and identical terminology should be used. Gambles should be regarded, not as speculative investments to be valued on the basis of interest yields, but as "2 to 1 bets" or "100 to 1 shots" and a different type of valuation is required, based upon investigation of the probability of success which attends the venture. A gamble is frequently of such short duration that the consideration of interest becomes an insignificant factor; but, when the results of the gamble will not be apparent for a period of years, interest becomes an important secondary consideration. The discount for hazard, or factor of safety, method of valuation described in Chapter VI, combines the primary consideration of the risk of loss with the secondary consideration of interest on the sum ventured.

It follows logically that different methods of valuation are applicable, dependent upon whether the earning of interest is the predominant element of investment or the risk of loss is the major consideration. For the valuation of the less speculative types of income by discounting at a rate of interest, there will be developed six methods which are mathematically correct and which are in accord either with existing, or possible, commercial practice, while the factor of safety method, or discount for hazard method, is developed for valuing income subject to great hazard of receipt. Between the exclusive fields of usefulness for these methods, there is a middle ground of speculative investment in which either type of valuation could be used, or where both might be utilized in comparative valuations. A basis for determining factors

of safety equivalent to speculative rates of interest, or vice versa, will be found in Chapter VI.

No definite limits can be set for the distinction between such descriptive terms as "safe investments," "semispeculative investments," "speculative investments," "speculations," and "gambles," but certain general principles apply. If slight changes in economic conditions will greatly affect the income yield or the periods of its receipt, and hence its value, the degree of speculation is greater than is the case when a material change in economic conditions may occur without substantially altering the income which a property will yield. Another general rule which may be stated is that, when the estimates of future income are predicated on past experience over a sufficient period of years, and present and anticipated conditions cannot be differentiated from those of the past, the uncertainty is a minimum and the speculative nature of the investment is relatively small. If the future expectancy is purely an estimate unsupported by prior history of the identical property, but substantiated to some extent by experience with similar ventures in the same vicinity, the speculative element is greater but not prohibitive. This may be regarded as the extreme limit of speculative investment, as the purchase of future income from a source for which no basis of comparison exists must be regarded as a speculation or a gamble rather than as an investment.

Turning now from the principles and theory of valuation to its practice, the authors will develop and present, in a manner which permits ready comparison, the compound interest deferment method, the compound interest actuarial method, and the compound interest sinking fund method, in all of which a single rate of interest is employed in discounting to present worth; the Hoskold and Morkill types of two interest rate sinking fund methods; and one triple interest rate sinking fund method of valuation as developed by the authors. All of these methods are expressed in algebraic formulas suitable to the valuation of income having any time distribution, that is, equal annual income, income increasing or decreasing by equal periodic amounts or percentages, deferred income, income changing every year, and other types. In fact, with slight adaptation, the formulas presented are suited to the valuation of any type and time distribution of income. The formulas

are also solved for other unknowns, such as years of duration of income necessary to obtain a stated value, and, in the case of the Hoskold type of sinking fund formulas, the rate of interest which stipulated income will yield on a fixed value. Other features, such as the effects of income taxes, are fully covered. The evaluator is thus presented with a number of alternative methods for valuing future income. The advantages of each method are indicated in the chapter devoted to the presentation of the authors' conclusions; arithmetical illustrations of the applications of all methods are included, and tables of valuation factors have been prepared to facilitate the use of any method to the greatest possible extent; but the selection of a method of valuation will depend upon the nature of the income valued and upon the financial policy of the investor, and is a matter which must be left to the discretion of the evaluator.

Chapter II

PREMISES AND MATHEMATICS OF VALUATION

Income received at equal intervals of time, either for terminable or perpetual periods, either in regular or irregular amounts, and either of a safe or a speculative nature may be valued according to any known mathematical premises by means of a single and rather simple algebraic series and formula hereafter developed, or a derivation of this formula may be obtained for the purpose of valuing any specific type of income according to a stated premise. It is also possible to develop formulas, under each of the several premises of valuation, which show the yearly allocation of income to the payment of interest on the determined value, and the return of that value to the investor within the period of income receipt. Income is defined as that which comes in, which includes both interest on capital value and the return of that value. The time distribution of income refers to the manner of its receipt, such as equal annual amounts, regularly varying annual amounts, or irregular annual amounts. The several premises will be stated from the viewpoint of an investor purchasing future income to yield a specified rate of interest and at the same time to return his investment intact within a finite term of years.

The common procedure which will be followed according to each premise of valuation is the development of an algebraic series with an algebraic expression for each year, showing either the amount of capital return or the amount added to the sinking fund for the year, although, as an alternative with sinking fund premises, the series may show the amount of each year's contribution to the sinking fund, together with its interest accretions to the end of the investment period. The total capital returned during the term of investment or the total amount in the sinking fund and available for the return of capital at the end of the investment period must, under any of the several premises, equal the capital invested, which is

the value at date of valuation. This method of presentation has been found to be the least involved of any of those considered and to enable comparisons between premises to be made with the greatest facility.

All valuation and other formulas are developed according to the conception that the receipt of income will terminate after a finite period of years, the valuation of perpetual income being regarded as a special phase of the valuation of terminable income. The formulas for the valuation of perpetuities may be derived from those for the valuation of terminable income by assuming the years' period of investment denoted by the symbol n to be infinite. Perpetuity formulas may be developed for types of income which increase or decrease regularly, either by equal periodic amounts or by percentages; but irregular income cannot be so valued. The perpetuity formulas will be stated but not derived, whenever the type of income is susceptible to valuation in perpetuity. The formulas for both terminable and perpetual income are all developed for the valuation of speculative income by means of speculative rates of interest, and formulas under all premises reduce to identity when safe (or sinking fund) rates of interest are substituted for speculative rates, or vice versa, provided the time distribution of the income is the same. The same statement may be made as to allocation formulas for all sinking fund premises of valuation.

Both the safe and the speculative rates of interest are assumed to remain constant throughout the investment period, although it would be possible to develop formulas that provide for periodic changes in the rate of interest or a different rate of interest for each year of the investment period. For reasons stated in Chapter VII, it is believed that the valuation of future income with the use of constant rates of interest is a sounder basis.

The receipt of income is assumed to take place once a year, at the end of the year, the initial amount becoming available at the end of the first year of income receipt—that is, at the end of the first year after the period of deferment, when deferment is involved, and otherwise at the end of the first year of the period of investment. Interest payments to the investor, and the increase or withdrawal of capital or contribution to the sinking fund for the retirement of capital, as the

case may be, take place at the end of each year of the period of investment, including any deferment period. Interest earned by a sinking fund during any year is credited to the fund at the end of that year.

Interest compounded at intervals other than yearly intervals.—No attempt is made to present valuation formulas involving the valuation of income received and distributed more or less frequently than at yearly intervals, for several reasons, but principally because the valuation of income in semiannual, quarterly, or monthly intervals, according to the formulas customarily employed, means that interest is compounded at the same intervals. While very low rates of interest are sometimes compounded semiannually or even quarterly, speculative rates of interest are rarely, if ever, compounded more frequently than at annual intervals. When payments on both interest and principal are made periodically during the year and the capital upon which interest must be paid at a stipulated annual rate is reduced with each payment according to the compound interest actuarial premises of valuation, the use of valuation formulas for income received a number of times a year is proper; but otherwise there is little application for fractional year valuation formulas in commercial practice. Installment sales of real estate, or similar transactions involving periodic payments during each year of a term of years, are the exceptions. It is also commercial practice to demand the payment of interest annually, and it is improbable that many financial transactions forego the annual compounding of interest in favor of longer periods. It is likewise a common business custom to close the books once a year rather than more or less frequently, and financial statements, such as balance sheets and profit and loss accounts, are usually prepared at the time of closing the books. However, the formulas developed for the valuation of income received annually, and with interest compounded annually, may be readily adapted to the valuation of income received at more or less frequent intervals, with interest compounded at the intervals of income receipt, by the following very simple rules:

1. The yearly interest rates are multiplied by the fraction or multiple of a year at which the premises require the compounding of interest.

2. The yearly time intervals are divided by the fraction or multiple of a year at which the premises require the compounding of interest.

In other words:

1. The rates of interest per year as indicated by the symbols, r, r_1, e, or j, in the valuation formulas, must be multiplied by the number of years between the dates at which interest is compounded, in order to find the effective rate of interest to be substituted in the valuation formula. Thus, if 10% is the yearly rate of interest in a single rate of interest formula for valuation, and interest is compounded biennially, the effective rate of interest is $2 \times 10\%$, or 20%. If interest is compounded semiannually the effective rate of interest is $\frac{1}{2}$ of 10%, or 5%. If more than one rate of interest appears in the valuation formula, each rate must be similarly adjusted.

2. The symbols representing periods of years in the notation are n, d, m, and k. For the number of years represented by each of these symbols appearing in the valuation formula, there must be substituted the number of times that interest is compounded during the corresponding period. Thus, if interest is compounded semiannually, the numerical quantity to be substituted for any symbol indicating a period of years is twice the number of years in that period. If interest is compounded biennially the numerical quantity to be substituted is one half of the period of years indicated by the algebraic symbol.

Combinations of full years and fractional years.—In cases compounding at yearly intervals, where the period of investment is a whole number of years plus a fractional year, no mathematical law can be asserted for the application of interest which is compounded annually, since the matter becomes a question of the definition of compounding as applied to the fractional year. Four approximate methods, each giving a slightly different result, any one of which may be used at the discretion of the evaluator, are available. Descriptions of these methods follow:

1. The fractional year may be introduced into the valuation formula directly by substitution in the notation. Thus, if the investment period is $10\frac{3}{4}$ years, $10\frac{3}{4}$ would be sub-

stituted for n. This method involves the computation of discount factors by means of logarithmic tables, as the valuation tables of Appendix B are limited to compounding at yearly intervals, and as the time intervals invariably appear as exponents in valuation formulas.

2. The duration of the investment period may be changed to the nearest full year, or, preferably, to the next greatest full year. This method is recommended for valuation when the duration of the investment period is an estimate.

3. The fractional year may be regarded as entering the computation either at the beginning or at the end of the period of investment. The value of the income for the period of complete years may be determined, that value being further discounted at compound interest for the fractional year, and then added to the dicounted value of the income for such fractional year. This method is chiefly applicable to premises of valuation using a single rate of interest, as complications enter when the premises require double and triple rates of interest.

4. The safe interest value may be determined according to the method first described, which involves the use of fractional exponents, and that value further reduced by a discount for hazard equivalent to the selected speculative rate of interest for the full years plus the fractional year. The complement of the discount for hazard, which is called the "factor of safety," may be determined by interpolation in the factor of safety tables of Appendix B, or it may be read directly from the factor of safety curves of Chapter VI.

Positive and negative amounts.—The total anticipated income from an investment must be positive in amount, as otherwise it has no value; but there may be years of predicted negative income, or losses, and other years in which the amount of income is expected to be insufficient for the payment of interest to the investor. Throughout this work positive and negative amounts of income are accorded identical treatment, according to the algebraic rules that the addition of a negative amount is equivalent to the subtraction of a positive amount, and the subtraction of a negative amount is equivalent to the addition of a positive amount. Capital additions after date of investment are treated as negative income. It is, therefore, necessary to regard the minus signs

in the arithmetical tabulations, as with few exceptions negative income, or a deficiency of income below the amount required for the payment of interest to the investor, results in an increase of the capital in the speculative investment either by direct addition to capital or by decrease of the amount of capital retired to the sinking fund.

These general premises should be borne in mind in following the statements of the specific premises of each method of valuation.

The development of formulas for valuing irregular terminable income will now be presented, as these are general formulas of which those for valuing income of a periodic nature as to time distribution are but special cases. As the development of these general formulas proceeds in accordance with the stated premises it will be noted that the same valuation formula is derived with four of the seven mathematical premises. The difference between these premises appears in the formulas for allocating the income for any year to the payment of interest and the return of capital, which are stated following the valuation formulas. The mathematical derivations of these allocation formulas will be omitted, as anyone interested may evolve them by the simple process of determining for any year (y), either the interest payment (Iy) or the capital return payment (Cy) from the series by which the valuation formula was developed; then, subtracting whichever is the easiest to determine from (Ay), the income for any year (y); and, finally, by substituting its previously determined value in other terms for V and summing, solving, and simplifying the resultant equation. The allocation formulas for all sinking fund premises of valuation may be derived from the allocation formulas of Division VI, but the allocation formulas for premises I and II must be independently derived.

In the development of the general valuation formulas of this chapter, one or more of the following elementary algebraic processes are repeatedly used in summing the terms of series and in solving equations for the value (V).

Let S equal the sum of a series of terms, let R equal $1 + r$, and let

$$S = A + AR + AR^2 + AR^3 + AR^4 + \cdots AR^{n-1}$$

then

$$RS = AR + AR^2 + AR^3 + AR^4 + \cdots AR^{n-1} + AR^n$$

and

$$RS - S = AR^n - A$$

or

$$S = \frac{A(R^n - 1)}{R - 1} = \frac{A(R^n - 1)}{r} \qquad \text{(Equation 1)}$$

Let

$$X = V + Vr\frac{R^n - 1}{r}$$

then

$$X = VR^n$$

or

$$V = \frac{X}{R^n} \qquad \text{(Equation 2)}$$

Let

$$\frac{X}{e} = V + Vr\frac{E^n - 1}{e}$$

then

$$V = \frac{X}{rE^n - r + e} \qquad \text{(Equation 3)}$$

Those shown above and similar processes of summation of series or solution of equations will not be repeated in the following text.

With the preceding general premises and equations in mind, the development of formulas according to specific premises of valuation will follow in regular order, the first of which is to find the amount of V in n years at a rate of interest (r) per year, compounded annually, where $R = 1 + r$.

Amount, beginning of first year = V
Interest added during first year = Vr
Amount, beginning of second year = $V + Vr = VR$
Interest added during second year = VRr
Amount, beginning of third year = $VR + VRr = VR^2$
Interest added during third year = VR^2r
Amount, beginning of fourth year = $VR^2 + VR^2r = VR^3$

Similarly, if the same process were continued for n years, there would be obtained:

Amount, beginning of nth year = VR^{n-1}
Interest added during nth year = $VR^{n-1}r$
Amount at end of n years = $VR^{n-1} + VR^{n-1}r = VR^n$

The amount of V in n years with interest at the rate r, compounded annually, is, therefore, VR^n.

Let

$$A = VR^n$$

then

$$V = \frac{A}{R^n}$$

that is, the value (V) of A due n years hence is $\frac{A}{R^n}$, when interest is to be paid at the rate r, compounded annually.

Let $A = 1$, and it becomes apparent that the value of 1 due n years hence, with interest at the rate r, compounded annually, is $\frac{1}{R^n}$.

Let $V = 1$, and it is evident that the amount of 1 in n years, with interest at the rate r, compounded annually, is $R^n = (1 + r)^n$.

The relations just stated are the basis of the entire structure for the valuation of income by mathematical premises.

Algebraic Notation and Numerical Quantities Used in Derivation of Formulas and Substituted in Arithmetical Solutions of This Chapter.

In general the algebraic formulas about to be derived hold true for substitutions of any numerical values, positive, negative, or zero, except where otherwise specified. The numerical quantities stated are those selected by the authors for purposes of illustration.

A_1, A_2, A_3, A_4, . . . A_y . . . A_n = variable annual amounts of income for each of the years indicated by the subscripts to the letter A. In the arithmetical solutions, $A_1 = 1{,}000$; $A_2 = 1{,}200$; $A_3 = 500$; $A_4 = 800$; $A_5 = 1{,}600$; $A_6 = 1{,}300$; $A_7 = 900$; $A_8 = 600$; $A_9 = 700$; and $A_{10} = 1{,}400$.

$r = 0.10$ = the speculative rate of interest to be earned by the investment. The decimal 0.10 is equivalent to 10%.

$R = 1 + r = 1.10$.

$e = 0.04$ = the safe rate of interest to be earned by the sinking fund, and, under the premises of Division V, to be paid to the investor upon the amount accumulated in the sinking fund. This notation is used in Divisions IV, V, and VI.

$E = 1 + e = 1.04$.

$r_1 = 0.08$ = the semispeculative rate of interest to be paid to the investor upon the amount accumulated in the sinking fund. This notation is used only in Division VI.

$j = r - r_1 + e = 0.06$. This notation is used only in Division VI.

$J = 1 + j = 1.06$. This notation is used only in Division VI.

$n = 10$ = the life in years, or the total period from date of investment to the end of the period when all transactions, involving the payment of interest and the retirement of capital, are closed.

y = any year within the term of the investment. The seventh year has been selected for the purposes of arithmetical solutions of algebraic income allocation formulas.

V = the value or present worth of the total income to be determined as at date of valuation or investment.

C_y = the amount of the income allocated to the retirement of capital in any year (y) of the period of investment. This is an amount determined by solution of the allocation formulas for C_y.

I_y = the amount of income allocated to the payment of interest to the investor in any year (y) of the period of investment. This is also an amount to be determined by solution of the allocation formulas for I_y.

A_y = the amount of income for any year (y) of the period of investment. A_y is always equal to $C_y + I_y$, and, since the seventh year has been selected for purposes of illustrations of arithmetical solutions of income allocation formulas, then $A_y = 900$ in such illustrations.

$S_1, S_2, S_3, S_4, \ldots S_n$ = symbols of convenience used in the derivation of valuation formulas. These symbols have slightly different meanings under the differing premises of the several divisions, as follows:

For Division II they represent the amounts remaining each year (indicated by the subscripts to the letter S) after interest paid to the investor on capital outstanding at the beginning of the year has been deducted from the income for that year.

For all divisions under sinking fund premises they represent the contribution to the sinking fund from income in the years indicated by the subscripts plus the interest earned during that year by the total amount in the sinking fund at the beginning of the year.

THE SIX MATHEMATICAL PREMISES OF VALUATION

The premises are given six division numbers, which have a relation to the number of interest rates used in valuation. Under the premises of Divisions I, II, and III, one interest rate only is involved; under the premises of Divisions IV and V two interest rates are required, and under the premises of Division VI three interest rates are necessary.

The premises of Divisions III, IV, V, and VI are all known as sinking fund premises, and the formulas of Divisions III, IV, and V may be derived directly from the formula of Division VI by assuming various equalities among the interest rates. Thus, when $r_1 = r$, the formula of Division VI reduces to that of Division IV; when $r_1 = e$ the formula of Division VI reduces to that of Division V; and when r_1 and e are both made equal to r the formula of Division VI becomes that of Division III. Still another variation is possible by assuming that $r = e$ or $e = r$ without changing r_1, but such a premise of valuation would be commercially absurd, since it presumes that, either an investor would be satisfied with a lower rate of interest on capital outstanding in the speculative enterprise than upon capital accumulated in the sinking fund, or he would sanction the speculative investment of a sinking fund and be satisfied with less than a speculative return on such investment. For these reasons this last mathematically possible premise of valuation is given no further consideration. The general formula of Division VI is an excellent indicator of the source of error in erroneous valuation formulas, as such errors very often involve the fallacy of one or even two zero rates of interest.

The method of presentation followed under each premise of valuation will be:

1. A statement of the premises of valuation.
2. The derivation of a formula for valuing irregular annual income.
3. The statement of the arithmetical solution of the valuation formula, when the numerical quantities indicated in the notation are substituted.
4. The statement of the two income allocation formulas and their arithmetical solutions, when the numerical quantities indicated in the notation are substituted.

5. The tabular arithmetical demonstration of the yearly allocation of income to payment of interest and the retirement of capital for the entire period of investment.

It should be noted that all formulas given in this chapter are general formulas applicable to any type of income distribution to be valued according to the stated premises.

It will also be noted that for all derivations of general formulas in this chapter (except the special formulas under Division IV for sinking fund deficiencies, where only a portion of the series is shown) a year by year series is employed, which shows the disposition each year of the income for that year in such a manner that the sum of all the terms of the series is always equal to the present worth value sought, or the value (V).

DIVISION I

COMPOUND INTEREST DEFERMENT METHOD OF VALUATION

Premises.—The investor acts as banker for the enterprise in which he is placing his capital, furnishing any additional capital required, and having capital returned to him as rapidly as the receipt of income from the investment will permit. As used in this text the term "banker" means one advancing necessary additional amounts for the improvement of the property or the payment of interest to the investor, when income for any year is deficient for those purposes, and also the one to whom yearly payments toward the retirement of the investment are intrusted.

Each year's income is regarded as a separate financial transaction and is valued separately, the algebraic sum of the yearly values comprising the total value of the income for the life of the investment.

Each year's income consists in part of a return of capital and in part of a payment of interest. The allocation of the income for any year to capital and interest accounts is independent of a similar allocation for any other year, and the amounts allocated to these accounts are independent both of the time distribution of the income for other years and of the total value of the income for the term of the investment.

The income for any year comprises a return of the investment value determined for that year's income at date of

valuation (the term "date of valuation" in this text meaning the date of investment or the beginning of the period of investment) plus interest on that value compounded annually at the stipulated rate from the date of valuation to the date at which the income is received.

The initial capital is reduced in each succeeding year by progressively smaller percentages of the yearly receipts of income; and interest is paid to the investor, not upon the total outstanding capital in any year, but upon the amount of the capital returned to the investor during the year.

The sum of the amounts of capital returned yearly during the life of the investment will equal the value determined for the total income for the term of the investment. If a loss or negative amount of income is anticipated for any year, the value of that loss at the date of investment has been deducted from the value of the profits or positive income, also as at the date of valuation. The amount of such deduction from the investment value at the date of valuation plus interest compounded annually at the stipulated rate to the end of the year of negative income equals the amount of negative income for that year.

The capital returned in any year is paid to the investor. Capital added after the initial investment is treated in the same manner as the initial investment.

Valuation and Allocation Formulas of Division I.

It has been shown that the present value (V) of A due n years hence at the interest rate (r), compounded annually, is $\frac{A}{R^n}$. According to the same law, the present worth value of each year's income of the irregular annual income (A_1, A_2, A_3, A_4, . . . A_n) will be as shown in the following series:

1st year: $\frac{A_1}{R}$

2nd year: $\frac{A_2}{R^2}$

3rd year: $\frac{A_3}{R^3}$

4th year: $\frac{A_4}{R^4}$

nth year: $\frac{A_n}{R^n}$

The sum of these separate present worth values is equal to the present worth (V) of the total income, therefore:

$$V = \frac{A_1}{R} + \frac{A_2}{R^2} + \frac{A_3}{R^3} + \frac{A_4}{R^4} + \cdots \frac{A_n}{R^n}$$

The above will be referred to as the final form of this general formula, but it may be written in the form:

$$V = \frac{1}{R^n}(A_1R^{n-1} + A_2R^{n-2} + A_3R^{n-3} + A_4R^{n-4} + \cdots A_n)$$

Arithmetical solution: . . . $V = 6{,}128.561$

$$C_y = \frac{A_y}{R^y}$$

Arithmetical solution: . . . $C_y = 461.842$ (see Column G of the following tabulation for the seventh year).

$$I_y = A_y\frac{R^y - 1}{R^y}$$

Arithmetical solution: . . . $I_y = 438.158$ (see Column H of the following tabulation for the seventh year).

INCOME ALLOCATION TABULATION, PREMISES I

Year	Capital Account			Income Allocation				Interest rate paid on outstanding capital: per cent
	Capital outstanding at beginning of year	Capital returned each year	Capital outstanding at end of year	Yearly income	Discount factor	Yearly capital return	Yearly interest paid to investor	
Column.....	A	B	C	D	F	G	H	I
Process.....	.....	B = G	A − B	Per notation	Table 2 App. B	D × F	D − G	H ÷ A
1	6,128.56	909.09	5,219.47	1,000.00	0.909091	909.09	90.91	1.483
2	5,219.47	991.74	4,227.73	1,200.00	0.826446	991.74	208.26	3.990
3	4,227.73	375.66	3,852.07	500.00	0.751315	375.66	124.34	2.941
4	3,852.07	546.41	3,305.66	800.00	0.683013	546.41	253.59	6.583
5	3,305.66	993.47	2,312.19	1,600.00	0.620921	993.47	606.53	18.348
6	2,312.19	733.82	1,578.37	1,300.00	0.564474	733.82	566.18	24.487
7	1,578.37	461.84	1,116.53	900.00	0.513158	461.84	438.16	27.760
8	1,116.53	279.90	836.63	600.00	0.466507	279.90	320.10	28.669
9	836.63	296.87	539.76	700.00	0.424098	296.87	403.13	48.185
10	539.76	539.76	000.00	1,400.00	0.385543	539.76	860.24	159.374
Totals.....	29,116.97	6,128.56	.....	10,000.00	.....	6,128.56	3,871.44	*13.296

* Average.

DIVISION II

COMPOUND INTEREST ACTUARIAL METHOD OF VALUATION

Premises.—The investor acts as banker for the enterprise in which he is placing his capital, furnishing any additional capital required and having capital returned to him as rapidly as the receipt of income from the investment will permit.

The income for the term of the investment is valued as a whole. The first charge against the income for any year is interest at the stipulated rate upon the total capital outstanding at the beginning of the year. That portion of the income for any year which remains after provision is made for the payment of interest is deducted from the capital outstanding at the end of the year. If the income for any year is insufficient to provide for the payment of interest, a negative amount of income remains for deduction from capital outstanding at the beginning of the year, and, as the deduction of a negative quantity is equivalent to an addition of a positive quantity, it follows that, in any year when the income is insufficient to pay interest on outstanding capital, the deficiency of interest is added to the amount of capital outstanding at the beginning of the year. If there are no interest deficiencies, the arithmetical illustrations will afford no indication that compound interest is involved, as simple interest is paid yearly on outstanding capital, when the income for each and every year is in excess of the interest requirement.

The sum of the amounts of capital returned in years when income is in excess of the charges for interest will equal the initial value of the investment plus all additions to capital during the term of the investment, including the addition of any unpaid interest.

The allocation of the income for any year depends upon the total value of the income for all years, as well as upon the time distribution over the life of the investment of the income which is valued.

The capital returned in any year is paid to the investor and capital added in any year is credited to the amount of the outstanding investment, both at the end of the year.

Valuation and Allocation Formulas of Division II.

The amounts available each year for the retirement of capital, after interest paid to the investor on the diminished capital is taken from the yearly income, may be algebraically expressed as shown in the following series:

1st year: $A_1 - Vr = S_1$
2nd year: $A_2 - (V - S_1)r = S_2$
3rd year: $A_3 - (V - [S_1 + S_2])r = S_3$
4th year: $A_4 - (V - [S_1 + S_2 + S_3])r = S_4$
nth year: $A_n - (V - [S_1 + S_2 + S_3 + \cdots S_{n-1}])r = S_n$

From an inspection of the above series, the following relations may be written at once:

$$S_1 = S_1$$
$$S_1 + S_2 = S_1R + A_2 - Vr$$
$$S_1 + S_2 + S_3 = (S_1 + S_2)R + A_3 - Vr$$
$$S_1 + S_2 + S_3 + S_4 = (S_1 + S_2 + S_3)R + A_4 - Vr$$
$$S_1 + S_2 + S_3 + S_4 + \cdots S_n = (S_1 + S_2 + S_3 + S_4 + \cdots S_{n-1})R + A_n - Vr$$

From the above it follows that:

$$S_1 = A_1 - Vr$$
$$S_1 + S_2 = (A_1 - Vr)R + (A_2 - Vr)$$
$$S_1 + S_2 + S_3 = (A_1 - Vr)R^2 + (A_2 - Vr)R + (A_3 - Vr)$$
$$S_1 + S_2 + S_3 + S_4 = (A_1 - Vr)R^3 + (A_2 - Vr)R^2 + (A_3 - Vr)R + (A_4 - Vr)$$
$$S_1 + S_2 + S_3 + S_4 + S_5 = (A_1 - Vr)R^4 + (A_2 - Vr)R^3 + (A_3 - Vr)R^2 + (A_4 - Vr)R + (A_5 - Vr)$$
$$S_1 + S_2 + S_3 + \cdots S_n = (A_1 - Vr)R^{n-1} + (A_2 - Vr)R^{n-2} + (A_3 - Vr)R^{n-3} + \cdots (A_n - Vr)$$

Since, under the premises, $S_1 + S_2 + S_3 + S_4 + \cdots S_n$ is equal to V, then,

$$V = (A_1 - Vr)R^{n-1} + (A_2 - Vr)R^{n-2} + (A_3 - Vr)R^{n-3} + \cdots (A_n - Vr)$$

or

$$V = A_1R^{n-1} + A_2R^{n-2} + A_3R^{n-3} + \cdots A_n - Vr(R^{n-1} + R^{n-2} + R^{n-3} + \cdots 1)$$

Solve the above for V, after summing the series within the brackets of the Vr term, in the manner already indicated, and obtain,

$$V = \frac{1}{R^n}(A_1R^{n-1} + A_2R^{n-2} + A_3R^{n-3} + A_4R^{n-4} + \cdots A_n)$$

or, finally,

$$V = \frac{A_1}{R} + \frac{A_2}{R^2} + \frac{A_3}{R^3} + \frac{A_4}{R^4} + \cdots \frac{A_n}{R^n}$$

It will be noted that the above formula is identical with that shown under Division I.

Arithmetical solution: . . . $V = 6{,}128.561$

$$C_y = A_y - r\left(\frac{A_y}{R} + \frac{A_{y+1}}{R^2} + \frac{A_{y+2}}{R^3} + \frac{A_{y+3}}{R^4} + \cdots \frac{A_n}{R^{n-y+1}}\right)$$

Arithmetical solution: . . . $C_y = 620.381$ (see Column G of the following tabulation for the seventh year).

$$I_y = r\left(\frac{A_y}{R} + \frac{A_{y+1}}{R^2} + \frac{A_{y+2}}{R^3} + \frac{A_{y+3}}{R^4} + \cdots \frac{A_n}{R^{n-y+1}}\right)$$

Arithmetical solution: . . . $I_y = 279.619$ (see Column F of the following tabulation for the seventh year).

INCOME ALLOCATION TABULATION, PREMISES II

Year	Capital Account			Income Allocation			Interest rate paid on outstanding capital: per cent
	Capital outstanding at beginning of year	Capital returned each year	Capital outstanding at end of year	Yearly income	Yearly interest due investor	Yearly capital return	
Column.....	A	B	C	D	F	G	H
Process.....	.....	B = G	A − B	Per notation	10% of A	D − F	F ÷ A
1	6,128.56	387.14	5,741.42	1,000.00	612.86	387.14	10
2	5,741.42	625.86	5,115.56	1,200.00	574.14	625.86	10
3	5,115.56	−11.56	5,127.12	500.00	511.56	−11.56	10
4	5,127.12	287.29	4,839.83	800.00	512.71	287.29	10
5	4,839.83	1,116.02	3,723.81	1,600.00	483.98	1,116.02	10
6	3,723.81	927.62	2,796.19	1,300.00	372.38	927.62	10
7	2,796.19	620.38	2,175.81	900.00	279.62	620.38	10
8	2,175.81	382.42	1,793.39	600.00	217.58	382.42	10
9	1,793.39	520.66	1,272.73	700.00	179.34	520.66	10
10	1,272.73	1,272.73	000.00	1,400.00	127.27	1,272.73	10
Totals.....	38,714.42	6,128.56	.....	10,000.00	3,871.44	6,128.56	*10

* Average.

Sinking Fund Valuation Methods.

(Covers Divisions III, IV, V, and VI)

Premises common to all sinking fund methods of valuation.—This general statement of the premises common to every sinking fund method of valuation will be followed by individual statements of the premises peculiar to each of the four types.

The distinguishing feature of all sinking fund valuation methods following unmixed premises is that no capital may be returned to the investor during the period of investment. A sinking fund is created to act as banker for the enterprise in which capital is placed; the investor makes payment to the vendor for the estimated value of the expected future income at date of investment, and this value is returned by annual payments to a sinking fund, which, with interest accumulated at the sinking fund rate, will also provide for all future estimated capital requirements at estimated dates of expenditure.

The sinking fund is the agency which attends to both the investment of the necessary additional capital in the enterprise, and the withdrawal of capital from the enterprise as rapidly as the receipt of income will permit. The investor receives interest at the stipulated rate or rates upon his initial investment during the entire period of investment and a return of that investment at the termination of the investment period from amounts accumulated in the sinking fund. The income for the entire duration of the term of investment is valued as a whole; but each year's income may be valued separately, if provision is made to pay the risk rate of interest for the entire period of the investment upon the value so determined. The failure of numerous algebraic sinking fund valuation formulas to stand the acid test of arithmetical solution, in the valuation of income which changes in yearly amounts during the term of the investment, may be traced to the false premise that a speculative rate of interest need be paid on the value of some of the income for but a portion of the investment period. The first charge against the income for each year consists of a deduction for interest due to the investor upon his unchanged initial investment. The difference between the amount of income received in any year

and the amount of interest due to the investor in that year is contributed to the sinking fund. These contributions may be positive, negative, or zero in amount; indeed, so may income. This is a most important item to bear in mind, for the reason that one of the principal mathematical errors of the past in the development and application of sinking fund methods of valuation has arisen from the failure to treat a deficiency of income below the amount required for the payment of interest to the investor in the same manner as a surplus of income above the amount of interest due to the investor. A deficiency was customarily added to outstanding capital while a surplus was contributed to the sinking fund. The only proper and possible mathematical treatment is to add to the sinking fund according to common algebraic procedure, the difference between the income received and the interest charge during each year, whether such amount is a surplus or a deficiency.

The sinking fund is invested to earn interest compounded annually, and the algebraic sum of the amounts contributed to the sinking fund, including any contribution by the investor at the date of investment, together with the algebraic sum of the interest earned by these amounts, must equal the initial investment at the termination of the period of investment. It is immaterial whether each contribution to the sinking fund is considered to earn interest compounded annually from the date of its entry into the sinking fund to the termination of the period of investment and the algebraic sum of each of the contributions and its interest accretion is determined at the latter date, or whether the sinking fund as a whole is considered to increase or to decrease during each year of the life of the investment by the algebraic sum of the contribution to the sinking fund for the particular year plus the interest earned by the entire amount in the sinking fund at the beginning of that year. The final result is the same with either method of consideration and both are used in this book.

To summarize.—The initial investment includes the amount paid to the vendor plus a possible payment to the sinking fund to provide for any anticipated additional capital expenditures; interest is paid to the investor yearly on his undiminished initial investment for the entire term for which it is placed; return of capital at the end of the investment

period is assured partially by contributions to the sinking fund from yearly income and partially by the interest earned by the investment of such contributions while in the sinking fund. When the income for any year is insufficient to pay interest due to the investor, such deficiency in interest is paid from the sinking fund to the investor; or, if the amount in the sinking fund is insufficient to make this interest payment when due, the interest is paid to the investor as soon as contributions to the sinking fund become available for that purpose, and interest upon unpaid interest due the investor is paid at rates stipulated in the premises of valuation. Additions to capital from income are treated as negative income and are cleared through the sinking fund in the same manner as that just described for interest, in the event that income for the year of capital addition is not adequate to provide for such addition, together with interest due the investor.

DIVISION III

COMPOUND INTEREST SINKING FUND METHOD OF VALUATION

Premises.—The distinguishing feature of this sinking fund valuation method is that the rate of interest earned by the sinking fund is the same as the rate of interest paid to the investor. As the initial investment is undisturbed for the term of the investment, it follows that the yearly amount of interest paid to the investor is constant.

The assumption that the interest rate earned by the investment and the interest rate earned by the sinking fund are identical is reasonable only in the event that this rate is one which may be expected from investments of a safe type. The purpose of a sinking fund is to assure the return of capital; the return of capital cannot be assured, when there is risk of the loss of either principal or interest from the sinking fund; and higher speculative rates of interest may not be expected from investments characterized by a minimum possibility of loss.

Valuation and Allocation Formulas of Division III.

The following valuation formula for Division III premises is derived from the valuation formula of Division VI, which

is hereinafter evolved, by making both r_1 and e equal to r, whereupon $j = r$ and $J = R$, and

$$V = \frac{A_1R^{n-1} + A_2R^{n-2} + A_3R^{n-3} + A_4R^{n-4} + \cdots A_n}{R^n}$$

or, finally,

$$V = \frac{A_1}{R} + \frac{A_2}{R^2} + \frac{A_3}{R^3} + \frac{A_4}{R^4} + \cdots \frac{A_n}{R^n}$$

It will be noted that the above formula is identical with those derived under Divisions I and II

Arithmetical solution: . . . $V = 6{,}128.561$

$$C_y = A_y - r\left(\frac{A_1}{R} + \frac{A_2}{R^2} + \frac{A_3}{R^3} + \cdots \frac{A_y}{R^y} + \cdots \frac{A_n}{R^n}\right)$$

Arithmetical solution: . . . $C_y = 287.144$ (see Column F of the following tabulation for the seventh year).

$$I_y = r\left(\frac{A_1}{R} + \frac{A_2}{R^2} + \frac{A_3}{R^3} + \cdots \frac{A_y}{R^y} + \cdots \frac{A_n}{R^n}\right)$$

Arithmetical solution: . . . $I_y = 612.856$ (see Column E of the following tabulation for the seventh year).

The valuation formula just derived under this division by substitutions in the formula for Division VI may be derived, independently of the latter formula, by forming a year by year series, where the yearly contribution to the sinking fund (the yearly income less interest due to the investor at the rate r) is increased by compound interest at the same rate r from the year contributed to the end of the investment period. Such a series will appear as follows:

1st year: $(A_1 - Vr)R^{n-1}$
2nd year: $(A_2 - Vr)R^{n-2}$
3rd year: $(A_3 - Vr)R^{n-3}$
nth year: $(A_n - Vr)R^o = A_n - Vr$

Under the premises of this division the sum of the terms of the above series must be equal to V, whereupon,

$$V = \frac{A_1R^{n-1} + A_2R^{n-2} + A_3R^{n-3} + A_4R^{n-4} + \cdots A_n}{R^n}$$

as shown before.

INCOME ALLOCATION TABULATION, PREMISES III

Year	Capital Account at Beginning of Year			Income Allocation at End of Year			Sinking Fund Account during Year			Interest rate paid on outstanding capital: per cent
	Total capital	Portion of capital in sinking fund	Portion of capital in speculative enterprise	Income	Interest due investor	Contributions to sinking fund	Amount in fund at beginning of year	Interest earned during year	Amount in fund at end of year	
Column	A	B	C	D	E	F	G	I	J	K
Process	From valuation formula	B = G	A − B	Per notation	10% of A	D − E	From J preceding year	10% of G	F + G + I	E ÷ A
1	6,128.56	000.00	6,128.56	1,000.00	612.86	387.14	000.00	000.00	387.14	10
2	6,128.56	387.14	5,741.42	1,200.00	612.85	587.15	387.14	38.71	1,013.00	10
3	6,128.56	1,013.00	5,115.56	500.00	612.85	−112.85	1,013.00	101.30	1,001.45	10
4	6,128.56	1,001.45	5,127.11	800.00	612.86	187.14	1,001.45	100.14	1,288.73	10
5	6,128.56	1,288.73	4,839.83	1,600.00	612.85	987.15	1,288.73	128.87	2,404.75	10
6	6,128.56	2,404.75	3,723.81	1,300.00	612.86	687.14	2,404.75	240.48	3,332.37	10
7	6,128.56	3,332.37	2,796.19	900.00	612.86	287.14	3,332.37	333.24	3,952.75	10
8	6,128.56	3,952.75	2,175.81	600.00	612.86	−12.86	3,952.75	395.28	4,335.17	10
9	6,128.56	4,335.17	1,793.39	700.00	612.86	87.14	4,335.17	433.52	4,855.83	10
10	6,128.56	4,855.83	1,272.73	1,400.00	612.85	787.15	4,855.83	485.58	6,128.56	10
Totals	61,285.60	.....	.....	10,000.00	6,128.56	3,871.44	.....	2,257.12	.....	*10

* Average.

DIVISION IV

SINKING FUND VALUATION METHOD WITH TWO RATES OF INTEREST (Hoskold Type)

Premises.—In this method of valuation the initial investment earns interest for the entire term of the investment at a speculative rate, while the sinking fund earns interest at a safe rate. As the initial capital is undisturbed for the period of investment, it follows that the yearly payments of interest to the investor are constant in amount.

This sinking fund method of valuation is sound in theory, and is a type frequently used in the commercial valuation of income subject to a material risk of loss. The investor receives a speculative rate of interest upon his initial investment; but, in the event that there is a negative amount in the sinking fund at any time, the investor receives but a safe rate of interest upon such sinking fund deficiency. These premises conform to the O'Donohue modification of the Hoskold formula for deferment.

Subdivision H of Division IV

Commercial modifications of the Hoskold formula.—An investor customarily regards unpaid interest as an involuntary addition to the amount of his investment and demands the same speculative rate of interest upon unpaid interest that is paid upon his previous investment. The method advocated by Hoskold for valuing equal annual income, the receipt of which is deferred, recognized the right of the investor to such speculative interest. The addition of sinking fund deficiencies to capital accomplishes this result, but violates the fundamental conception of sinking fund premises requiring that initial capital shall remain constant for the period of investment. This addition of sinking fund deficiencies—that is, negative total sinking fund accumulations—to capital for the remaining portion of the investment period is a combination of the Hoskold premises of Division IV with the compound interest actuarial premises of Division II applied to sinking fund deficiencies. These mixed or "hybrid" premises can only be developed algebraically as modifications of the formulas developed for regular Division

IV premises, and the algebraic processes involved are therefore treated as a subdivision of Division IV formulas—no other division of the premises having this Subdivision H. These mixed premises of valuation must be excluded from any generalizations with respect to sinking fund methods, particularly those with respect to Division IV formulas, as they do not conform to any single premise of valuation.

Several alternative methods are omitted as too unimportant, commercially, to justify extended discussion.

Subdivision G of Division IV

Alternative premises for treatment of capital additions after date of initial investment.—These alternatives apply to expenditures which can be definitely anticipated both as to date and amount, as at date of valuation, while the premises of Subdivision H apply to future sinking fund deficiencies which can be anticipated as to date but not as to amount. The modifications of the regular formulas of Division IV required for the alternative premises are stated with the algebraic derivation of the formulas.

Valuation and Allocation Formulas of Division IV.

The following valuation formula for Division IV premises is derived from the valuation formula of Division VI, which is hereinafter evolved, by making $r_1 = r$, whereupon, $j = e$ and $J = E$, and,

$$V = e\,\frac{A_1E^{n-1} + A_2E^{n-2} + A_3E^{n-3} + \cdots A_n}{rE^n - r + e}$$

The above formula may also be written in the form:

$$V = \frac{eE^n}{rE^n - r + e}\left(\frac{A_1}{E} + \frac{A_2}{E^2} + \frac{A_3}{E^3} + \cdots \frac{A_n}{E^n}\right)$$

In the latter form, the expression, $\dfrac{eE^n}{rE^n - r + e}$, is referred to as the factor of safety, which is discussed at length in Chapter VI.

Arithmetical solution: . . . $V = 5{,}449.676$

$$C_y = A_y - re\,\frac{A_1E^{n-1} + A_2E^{n-2} + A_3E^{n-3} + \cdots A_n}{rE^n - r + e}$$

Arithmetical solution: . . . $C_y = 355.032$ (see Column F in the following tabulation for the seventh year).

$$I_y = re\frac{A_1E^{n-1} + A_2E^{n-2} + A_3E^{n-3} + \cdots A_n}{rE^n - r + e}$$

Arithmetical solution: . . . $I_y = 544.968$ (see Column E in the following tabulation for the seventh year).

The valuation formula just derived under this division by substitutions in the formula for Division VI may be derived, independently of the latter formula, by forming a year by year series, where the yearly contribution to the sinking fund (the yearly income less interest due to the investor at the speculative rate, r) is increased by compound interest at the safe rate (e) from the year contributed to the end of the investment period. Such a series will appear as follows:

1st year: $(A_1 - Vr)E^{n-1}$
2nd year: $(A_2 - Vr)E^{n-2}$
3rd year: $(A_3 - Vr)E^{n-3}$
nth year: $(A_n - Vr)E^o = A_n - Vr$

Under the premises of this division the sum of the terms of the above series must be equal to V, whereupon,

$$V = e\frac{A_1E^{n-1} + A_2E^{n-2} + A_3E^{n-3} + \cdots A_n}{rE^n - r + e}$$

as shown before.

It will be noted that the above series will, if E is replaced by J, and e is replaced by j, be the same as the J series of Division VI.

If the present worth values of each year's income is regarded separately, so that the sum of these separate present worth values shall equal the present worth value (V) of the combined income as shown by the formula just derived, and assuming these separate present worth values to be represented by V_1, V_2, V_3, . . . V_n; then:

$$V_1 + V_2 + V_3 + \cdots V_n = V$$

Let V_y represent the present worth value of the income (A_y) in any year (y). Since all other years except the year y are years of zero income, since under this sinking fund law the investor is entitled to interest at the speculative rate (r) on the entire investment (V) (and, therefore, on every portion

INCOME ALLOCATION TABULATION, PREMISES IV

Year	Capital Account at Beginning of Year			Income Allocation at End of Year			Sinking Fund Account during Year			Interest rate paid on outstanding capital: per cent
	Total capital	Portion of capital in sinking fund	Portion of capital in speculative enterprise	Income	Interest due investor	Contributions to sinking fund	Amount in fund at beginning of year	Interest earned during year	Amount in fund at end of year	
Column	A	B	C	D	E	F	G	I	J	K
Process	From valuation formula	B = G	A − B	Per notation	10 % of A	D − E	From J preceding year	4 % of G	F + G + I	E ÷ A
1	5,449.68	000.00	5,449.68	1,000.00	544.97	455.03	000.00	000.00	455.03	10
2	5,449.68	455.03	4,994.65	1,200.00	544.96	655.04	455.03	18.20	1,128.27	10
3	5,449.68	1,128.27	4,321.41	500.00	544.97	−44.97	1,128.27	45.13	1,128.43	10
4	5,449.68	1,128.43	4,321.25	800.00	544.97	255.03	1,128.43	45.14	1,428.60	10
5	5,449.68	1,428.60	4,021.08	1,600.00	544.96	1,055.04	1,428.60	57.14	2,540.78	10
6	5,449.68	2,540.78	2,908.90	1,300.00	544.97	755.03	2,540.78	101.63	3,397.44	10
7	5,449.68	3,397.44	2,052.24	900.00	544.97	355.03	3,397.44	135.90	3,888.37	10
8	5,449.68	3,888.37	1,561.31	600.00	544.97	55.03	3,888.37	155.54	4,098.94	10
9	5,449.68	4,098.94	1,350.74	700.00	544.97	155.03	4,098.94	163.96	4,417.93	10
10	5,449.68	4,417.93	1,031.75	1,400.00	544.97	855.03	4,417.93	176.72	5,449.68	10
Totals	54,496.80	.....	.....	10,000.00	5,449.68	4,550.32	.....	899.36	.....	*10

* Average.

thereof) each and every year of the entire investment period, and since no part of the investment is returnable until the end of the investment period, the series, the sum of which will represent the present worth value (V_y), may be written as shown below, following the form of the series just shown:

1st year: $(O - V_y r)E^{n-1}$
2nd year: $(O - V_y r)E^{n-2}$
3rd year: $(O - V_y r)E^{n-3}$
yth year: $(A_y - V_y r)E^{n-y}$
nth year: $(O - V_y r)E^{o} = -V_y r$

Under the premises the sum of the terms of the above series must equal V_y, hence:

$$V_y = A_y E^{n-y} - V_y r(E^{n-1} + E^{n-2} + E^{n-3} + \cdots 1)$$

Summing the series in parentheses and solving for V_y we obtain:

$$V_y = e\frac{A_y E^{n-y}}{rE^n - r + e}$$

Since, under the hypothesis, y represents any year, substitute for y in the above equation the consecutive years 1, 2, 3, . . . n and obtain:

$$V_1 + V_2 + V_3 + \cdots V_n = e\frac{A_1 E^{n-1} + A_2 E^{n-2} + A_3 E^{n-3} + \cdots A_n}{rE^n - r + e} = V$$

The above is seen to be the same value for V as that shown for the present worth value of the combined income.

The analysis just concluded as a supplement to the derivation of the general formula of this division has been introduced for the purpose of clarifying the source of certain mathematical errors which customarily have been made when the double interest rate sinking fund formula has been used in valuing this type of income, and for the purpose of emphasizing the necessity for a strict adherence to the sinking fund premises. The errors made have been largely on account of: (1) the practice of valuing a portion of the income on the basis of a partial capital retirement before the termination of the investment period, while interest payment to the investor on the total initial value was continued to the end of the investment period; and (2) considering the difference between yearly income and yearly interest due to the investor as a payment

to the sinking fund only when the amount of the difference was positive, whereas, if this amount is negative, it also should be taken into account as a charge against the sinking fund.

Mathematical Adjustments Necessitated by Commercial Modifications of Division IV Premises of Valuation.

There are certain commercial practices in valuation to which the preceding formulas can be adapted, and the following analysis and derivations, intended to cover this phase, will be referred to as Subdivisions G and H of Division IV.

Subdivision G

If Q_1, Q_2, Q_3, . . . Q_n represent expected irregular capital expenditures at the end of each of the years indicated by the subscripts to the letter Q, and if it is agreed that the investor is entitled to the speculative rate of interest on each additional capital expenditure from the date it is made until the end of the investment period, then the value of the income subject to such capital expenditures at the date of the investment is determined by deducting from the value of the same income without allowance for the capital expenditures the following expression:

$$\frac{Q_1(rE^{n-1} - r + e) + Q_2(rE^{n-2} - r + e) + Q_3(rE^{n-3} - r + e) + \cdots Q_n(rE^0 - r + e)}{E^n - r + e}$$

The above deduction holds true for any type of income distribution formula which follows the regular Division IV premises.

If it is a condition of valuation that the initial investment subject to a speculative rate of interest is divided into two parts, one of which is the initial investment in the speculative enterprise, while the other is a fund earning a safe rate of interest until exhausted by withdrawals to meet required additional expenditures of capital in years after initial date of investment, then there should be deducted from the value determined for the income (without allowance for such capital expenditures) by means of the regular Division IV formulas the following expression:

$$\text{Value of fund} = \frac{Q_1}{E} + \frac{Q_2}{E^2} + \frac{Q_3}{E^3} + \frac{Q_4}{E^4} + \cdots \frac{Q_n}{E^n}$$

Subdivision H

All the preceding formulas of Division IV might be said to follow the O'Donohue modification of the Hoskold premises of valuation, since, in the event that the total amount accumulated in the sinking fund is negative in value at any time, the investor receives interest upon such negative amount or sinking fund deficiency at the safe or sinking fund rate rather than at the speculative rate. Such negative accumulations in the sinking fund represent either additional capital advanced by the investor after initial date of investment, or interest due to the investor and unpaid for lack of funds. The true Hoskold premises provide that the investor shall receive the speculative rate of interest upon further capital advances, including unpaid interest in this category. Formulas will next be derived which follow the premises that sinking fund deficiencies are added to capital as soon as they occur and that this added capital earns a speculative rate of interest from the time it is added to the end of the investment period. The exact years in which the sinking fund deficiencies will develop must be known, in order that the following formulas may be applied, but it is not necessary that the amounts of such deficiencies should be known. The years in which the deficiency will occur must be determined by solving the preceding general formula for Division IV and developing an income allocation table for the period of investment, unless all the years of sinking fund deficiency may be otherwise determined, as in the case of deferred equal annual income. The algebraic derivation of the succeeding formulas are exceedingly complex and the details as indicated by a complete year by year series will be omitted; but a portion of the series is given which is representative of all similar periods, as they all follow the same form. The arithmetical tabulations given hereinafter in connection with these formulas are indicative of a complete year by year analysis and further confirm the accuracy of the formulas. These formulas are derived for any time distribution of income within the period of investment, and are the true Hoskold type, as applied to sinking fund deficiencies of various number and known years of occurrence. They can be modified to suit specific time distribution of income, but in Chapter III all modifications of

Division IV formulas as to time distribution of income apply to the preceding and not to the formulas about to be derived.

Additional Notation.

As in preceding notation let S_1, S_2, S_3, . . . S_n represent the yearly additions to the sinking fund, each addition to the sinking fund including the sinking fund contribution for the designated year plus interest earned at the safe rate for that year by the total amount in the sinking fund at the beginning of the year. Then, $S_1 + S_2 + S_3 + \ldots S_y$ represents the total amount in the sinking fund at the end of the year (y) or the beginning of the year $(y + 1)$.

Let:

$a, b, c, d, \ldots z$ = periods of years at the end of which sinking fund deficiencies are added to outstanding capital, each period being reckoned from the beginning of the first year, each consecutive period being greater than the preceding in the order indicated, and all of these periods being within and less than the total investment period (n):

$\Sigma_a, \Sigma_b, \Sigma_c, \Sigma_d, \ldots \Sigma_z$ = the algebraic sum in each case of all prior sinking fund contributions plus their interest accretions at the safe rate to the end of the years $a, b, c, d, \ldots z$, respectively.

Then:

$$
\begin{aligned}
&S_1 + S_2 + S_3 + \cdots S_a = \Sigma_a \\
&S_1 + S_2 + S_3 + \cdots S_a + \cdots S_b = \Sigma_b \\
&S_1 + S_2 + S_3 + \cdots S_a + \cdots S_b + \cdots S_c = \Sigma_c \\
&S_1 + S_2 + S_3 + \cdots S_a + \cdots S_b + \cdots S_c + \cdots S_d = \Sigma_d \\
&S_1 + S_2 + S_3 + \cdots S_w + \cdots S_x + \cdots S_y = \Sigma_y \\
&S_1 + S_2 + S_3 + \cdots S_w + \cdots S_x + \cdots S_y + \cdots S_z = \Sigma_z
\end{aligned}
$$

As indicated by the notation, any number of sinking fund deficiencies is assumed. Take, for example, that which occurs in the year (z). Under the premises the terms of the series for the year z, the year immediately preceding, the three years immediately following, and the year n, which represent the yearly additions to the sinking fund, including interest earned for that year by the total amount in the sinking fund at the beginning of the year, will appear as follows:

$(z-1)$th year: $A_{z-1} - (V - \Sigma_y)r + (S_{y+1} + S_{y+2} + S_{y+3} + \cdots + S_{z-2})e = S_{z-1}$

zth year: $A_z - (V - \Sigma_y)r + (S_{y+1} + S_{y+2} + S_{y+3} + \cdots S_{z-1})e = S_z$

$(z+1)$th year: $A_{z+1} - (V - \Sigma_z)r = S_{z+1}$

$(z+2)$th year: $A_{z+2} - (V - \Sigma_z)r + S_{z+1}e = S_{z+2}$

$(z+3)$th year: $A_{z+3} - (V - \Sigma_z)r + (S_{z+1} + S_{z+2})e = S_{z+3}$

nth year: $A_n - (V - \Sigma_z)r + (S_{z+1} + S_{z+2} + S_{z+3} + \cdots S_{n-1})e = S_n$

Since the terms of the series for all sinking fund deficiencies in other years follow precisely the same form as that shown for the year z and the years preceding and following, terms corresponding to those above may be written by inspection for all years. The years prior to the first deficiency (or the year a) will follow the same form except that Σ_a will be replaced by zero.

From an inspection of the terms of the series given above, the total amount in the sinking fund at the end of the year may be written as follows:

For convenience in notation, assume that $A_1 - Vr = X_1$; $A_2 - Vr = X_2$; $A_3 - Vr = X_3$; $A_4 - Vr = X_4$; . . . $A_n - Vr = X_n$

$$S_1 + S_2 + S_3 + \cdots S_{z-1} = X_{y+1}E^{n-y-2} + X_{y+2}E^{n-y-3} + X_{y+3}E^{n-y-4} + \cdots X_{z-1} + \Sigma_y + \Sigma_y r(E^{n-y-2} + E^{n-y-3} + E^{n-y-4} + \cdots 1)$$

$$S_1 + S_2 + S_3 + \cdots S_z = X_{y+1}E^{n-y-1} + X_{y+2}E^{n-y-2} + X_{y+3}E^{n-y-3} + \cdots X_z + \Sigma_y + \Sigma_y r(E^{n-y-1} + E^{n-y-2} + E^{n-y-3} + \cdots 1) = \Sigma_z$$

$$S_1 + S_2 + S_3 + \cdots S_{z+1} = X_{z+1} + \Sigma_z + \Sigma_z r$$

$$S_1 + S_2 + S_3 + \cdots S_{z+2} = X_{z+1}E + X_{z+2} + \Sigma_z + \Sigma_z r(E+1)$$

$$S_1 + S_2 + S_3 + \cdots S_{z+3} = X_{z+1}E^2 + X_{z+2}E + X_{z+3} + \Sigma_z + \Sigma_z r(E^2 + E + 1)$$

$$S_1 + S_2 + S_3 + \cdots S_{z+4} = X_{z+1}E^3 + X_{z+2}E^2 + X_{z+3}E + X_{z+4} + \Sigma_z + \Sigma_z r(E^3 + E^2 + E + 1)$$

$$S_1 + S_2 + S_3 + \cdots S_n = X_{z+1}E^{n-z-1} + X_{z+2}E^{n-z-2} + X_{z+3}E^{n-z-3} + \cdots X_n + \Sigma_z + \Sigma_z r(E^{n-z-1} + E^{n-z-2} + E^{n-z-3} + \cdots 1)$$

The sum $S_1 + S_2 + S_3 + \cdots S_n$, or the total amount in the sinking fund at the end of the nth year, must be equal to V, or:

$$V = X_{z+1}E^{n-z-1} + X_{z+2}E^{n-z-2} + X_{z+3}E^{n-z-3} + \cdots X_n + \Sigma_z + \Sigma_z r(E^{n-z-1} + E^{n-z-2} + E^{n-z-3} + \cdots 1)$$

Summing the series in the parentheses above, then:

$$V = X_{z+1}E^{n-z-1} + X_{z+2}E^{n-z-2} + X_{z+3}E^{n-z-3} + \cdots X_n + \Sigma_z \frac{rE^{n-z} - r + e}{e}$$

Since, in the above equation, Vr terms are contained in the right hand member of the equation, the final solution for V, for one, two, three, or more sinking fund deficiencies, may be accomplished as follows:

From what has preceded the following relations may be written, as the same form is followed for all sinking fund deficiency years:

$$\Sigma_a = X_1E^{a-1} + X_2E^{a-2} + X_3E^{a-3} + \cdots X_a$$

$$\Sigma_b = X_{a+1}E^{b-a-1} + X_{a+2}E^{b-a-2} + X_{a+3}E^{b-a-3} + \cdots X_b + \Sigma_a\frac{rE^{b-a} - r + e}{e}$$

$$\Sigma_c = X_{b+1}E^{c-b-1} + X_{b+2}E^{c-b-2} + X_{b+3}E^{c-b-3} + \cdots X_c + \Sigma_b\frac{rE^{c-b} - r + e}{e}$$

$$\Sigma_z = X_{y+1}E^{z-y-1} + X_{y+2}E^{z-y-2} + X_{y+3}E^{z-y-3} + \cdots X_z + \Sigma_y\frac{rE^{z-y} - r + e}{e}$$

By substituting the above equivalents in the V equation just shown, a formula for one or more sinking fund deficiencies may be obtained as shown below:

Let x, y, and z be the years in which the sinking fund deficiencies occur, then:

For one sinking fund deficiency in the year x:

$$V = \left[\frac{e}{rE^{n-x} - r + e}\right]\left[\frac{e}{rE^{x} - r + e}\right]\left[A_{x+1}E^{n-x-1} + A_{x+2}{}^{n-x-2} + A_{x+3}E^{n-x-3} + \cdots A_n\right] + \left[\frac{e}{rE^{x} - r + e}\right]\left[A_1E^{x-1} + A_2E^{x-2} + A_3E^{x-3} + \cdots A_x\right]$$

For two sinking fund deficiencies, one in the year x and another in a later year y:

$$V = \left[\frac{e}{rE^{n-y} - r + e}\right]\left[\frac{e}{rE^{y-x} - r + e}\right]\left[\frac{e}{rE^{x} - r + e}\right]\left[A_{y+1}E^{n-y-1} + A_{y+2}E^{n-y-2} + A_{y+3}E^{n-y-3} + \cdots A_n\right] + \left[\frac{e}{rE^{y-x} - r + e}\right]\left[\frac{e}{rE^{x} - r + e}\right]\left[A_{x+1}E^{y-x-1} + A_{x+2}E^{y-x-2} + A_{x+3}E^{y-x-3} + \cdots A_y\right] + \left[\frac{e}{rE^{x} - r + e}\right]\left[A_1E^{x-1} + A_2E^{x-2} + A_3E^{x-3} + \cdots A_x\right]$$

For three sinking fund deficiencies in the years x, y, and z, x being the year in which the first deficiency occurs, y the

year in which the second deficiency occurs, and z the year of the third deficiency:

$$V = \left[\frac{e}{rE^{n-z} - r + e}\right]\left[\frac{e}{rE^{z-y} - r + e}\right]\left[rE^{y-z} \frac{e}{-r+e}\right]\left[\frac{e}{rE^{z} - r + e}\right]$$
$$\left[A_{z+1}E^{n-z-1} + A_{z+2}E^{n-z-2} + \cdots A_n\right] + \left[\frac{e}{rE^{z-y} - r + e}\right]$$
$$\left[\frac{e}{rE^{y-x} - r + e}\right]\left[rE^{z} \frac{e}{-r+e}\right]\left[A_{y+1}E^{z-y-1} + A_{y+2}E^{z-y-2} + \cdots A_z\right] +$$
$$\left[\frac{e}{rE^{y-x} - r + e}\right]\left[\frac{e}{rE^{z} - r + e}\right]\left[A_{x+1}E^{y-x-1} + A_{x+2}E^{y-x-2} +\right.$$
$$\left.\cdots A_y\right] + \left[\frac{e}{rE^{z} - r + e}\right]\left[A_1E^{z-1} + A_2E^{z-2} + \cdots A_z\right]$$

The formulas for any number of deficiencies in the sinking fund may be written by inspection of the above, following the forms just stated for one, two, and three deficiencies. For sinking fund deficiencies for d years of deferment, followed by $n-d$ years of equal annual income in the amount A, the deficiency formula becomes:

$$V = A\frac{E^{n-d} - 1}{rE^{n-d} - r + e}\frac{1}{R^d}$$

The above formula is equivalent to the formula used by Hoskold in valuing equal annual income, the receipt of which is deferred. This formula is derived directly as follows:

According to the Hoskold formula for valuing equal annual income, the present worth value (V_d) of an income (A) each year for $n - d$ years would be as shown by the formula:

$$V_d = A\frac{E^{n-d} - 1}{rE^{n-d} - r + e}$$

The present worth value (V), of an amount (V_d), due in d years, at the interest rate r, compounded annually, would be as shown by the formula:

$$V = \frac{V_d}{R^d}$$

Substitute in the latter formula the value (V_d) as shown by the former and obtain,

$$V = A\frac{E^{n-d} - 1}{rE^{n-d} - r + e}\frac{1}{R^d}$$

as shown before.

INCOME ALLOCATION TABULATION, PREMISES IV, MODIFIED TO ALLOW A SPECULATIVE RATE OF INTEREST ON SINKING FUND DEFICIENCIES

Year	Capital Account			Income Allocation at End of Year			Sinking Fund Account during Year			Sinking fund deficiencies added to capital	Interest rate paid on outstanding capital: per cent
	Capital outstanding at beginning of year	Added to capital at end of year	Capital outstanding at end of year	Income	Interest due investor	Contributions to sinking fund	Amount in fund at beginning of year	Interest earned during year	Amount in fund at end of year		
Column	A	B	C	D	E	F	G	I	J	K	L
Process	From val. formula and C preceding yr.	From K same yr.	A + B	Per notation	10% of A	D − E	Positive amts. from J preceding year	4% of G	F + G + I	Negative amounts in column J	E ÷ A
1	4,410.37	441.04	4,851.41	000.00	441.04	− 441.04	000.00	000.00	− 441.04	441.04	10
2	4,851.41	000.00	4,851.41	1,000.00	485.14	514.86	000.00	000.00	514.86	000.00	10
3	4,851.41	000.00	4,851.41	1,000.00	485.14	514.86	514.86	20.59	1,050.31	000.00	10
4	4,851.41	2,392.81	7,244.22	−3,000.00	485.14	−3,485.14	1,050.31	42.02	−2,392.81	2,392.81	10
5	7,244.22	000.00	7,244.22	1,000.00	724.42	275.58	000.00	000.00	275.58	000.00	10
6	7,244.22	000.00	7,244.22	2,000.00	724.42	1,275.58	275.58	11.02	1,562.18	000.00	10
7	7,244.22	000.00	7,244.22	2,000.00	724.43	1,275.57	1,562.18	62.49	2,900.24	000.00	10
8	7,244.22	000.00	7,244.22	2,000.00	724.42	1,275.58	2,900.24	116.01	4,291.83	000.00	10
9	7,244.22	000.00	7,244.22	2,000.00	724.42	1,275.58	4,291.83	171.67	5,739.08	000.00	10
10	7,244.22	000.00	7,244.22	2,000.00	724.42	1,275.58	5,739.08	229.56	7,244.22	000.00	10
Totals	62,429.92	2,833.85	65,263.77	10,000.00	6,242.99	3,757.01	.....	653.36	.....	2,833.85	*10

* Average.

The preceding arithmetical substantiation of the formulas applying to sinking fund deficiencies is based upon the substitution of the numerical quantities shown below:

$A_1 = O$:	$A_2 = 1{,}000$:	$A_3 = 1{,}000$:	$A_4 = -3{,}000$:	$A_5 = 1{,}000$:
$A_6 = 2{,}000$:	$A_7 = 2{,}000$:	$A_8 = 2{,}000$:	$A_9 = 2{,}000$:	$A_{10} = 2{,}000$:
	$n = 10$	$r = 0.10$	$e = 0.04$	$E = 1.04$

By inspection it may be seen that sinking fund deficiencies will surely develop in the first and the fourth years. If unable to determine such years of sinking fund deficiencies by inspection, the evaluator should solve the general formula of Division IV and work out an income allocation tabulation, similar to the one shown in connection with that formula.

It then develops that $A_x = A_1 = O$, and that $A_y = A_4 = -3{,}000$. Since there are two deficiencies, this fact governs the selection of a formula for valuation, as there is a different formula with these premises for each number of sinking fund deficiencies.

Substituting the numerical quantities and solving:

$$V = 4{,}410.370$$

Note.—With the general formula of Division IV the value would be 4,970.366.

DIVISION V

SINKING FUND VALUATION METHOD WITH TWO RATES OF INTEREST (Morkill Type)

Premises.—With this method of valuation the investor receives a speculative rate of interest on capital outstanding in the speculative enterprise at the beginning of the year. The capital outstanding in the speculative enterprise is the algebraic difference between the amount of the initial investment and the amount accumulated in the sinking fund at the beginning of the year, including both the contributions to the sinking fund and their interest accretions. Thus, when the sinking fund contains a negative amount, a speculative rate of interest is paid to the investor on such negative amount, which has been temporarily loaned to the sinking fund by him.

The sinking fund earns a safe rate of interest and interest is also paid to the investor at a safe rate on positive amounts of capital withdrawn from the speculative enterprise by accumulation in the sinking fund. If the investor has advanced money to the sinking fund at the date of investment to cover future anticipated capital expenditures, he receives a safe rate of interest upon such advances to the sinking fund until the time when the anticipated capital expenditures are made, at which time he begins to receive a speculative rate of interest upon this portion of his initial investment, as it then becomes capital outstanding in the speculative enterprise.

The effect of this method of valuation is a division of the total investment into two separate and distinct investments, the one a safe investment earning a safe rate of interest, and the other a speculative investment earning a speculative rate of interest, the safe and speculative portions from year to year being variable parts of the constant total investment.

Valuation and Allocation Formulas of Division V.

The following valuation formula for Division V premises is derived from that of Division VI, which is hereinafter evolved, by making r_1 equal e. Then:

$$V = \frac{A_1}{R} + \frac{A_2}{R^2} + \frac{A_3}{R^3} + \frac{A_4}{R^4} + \cdots \frac{A_n}{R^n}$$

It will be noted that the above formula is identical with those derived under Divisions I, II, and III.

Arithmetical solution: . . . $V = 6{,}128.561$

$$C_y = A_y - \left(\frac{A_y}{R} + \frac{A_{y+1}}{R^2} + \frac{A_{y+2}}{R^3} + \cdots \frac{A_n}{R^{n-y+1}}\right)(r - e) - e\left(\frac{A_1}{R} + \frac{A_2}{R^2} + \frac{A_3}{R^3} + \cdots \frac{A_n}{R^n}\right)$$

Arithmetical solution: . . . $C_y = 487.086$

$$I_y = \left(\frac{A_y}{R} + \frac{A_{y+1}}{R^2} + \frac{A_{y+2}}{R^3} + \cdots \frac{A_n}{R^{n-y+1}}\right)(r - e) + e\left(\frac{A_1}{R} + \frac{A_2}{R^2} + \frac{A_3}{R^3} + \cdots \frac{A_n}{R^n}\right)$$

Arithmetical solution: . . . $I_y = 412.914$ (see Column G, seventh year for C_y and the sum of Columns E and F for I_y same year of the arithmetical tabulation which follows).

The term by term series showing yearly additions to the sinking fund, from which the above valuation formula may be obtained by summation, is as follows:

Year	Income	Less interest due investor at rate r on capital remaining in speculative business	Less interest due investor at rate e on capital accumulated in sinking fund*	Plus interest earned at rate e by sinking fund*	Balance, yearly additions to sinking fund
1st	A_1	$-Vr$	$-$zero	$+$zero	$= S_1$
2nd	A_2	$-[V - S_1]r$	$-S_1e$	$+S_1e$	$= S_2$
3rd	A_3	$-[V - (S_1 + S_2)]r$	$-(S_1 + S_2)e$	$+(S_1 + S_2)e$	$= S_3$
4th	A_4	$-[V - (S_1 + S_2 + S_3)]r$	$-(S_1 + S_2 + S_3)e$	$+(S_1 + S_2 + S_3)e$	$= S_4$
5th	A_5	$-[V - (S_1 + S_2 + S_3 + S_4)]r$	$-(S_1 + S_2 + S_3 + S_4)e$	$+(S_1 + S_2 + S_3 + S_4)e$	$= S_5$
nth	A_n	$-[V - (S_1 + S_2 + S_3 + \cdots + S_{n-1})]r$	$-(S_1 + S_2 + S_3 + \cdots + S_{n-1})e$	$+(S_1 + S_2 + S_3 + \cdots + S_{n-1})e$	$= S_n$

* It will be noted that the terms for each year under these two columns will cancel, as they are equal and of opposite sign, thus leaving a series identical with that shown for the derivation of Valuation Formula of Division II.

INCOME ALLOCATION TABULATION, PREMISES V

Year	Capital Account at Beginning of Year			Income Allocation at End of Year				Sinking Fund Account During Year			Interest rate paid on total capital: per cent
	Total capital	Portion of capital in sinking fund	Portion of capital in speculative enterprise	Income	Interest paid to investor on capital in		Contributions to sinking fund	Amount in fund at beginning of year	Interest earned during year	Amount in fund at end of year	
					Sinking fund	Speculative enterprise					
Column	A	B	C	D	E	F	G	H	J	K	L
Process	From valuation formula	B = H	A − B	Per notation	4% of B	10% of C	D − E − F	From K preceding year	4% of H	G + H + J	$\frac{E+F}{A}$
1	6,128.56	000.00	6,128.56	1,000.00	000.00	612.86	387.14	000.00	000.00	387.14	10.000
2	6,128.56	387.14	5,741.42	1,200.00	15.49	574.14	610.37	387.14	15.49	1,013.00	9.621
3	6,128.56	1,013.00	5,115.56	500.00	40.52	511.55	−52.07	1,013.00	40.52	1,001.45	9.008
4	6,128.56	1,001.45	5,127.11	800.00	40.05	512.72	247.23	1,001.45	40.05	1,288.73	9.020
5	6,128.56	1,288.73	4,839.83	1,600.00	51.55	483.98	1,064.47	1,288.73	51.55	2,404.75	8.738
6	6,128.56	2,404.75	3,723.81	1,300.00	96.19	372.38	831.43	2,404.75	96.19	3,332.37	7.646
7	6,128.56	3,332.37	2,796.19	900.00	133.29	279.62	487.09	3,332.37	133.29	3,952.75	6.738
8	6,128.56	3,952.75	2,175.81	600.00	158.11	217.58	224.31	3,952.75	158.11	4,335.17	6.130
9	6,128.56	4,335.17	1,793.39	700.00	173.41	179.34	347.25	4,335.17	173.41	4,855.83	5.756
10	6,128.56	4,855.83	1,272.73	1,400.00	194.24	127.27	1,078.49	4,855.83	194.24	6,128.56	5.246
Totals	61,285.60	22,571.19	38,714.41	10,000.00	902.85	3,871.44	5,225.71	.....	902.85	.....	*7.790

* Average.

DIVISION VI

SINKING FUND VALUATION METHOD WITH THREE RATES OF INTEREST (type developed by authors)

Premises.—With this method of valuation the investor receives a speculative rate of interest on capital outstanding in the speculative enterprise at the beginning of each year. The capital outstanding in the speculative enterprise is the algebraic difference between the amount of the initial investment and the amount accumulated in the sinking fund at the beginning of the year. The sinking fund is invested to earn a safe rate of interest. The investor is paid a semispeculative rate of interest—a rate of interest intermediate between the speculative and safe rates—upon the amount of capital in the sinking fund at the beginning of the year. The theory accounting for the semispeculative rate of interest is that the possible diversion of the sinking fund to other purposes than the return of capital, without the consent of the investor, justifies a demand for a rate of interest higher than the safe rate but less than the speculative rate upon unreturned capital.

Thus, the investor receives a speculative rate of interest upon negative amounts in the sinking fund, that is, upon amounts which he has temporarily loaned to the sinking fund to cover deficiencies in income below the amount required to pay interest to the investor. Upon amounts advanced to the sinking fund at the date of investment the investor receives a semispeculative rate of interest while such amounts remain in the sinking fund and until they are withdrawn from the sinking fund for investment in the speculative enterprise, after which he receives a speculative rate of interest upon these amounts. The investor is paid a semispeculative rate of interest upon positive amounts accumulated in the sinking fund, which represent capital withdrawn from the speculative investment but not yet returned to the investor.

The effect of this method of valuation is a division of the total investment into two separate and distinct parts, one a speculative investment at a speculative rate of interest and the other a semispeculative investment at a semispeculative rate of interest. The total investment is constant, but the

speculative and semispeculative portions vary from year to year.

Valuation and Allocation Formulas of Division VI.

As in the case of all sinking fund premises, the value (V) is the sum of the amounts contributed to the sinking fund plus interest earned at the rate (e) by each contribution. The amount contributed to the sinking fund in any year is the income for that year less interest paid to the investor. Interest is paid to the investor at one rate (r) on the portion of the initial investment not retired to the sinking fund, and at another rate (r_1) on the portion of the initial investment retired to the sinking fund. These relations may be expressed in algebraic series as follows:

Year	Income	Less interest due to investor at rate r on capital remaining in speculative business	Less interest due to investor at rate r_1 on capital accumulated in sinking fund	Plus interest earned at rate e by sinking fund, or by capital taken out of business	Balance, yearly additions to sinking fund
1st:	A_1	$-Vr$	$-$zero	$+$zero	$= S_1$
2nd:	A_2	$-[V - S_1]r$	$-S_1r_1$	$+S_1e$	$= S_2$
3rd:	A_3	$-[V - (S_1 + S_2)]r$	$-(S_1 + S_2)r_1$	$+(S_1 + S_2)e$	$= S_3$
4th:	A_4	$-[V - (S_1 + S_2 + S_3)]r$	$-(S_1 + S_2 + S_3)r_1$	$+(S_1 + S_2 + S_3)e$	$= S_4$
5th:	A_5	$-[V - (S_1 + S_2 + S_3 + S_4)]r$	$-(S_1 + S_2 + S_3 + S_4)r_1$	$+(S_1 + S_2 + S_3 + S_4)e$	$= S_5$
nth	A_n	$-[V - (S_1 + S_2 + S_3 + \cdots S_{n-1})]r$	$-(S_1 + S_2 + S_3 + \cdots S_{n-1})r_1$	$+(S_1 + S_2 + S_3 + \cdots S_{n-1})e$	$= S_n$

By inspection the above series may be written, term for term, as:

1st year: $A_1 - Vr = S$
2nd year: $A_2 - Vr + S_1(r - r_1 + e) = S_2$
3rd year: $A_3 - Vr + (S_1 + S_2)(r - r_1 + e) = S_3$
4th year: $A_4 - Vr + (S_1 + S_2 + S_3)(r - r_1 + e) = S_4$
5th year: $A_5 - Vr + (S_1 + S_2 + S_3 + S_4)(r - r_1 + e) = S_5$
nth year: $A_n - Vr + (S_1 + S_2 + S_3 + \cdots S_{n-1})(r - r_1 + e) = S_n$

In the above series write the composite interest rate j for $r - r_1 + e$ and obtain:

1st year: $A_1 - Vr = S_1$
2nd year: $A_2 - Vr + S_1 j = S_2$
3rd year: $A_3 - Vr + (S_1 + S_2)j = S_3$
4th year: $A_4 - Vr + (S_1 + S_2 + S_3)j = S_4$
5th year: $A_5 - Vr + (S_1 + S_2 + S_3 + S_4)j = S_5$
nth year: $A_n - Vr + (S_1 + S_2 + S_3 + \cdots S_{n-1})j = S_n$

From an inspection of the above series, the following relations may be written at once:

$$S_1 = S_1$$
$$S_1 + S_2 = S_1 J + A_2 - Vr$$
$$S_1 + S_2 + S_3 = (S_1 + S_2)J + A_3 - Vr$$
$$S_1 + S_2 + S_3 + S_4 = (S_1 + S_2 + S_3)J + A_4 - Vr$$
$$S_1 + S_2 + S_3 + S_4 + \cdots S_n = (S_1 + S_2 + S_3 + S_4 + \cdots S_{n-1})J + A_n - Vr$$

From the above it follows that:

$$S_1 = A_1 - Vr$$
$$S_1 + S_2 = (A_1 - Vr)J + (A_2 - Vr)$$
$$S_1 + S_2 + S_3 = (A_1 - Vr)J^2 + (A_2 - Vr)J + (A_3 - Vr)$$
$$S_1 + S_2 + S_3 + S_4 = (A_1 - Vr)J^3 + (A_2 - Vr)J^2 + (A_3 - Vr)J + (A_4 - Vr)$$
$$S_1 + S_2 + S_3 + S_4 + S_5 = (A_1 - Vr)J^4 + (A_2 - Vr)J^3 + (A_3 - Vr)J^2 + (A_4 - Vr)J + (A_5 - Vr)$$
$$S_1 + S_2 + S_3 + S_4 + \cdots S_n = (A_1 - Vr)J^{n-1} + (A_2 - Vr)J^{n-2} + (A_3 - Vr)J^{n-3} + \cdots (A_n - Vr)$$

Since, under the premises, $S_1 + S_2 + S_3 + S_4 + \cdots S_n$ is equal to V, then,

$$V = (A_1 - Vr)J^{n-1} + (A_2 - Vr)J^{n-2} + (A_3 - Vr)J^{n-3} + \cdots (A_n - Vr)$$

or

$$V = A_1 J^{n-1} + A_2 J^{n-2} + A_3 J^{n-3} + \cdots A_n - Vr(J^{n-1} + J^{n-2} + J^{n-3} + \cdots 1)$$

Solve the above for V after summing the series within the parentheses following the Vr term, in the manner already indicated, and obtain:

$$V = j\frac{A_1 J^{n-1} + A_2 J^{n-2} + A_3 J^{n-3} + A_4 J^{n-4} + \cdots A_n}{rJ^n - r + j}$$

It follows from what has been shown that a year by year series could be constructed on the basis that the yearly contribution to the sinking fund draws interest at the composite rate, j, from the date contributed to the end of the investment period. Such a series would appear as follows:

	J Series
1st year:	$(A_1 - Vr)J^{n-1}$
2nd year:	$(A_2 - Vr)J^{n-2}$
3rd year:	$(A_3 - Vr)J^{n-3}$
4th year:	$(A_4 - Vr)J^{n-4}$
nth year:	$(A_n - Vr)J^0 = A_n - Vr$

As in the case of Division IV, the sum of the terms of the above series must be equal to V; hence, as shown under the preceding derivation:

$$V = j\frac{A_1J^{n-1} + A_2J^{n-2} + A_3J^{n-3} + A_4J^{n-4} + \cdots A_n}{rJ^n - r + j}$$

Arithmetical solution: . . . $V = 5{,}676.761$

$$C_y = A_y - r\frac{[(J^{y-1} - 1)(r - r_1) + j](A_1J^{n-1} + A_2J^{n-2} + A_3J^{n-3} + \cdots A_n)}{rJ^n - r + j} + (A_1J^{y-2} + A_2J^{y-3} + A_3J^{y-4} + \cdots A_{y-1})(r - r_1)$$

Arithmetical solution: . . . $C_y = 400.001$ (see Column G of the following tabulation for the seventh year).

$$I_y = r\frac{[(J^{y-1} - 1)(r - r_1) + j](A_1J^{n-1} + A_2J^{n-2} + A_3J^{n-3} + \cdots A_n)}{rJ^n - r + j} - (A_1J^{y-2} + A_2J^{y-3} + A_3J^{y-4} + \cdots A_{y-1})(r - r_1)$$

Arithmetical solution: . . . $I_y = 499.999$ (see the sum of Columns E and F of the following tabulation for the seventh year).

Chapter summary.—It has been shown, by separate derivations of the general valuation formulas following the premises of Divisions I and II, that these premises result in identical valuations when applied to the same income. It was later shown that the values determined under the premises of Divisions III and V were also identical with those of Divisions I and II. These four premises of valuation, therefore, differ only as to the income allocation.

Statements have been made that the general formulas of Divisions III, IV, and V may be derived from the general formula of Division VI. In order to show these derivations

INCOME ALLOCATION TABULATION, PREMISES VI

Year	Capital Account at Beginning of Year			Income Allocation at End of Year				Sinking Fund Account During Year			Interest rate paid on total capital: per cent
	Total capital	Portion of capital in sinking fund	Portion of capital in speculative enterprise	Income	Interest paid to investor on capital in		Contributions to sinking fund	Amount in fund at beginning of year	Interest earned during year	Amount in fund at end of year	
					Sinking fund	Speculative enterprise					
Column	A	B	C	D	E	F	G	H	J	K	L
Process	From valuation formula	B = H	A − B	Per notation	8 % of B	10 % of C	D − E − F	From K preceding year	4 % of H	G + H + J	$\frac{E+F}{A}$
1	5,676.76	000.00	5,676.76	1,000.00	000.00	567.68	432.32	000.00	000.00	432.32	10.000
2	5,676.76	432.32	5,244.44	1,200.00	34.59	524.44	640.97	432.32	17.30	1,090.59	9.848
3	5,676.76	1,090.59	4,586.17	500.00	87.25	458.61	−45.86	1,090.59	43.62	1,088.35	9.616
4	5,676.76	1,088.35	4,588.41	800.00	87.07	458.84	254.09	1,088.35	43.53	1,385.97	9.617
5	5,676.76	1,385.97	4,290.79	1,600.00	110.88	429.08	1,060.04	1,385.97	55.44	2,501.45	9.512
6	5,676.76	2,501.45	3,175.31	1,300.00	200.12	317.53	782.35	2,501.45	100.06	3,383.86	9.119
7	5,676.76	3,383.86	2,292.90	900.00	270.71	229.29	400.00	3,383.86	135.36	3,919.22	8.808
8	5,676.76	3,919.22	1,757.54	600.00	313.54	175.75	110.71	3,919.22	156.77	4,186.70	8.619
9	5,676.76	4,186.70	1,490.06	700.00	334.94	149.01	216.05	4,186.70	167.47	4,570.22	8.525
10	5,676.76	4,570.22	1,106.54	1,400.00	365.62	110.65	923.73	4,570.22	182.81	5,676.76	8.390
Totals	56,767.60	22,558.68	34,208.92	10,000.00	1,804.72	3,420.88	4,774.40	.....	902.36	.....	*9.205

* Average.

very simply, it is advisable to present the general formula in a modified form, which follows by inspection from the valuation formula for Division VI just derived, as shown below:

$$V = \frac{jJ^n}{rJ^n - r + j}\left(\frac{A_1}{J} + \frac{A_2}{J^2} + \frac{A_3}{J^3} + \frac{A_4}{J^4} + \cdots \frac{A_n}{J^n}\right)$$

Since, in the above formula, $j = r - r_1 + e$, then, if r_1 and e are both made equal to r, or if r_1 is made equal to e, j is equal to r and J is equal to R. Also, if r_1 is made equal to r, j is equal to e and J is equal to E.

With these substitutions in the general formula for Division VI, the general formulas for all other sinking fund premises are readily derived, and, since it has been shown that the general formulas for the compound interest premises of valuation under Divisions I and II are identical with the general valuation formula of the compound interest sinking fund premises under Division III, it is evident that valuation formulas for all premises may be derived by simple substitutions from the triple interest rate sinking fund valuation formula of Division VI. In the next chapter it will be shown that formulas for valuing any time distribution of income within the investment period may be similarly derived from the corresponding formula of Division VI.

Chapter III

TYPES OF VALUATION FORMULAS FOR TERMINABLE INCOME AND FOR PERPETUAL INCOME, WHICH MAY BE DERIVED FROM THE GENERAL FORMULAS OF CHAPTER II, TOGETHER WITH ARITHMETICAL SOLUTIONS AND ILLUSTRATIONS OF INCOME ALLOCATION BY TABULATION

The formulas developed in this chapter are adaptations of those in Chapter II, for the valuation of a few of the more common types of income, including:

A. Equal annual income for the term of the investment.
B. Income increasing or decreasing by equal annual amounts for the term of investment.
C. Income increasing or decreasing by equal annual percentages for the term of investment.
D. Income changing intermittently from one amount of equal annual income for one period to another amount of equal annual income for a succeeding period.
E. A period of deferment, or zero income, followed by equal annual income for the remaining term of the investment.
F. A period of deferment, during which additional capital expenditure in equal annual amount (treated as negative income) is required, followed by equal annual income for the remaining term of the investment.
G. Formulas for determining adjustments to value attributable to the necessity of making future capital expenditures of unequal amounts and irregular time distribution, in order that the receipt of income may be maintained in the amount valued.

Consideration of terminable income under each of the above headings will be given primary attention by means

of valuation formulas applicable to each of the several premises, and by substitution of numerical amounts for algebraic symbols and solution of the valuation formulas, with accompanying tabulations showing income allocation year by year according to the several premises. Income allocation tabulations are given in this chapter for each of the six premises of valuation in the case of equal annual income, which is the type most frequently valued, and these tabulations, together with those of Chapter II for irregular annual income, are considered to be sufficient illustration of valuation premises which are not generally used in commercial practice. Tabulations showing the allocation of income to interest payment and retirement of investment will be given for each type of terminable income, both for the compound interest actuarial valuation premises of Division II and for the Hoskold type of sinking fund valuation premises of Division IV, as these are the more generally accepted commercial methods of valuation. Allocation tabulations, under the premises of Divisions I, II, III, and IV, are given for various types of regularly changing income and for deferred income.

Perpetual income formulas, with arithmetical solutions and illustrations, will be presented whenever the type of income under consideration may be valued in perpetuity. Not all of the perpetuity formulas will be accompanied by arithmetical tabulations showing income disposition from year to year, since many of these formulas, while correct mathematically, are commercial anachronisms. A perpetuity being defined as an everlasting investment, the creation of a sinking fund for the retirement of capital is out of place, but algebraically and arithmetically it is just as possible to value perpetual income by means of sinking fund premises, as it is by any other premises. The few arithmetical illustrations given for the sinking fund methods for valuing perpetual income are to be regarded as mathematical curiosities rather than as recommended practice. The commercially accepted method is by means of the premises of Division II, and emphasis will be given to arithmetical tabulations illustrating those premises as applied to perpetual income.

The valuation formulas will also be solved for the number of years (n) during which income of a stated rate of receipt

is required in order to return a stated value with an agreed rate of interest, or in some cases with an agreed discount for hazard deducted. Such solutions of valuation formulas in terms of n are possible in a limited number of cases, but cannot be determined for many types of income.

The solution of valuation formulas for the speculative rate of interest (r), other necessary quantities being known, is possible only with respect to valuation formulas of the Hoskold type of Division IV. With other premises corresponding solutions require "cut and try" methods. The statement of formulas of the Hoskold type solved in terms of r will be omitted, because r is never present in exponential form and the solution for r is, therefore, very simple.

Neither the solutions for n nor the solutions for r need arithmetical illustration, as values of V already determined may be used and the numerical values of the notation for n and r should be found when equations are solved for one or the other of them as the unknown quantity.

The only difference between the valuation formulas of this chapter and those of Chapter II lies in the time distribution of the income. Substitution of the proper symbols for yearly income in place of the A_1, A_2, A_3, etc., to A_n terms of the formulas for valuing irregular annual income, and summation of terms which follow any mathematical law of variation, will permit the derivation not only of all formulas of this chapter but of any other type of formula desired. For example, income might increase by equal annual amounts for a period of years after date of valuation, remain constant for a period of years following, and then decrease by equal annual percentages. A formula in three terms could be derived for valuing such income; but, unless the term of the investment should be of many years duration, it would be simpler and more expeditious to accomplish the valuation through direct use of the formula for irregular annual income rather than to obtain and use a derivation of another form of that formula.

In summation of the terms of the various series the following processes will be frequently encountered. The substitution of $1 + r$ for R, of $1 + e$ for E, and of $1 + j$ for J, or vice versa, will also be of frequent occurrence. These progressions will serve as illustrations:

$$1 + R + R^2 + R^3 + \cdots R^{n-1} = \frac{R^n - 1}{r}$$

$$1 + R + R^2 + R^3 + \cdots R^{n-2} = \frac{R^{n-1} - 1}{r}$$

$$\frac{1}{R} + \frac{1}{R^2} + \frac{1}{R^3} + \cdots \frac{1}{R^n} = \frac{R^n - 1}{rR^n}$$

$$W^{n-1} + W^{n-2}R + W^{n-3}R^2 + W^{n-4}R^3 + \cdots WR^{n-2} + R^{n-1} = \frac{R^n - W^n}{R - W}$$

$$W^{n-m} + W^{n-m-1}R + W^{n-m-2}R^2 + W^{n-m-3}R^3 + \cdots WR^{n-m-1} = W\frac{R^{n-m} - W^{n-m}}{R - W}$$

$$R^{n-1} + 2R^{n-2} + 3R^{n-3} + 4R^{n-5} + \cdots n - 1 = \frac{R^n - 1 - nr}{r^2}$$

$$R^{n-m-1} + 2R^{n-m-2} + 3R^{n-m-3} + 4R^{n-m-4} + \cdots n - m = \frac{R^{n-m+1} - R - (n - m)r}{r^2}$$

Equivalent terms, which occasionally will be found, are:

$$V + Vr\frac{R^n - 1}{r} = VR^n$$

$$V + Vr\frac{E^n - 1}{e} = V\frac{rE^n - r + e}{e}$$

At has been shown in Chapter II that the general valuation formulas of Premises I, II, III, and V are identical, hence, the formulas of these divisions for valuing similar types of income of any other time distribution must also be identical. The difference in the premises of these divisions could be shown by algebraic formulas for the allocation of income to payment of interest and retirement of capital; but it is shown to equal advantage in the income allocation tabulations which follow the statement of the algebraic formulas and their numerical solutions for the valuation of equal annual income.

Subdivision A

Valuation Formulas for Equal Annual Income.

Notation in addition to that given in Chapter II:

$A = 1{,}000 =$ the equal annual amount of income

Substitutions in the general formulas of Chapter II:

$$A = A_1 = A_2 = A_3 = A_4 = \cdots A_n$$

Divisions	Valuation formulas terminable income	Arithmetical solutions
I, II, III, V	$V = A\frac{R^n - 1}{rR^n}$	$V = 6{,}144.567$
IV	$V = A\frac{E^n - 1}{rE^n - r + e}$	$V = 5{,}455.807$
VI	$V = A\frac{J^n - 1}{rJ^n - r + j}$	$V = 5{,}686.084$

There are certain relations between income allocation formulas of Divisions I and II, which are most important, as knowledge of these relations will be of considerable advantage to the evaluator using the premises of Division II. These relations apply only to equal annual income beginning at the date of investment, and are not applicable to types of income other than the equal annual type. These relations are algebraically expressed as follows, notation being identical with that of Chapter II:

Returns	Division I	Division II
Capital return in yth year	$C_y = \frac{A}{R^y}$	$C_y = \frac{A}{R^{n-y+1}}$
Capital return in $n - y + 1$th year	$C_{n-y+1} = \frac{A}{R^{n-y+1}}$	$C_{n-y+1} = \frac{A}{R^y}$
Interest payment in yth year	$I_y = A\frac{R^y - 1}{R^y}$	$I_y = A\frac{R^{n-y+1} - 1}{R^{n-y+1}}$
Interest payment in $n - y + 1$th year	$I_{n-y+1} = A\frac{R^{n-y+1} - 1}{R^{n-y+1}}$	$I_{n-y+1} = A\frac{R^y - 1}{R^y}$

Let $y = 1$, and it follows that $n - y + 1 = n$. With these substitutions in the above formulas, they become:

Returns	Division I	Division II
Capital return in first year	$C_1 = \frac{A}{R}$	$C_1 = \frac{A}{R^n}$
Capital return in nth year	$C_n = \frac{A}{R^n}$	$C_n = \frac{A}{R}$
Interest payment in first year	$I_1 = A\frac{R - 1}{R}$	$I_1 = A\frac{R^n - 1}{R^n}$
Interest payment in nth year	$I_n = A\frac{R^n - 1}{R^n}$	$I_n = A\frac{R - 1}{R}$

The above formulas prove that the return of capital in the nth year under Division II premises is equivalent to the return of capital in the first year under Division I premises, that the return of capital in the second year under Division II premises is equivalent to the return of capital in the $(n - 1)$th year under Division I premises, and so on, until the return of capital in the nth year under Division I premises is equivalent

to the return of capital in the first year under Division II premises.

In other words, the tables for the value of 1 due n years hence may be used with Division II premises to show the return of capital in each year of the period of investment by reversing the order of the factors, stating the factor given in the nth year of the table as the first year's return of capital, and the factor stated in the first year of the table as the nth year's return of capital.

The interest payments by years under premises of Division II are likewise the same as those under premises of Division I with the order of the payments reversed.

INCOME ALLOCATION TABULATION, PREMISES I-A

Year	Capital Account			Income Allocation				Interest rate paid on outstanding capital: per cent
	Capital outstanding at beginning of year	Capital returned each year	Capital outstanding at end of year	Yearly income	Discount factor	Yearly capital return	Yearly interest paid to investor	
Column.....	A	B	C	D	F	G	H	I
Process.....	.....	B = G	A − B	Per notation	Table 2 App. B	D × F	D − G	H ÷ A
1	6,144.57	909.09	5,235.48	1,000.00	0.909091	909.09	90.91	1.480
2	5,235.48	826.45	4,409.03	1,000.00	0.826446	826.45	173.55	3.315
3	4,409.03	751.32	3,657.71	1,000.00	0.751315	751.32	248.68	5.640
4	3,657.71	683.01	2,974.70	1,000.00	0.683013	683.01	316.99	8.666
5	2,974.70	620.92	2,353.78	1,000.00	0.620921	620.92	379.08	12.743
6	2,353.78	564.47	1,789.31	1,000.00	0.564474	564.47	435.53	18.503
7	1,789.31	513.16	1,276.15	1,000.00	0.513158	513.16	486.84	27.208
8	1,276.15	466.51	809.64	1,000.00	0.466507	466.51	533.49	41.805
9	809.64	424.10	385.54	1,000.00	0.424098	424.10	575.90	71.130
10	385.54	385.54	000.00	1,000.00	0.385543	385.54	614.46	159.374
Totals.....	29,035.91	6,144.57	.....	10,000.00	.....	6,144.57	3,855.43	*13.278

* Average.

INCOME ALLOCATION TABULATION, PREMISES II-A

Year	Capital Account			Income Allocation			Interest rate paid on outstanding capital: per cent
	Capital outstanding at beginning of year	Capital returned each year	Capital outstanding at end of year	Yearly income	Yearly interest due investor	Yearly capital return	
Column.....	A	B	C	D	F	G	H
Process.....	.....	B = G	A − B	Per notation	10% of A	D − F	F ÷ A
1	6,144.57	385.54	5,759.02	1,000.00	614.46	385.54	10
2	5,759.02	424.10	5,334.93	1,000.00	575.90	424.10	10
3	5,334.93	466.51	4,868.42	1,000.00	533.49	466.51	10
4	4,868.42	513.16	4,355.26	1,000.00	486.84	513.16	10
5	4,355.26	564.47	3,790.79	1,000.00	435.53	564.47	10
6	3,790.79	620.92	3,169.87	1,000.00	379.08	620.92	10
7	3,169.87	683.01	2,486.85	1,000.00	316.99	683.01	10
8	2,486.85	751.32	1,735.54	1,000.00	248.68	751.32	10
9	1,735.54	826.45	909.09	1,000.00	173.55	826.45	10
10	909.09	909.09	000.00	1,000.00	90.91	909.09	10
Totals.....	38,554.34	6,144.57	.....	10,000.00	3,855.43	6,144.57	*10

* Average.

INCOME ALLOCATION TABULATION, PREMISES III-A

Year	Capital Account at Beginning of Year			Income Allocation at End of Year			Sinking Fund Account During Year			Interest rate paid on outstanding capital: per cent
	Total capital	Portion of capital in sinking fund	Portion of capital in speculative enterprise	Income	Interest due investor	Contributions to sinking fund	Amount in fund at beginning of year	Interest earned during year	Amount in fund at end of year	
Column	A	B	C	D	E	F	G	I	J	K
Process	From valuation formula	B = G	A − B	Per notation	10% of A	D − E	From J preceding year	10% of G	F + G + I	E ÷ A
1	6,144.57	000.00	6,144.57	1,000.00	614.46	385.54	000.00	000.00	385.54	10
2	6,144.57	385.54	5,759.03	1,000.00	614.45	385.55	385.54	38.55	809.64	10
3	6,144.57	809.64	5,334.93	1,000.00	614.46	385.54	809.64	80.97	1,276.15	10
4	6,144.57	1,276.15	4,868.42	1,000.00	614.46	385.54	1,276.15	127.62	1,789.31	10
5	6,144.57	1,789.31	4,355.26	1,000.00	614.46	385.54	1,789.31	178.93	2,353.78	10
6	6,144.57	2,353.78	3,790.79	1,000.00	614.46	385.54	2,353.78	235.38	2,974.70	10
7	6,144.57	2,974.70	3,169.87	1,000.00	614.45	385.55	2,974.70	297.47	3,657.72	10
8	6,144.57	3,657.72	2,486.85	1,000.00	614.46	385.54	3,657.72	365.77	4,409.03	10
9	6,144.57	4,409.03	1,735.54	1,000.00	614.45	385.55	4,409.03	440.90	5,235.48	10
10	6,144.57	5,235.48	909.09	1,000.00	614.46	385.54	5,235.48	523.55	6,144.57	10
Totals	61,445.70	.....	.....	10,000.00	6,144.57	3,855.43	.....	2,289.14	.....	*10

* Average.

INCOME ALLOCATION TABULATION, PREMISES IV-A

Year	Capital Account at Beginning of Year			Income Allocation at End of Year			Sinking Fund Account During Year			Interest rate paid on outstanding capital: per cent
	Total capital	Portion of capital in sinking fund	Portion of capital in speculative enterprise	Income	Interest due investor	Contributions to sinking fund	Amount in fund at beginning of year	Interest earned during year	Amount in fund at end of year	
Column	A	B	C	D	E	F	G	I	J	K
Process	From valuation formula	B = G	A − B	Per notation	10% of A	D − E	From J preceding year	4% of G	F + G + I	E ÷ A
1	5,455.81	000.00	5,455.81	1,000.00	545.58	454.42	000.00	000.00	454.42	10
2	5,455.81	454.42	5,001.39	1,000.00	545.58	454.42	454.42	18.18	927.02	10
3	5,455.81	927.02	4,528.79	1,000.00	545.58	454.42	927.02	37.08	1,418.52	10
4	5,455.81	1,418.52	4,037.29	1,000.00	545.58	454.42	1,418.52	56.74	1,929.68	10
5	5,455.81	1,929.68	3,526.13	1,000.00	545.59	454.41	1,929.68	77.19	2,461.28	10
6	5,455.81	2,461.28	2,994.53	1,000.00	545.58	454.42	2,461.28	98.45	3,014.15	10
7	5,455.81	3,014.15	2,441.66	1,000.00	545.58	454.42	3,014.15	120.57	3,589.14	10
8	5,455.81	3,589.14	1,866.67	1,000.00	545.58	454.42	3,589.14	143.56	4,187.12	10
9	5,455.81	4,187.12	1,268.69	1,000.00	545.58	454.42	4,187.12	167.49	4,809.03	10
10	5,455.81	4,809.03	646.78	1,000.00	545.58	454.42	4,809.03	192.36	5,455.81	10
Totals	54,558.10	.....	.....	10,000.00	5,455.81	4,544.19	.....	911.62	.....	*10

* Average.

INCOME ALLOCATION TABULATION, PREMISES V-A

Year	Capital Account at Beginning of Year			Income Allocation at End of Year				Sinking Fund Account During Year			Interest rate paid on total capital: per cent
	Total capital	Portion of capital in sinking fund	Portion of capital in speculative enterprise	Income	Interest paid to investor on capital in		Contributions to sinking fund	Amount in fund at beginning of year	Interest earned during year	Amount in fund at end of year	
					Sinking fund	Speculative enterprise					
Column	A	B	C	D	E	F	G	H	J	K	L
Process	From valuation formula	B = H	A − B	Per notation	4% of B	10% of C	D − E − F	From K preceding year	4% of H	G + H + J	$\frac{E + F}{A}$
1	6,144.57	000.00	6,144.57	1,000.00	000.00	614.46	385.54	000.00	000.00	385.54	10.000
2	6,144.57	385.54	5,759.03	1,000.00	15.42	575.90	408.68	385.54	15.42	809.64	9.624
3	6,144.57	809.64	5,334.93	1,000.00	32.39	533.49	434.12	809.64	32.39	1,276.15	9.209
4	6,144.57	1,276.15	4,868.42	1,000.00	51.05	486.84	462.11	1,276.15	51.05	1,789.31	8.754
5	6,144.57	1,789.31	4,355.26	1,000.00	71.57	435.53	492.90	1,789.31	71.57	2,353.78	8.253
6	6,144.57	2,353.78	3,790.79	1,000.00	94.15	379.08	526.77	2,353.78	94.15	2,974.70	7.702
7	6,144.57	2,974.70	3,169.87	1,000.00	118.99	316.99	564.02	2,974.70	118.99	3,657.71	7.095
8	6,144.57	3,657.71	2,486.86	1,000.00	146.31	248.68	605.01	3,657.71	146.31	4,409.03	6.428
9	6,144.57	4,409.03	1,735.54	1,000.00	176.36	173.55	650.09	4,409.03	176.36	5,235.48	5.695
10	6,144.57	5,235.48	909.09	1,000.00	209.42	90.91	699.67	5,235.48	209.42	6,144.57	4.888
Totals	61,445.70	22,891.34	38,554.36	10,000.00	915.66	3,855.43	5,228.91	.....	915.66	.....	*7.765

* Average.

INCOME ALLOCATION TABULATION, PREMISES VI-A

Year	Capital Account at Beginning of Year			Income Allocation at End of Year				Sinking Fund Account During Year			Interest rate paid on total capital: per cent
	Total capital	Portion of capital in sinking fund	Portion of capital in speculative enterprise	Income	Interest paid to investor on capital in		Contributions to sinking fund	Amount in fund at beginning of year	Interest earned during year	Amount in fund at end of year	
					Sinking fund	Speculative enterprise					
Column	A	B	C	D	E	F	G	H	J	K	L
Process	From valuation formula	B = H	A − B	Per notation	8 % of B	10 % of C	D − E − F	From K preceding year	4 % of H	G + H + J	$\frac{E+F}{A}$
1	5,686.08	000.00	5,686.08	1,000.00	000.00	568.61	431.39	000.00	000.00	431.39	10.000
2	5,686.08	431.39	5,254.69	1,000.00	34.51	525.47	440.02	431.39	17.26	888.67	9.848
3	5,686.08	888.67	4,797.41	1,000.00	71.09	479.74	449.17	888.67	35.54	1,373.38	9.687
4	5,686.08	1,373.38	4,312.70	1,000.00	109.87	431.27	458.86	1,373.38	54.93	1,887.17	9.517
5	5,686.08	1,887.17	3,798.91	1,000.00	150.97	379.89	469.14	1,887.17	75.48	2,431.79	9.336
6	5,686.08	2,431.79	3,254.29	1,000.00	194.54	325.43	480.03	2,431.79	97.27	3,009.09	9.145
7	5,686.08	3,009.09	2,676.99	1,000.00	240.73	267.70	491.57	3,009.09	120.37	3,621.03	8.942
8	5,686.08	3,621.03	2,065.05	1,000.00	289.68	206.51	503.81	3,621.03	144.84	4,269.68	8.726
9	5,686.08	4,269.68	1,416.40	1,000.00	341.57	141.64	516.79	4,269.68	170.79	4,957.26	8.498
10	5,686.08	4,957.26	728.82	1,000.00	396.58	72.89	530.53	4,957.26	198.29	5,686.08	8.256
Totals	56,860.80	22,869.46	33,991.34	10,000.00	1,829.54	3,399.15	4,771.31	.....	914.77	.....	*9.196

* Average.

Divisions	Valuation formulas, perpetual income	Arithmetical solutions
I, II, III, V	$V = \frac{A}{r}$	$V = 10{,}000$
IV	$V = \frac{A}{r}$	$V = 10{,}000$
VI	$V = \frac{A}{r}$	$V = 10{,}000$

According to any of the premises of valuation for perpetuities of equal annual income, the yearly income, or in this case 1,000, is just sufficient to pay the stipulated rate of interest, or 10%, on the value (V), or 10,000, and there will be neither increase nor decrease of capital. The fact that all contributions to the sinking fund will be zero under any sinking fund premises of valuation is equivalent to the elimination of the sinking fund for this particular type of income; but, in other types, such as deferred income, the sinking fund may become a dominant consideration in determination of value.

Solutions for n When V Is Known

Divisions	Formulas
I, II, III, V	$n = \frac{\log A - \log (A - Vr)}{\log R}$
IV	$n = \frac{\log A[- V(r - e)] - \log (A - Vr)}{\log E}$
VI	$n = \frac{\log [A - V(r - \mathrm{j})] - \log (A - Vr)}{\log J}$

Subdivision B

Valuation Formulas for Income Increasing or Decreasing by Equal Annual Amounts.

Notation in addition to that given in Chapter II:

A = the amount of income for the first year of the investment period, that is, for the first year after date of valuation.

U = the uniform amount of the yearly addition to, or deduction from, the yearly income.

Substitutions in the general formulas of Chapter II:

$A = A_1; A \pm U = A_2; A \pm 2U = A_3; A \pm 3U = A_4: \cdots$
$A \pm (n-1)U = A_n$

Numerical amounts used in arithmetical illustrations of valuation formulas are:

For terminable income, $A = 550$; $U = 100$
For perpetual income, $A = 1{,}000$; $U = 30$

The income in the arithmetical solutions is of the increasing type, where the amount (U) is added to income each year, both for the terminable and perpetual income valuation formula.

In any of the following formulas showing an alternative plus or minus sign, the plus sign applies to increasing income and the minus sign must be used with decreasing income.

Divisions	Valuation formulas terminable income	Arithmetical solutions
I, II, III, V	$V = A\frac{R^n - 1}{rR^n} \pm U\frac{R^n - 1 - nr}{r^2R^n}$	$V = 5{,}668.646$
IV	$V = A\frac{E^n - 1}{rE^n - r + e} \pm U\frac{E^n - 1 - en}{e(rE^n - r + e)}$	$V = 5{,}279.728$
VI	$V = A\frac{J^n - 1}{rJ^n - r + j} \pm U\frac{J^n - 1 - jn}{j(rJ^n - r + j)}$	$V = 5{,}414.293$

INCOME ALLOCATION TABULATION, PREMISES I-B

Year	Capital Account			Income Allocation				Interest rate paid on outstanding capital: per cent
	Capital outstanding at beginning of year	Capital returned each year	Capital outstanding at end of year	Yearly income	Discount factor	Yearly capital return	Yearly interest paid to investor	
Column.....	A	B	C	D	F	G	H	I
Process.....	.....	B = G	A − B	Per notation	Table 2 App. B	D × F	D − G	H ÷ A
1	5,668.65	500.00	5,168.65	550.00	0.909091	500.00	50.00	0.880
2	5,168.65	537.19	4,631.46	650.00	0.826446	537.19	112.81	2.183
3	4,631.46	563.49	4,067.97	750.00	0.751315	563.49	186.51	4.027
4	4,067.97	580.56	3,487.41	850.00	0.683013	580.56	269.44	6.623
5	3,487.41	589.88	2,897.53	950.00	0.620921	589.88	360.12	10.326
6	2,897.53	592.70	2,304.83	1,050.00	0.564474	592.70	457.30	15.782
7	2,304.83	590.13	1,714.70	1,150.00	0.513158	590.13	559.87	24.291
8	1,714.70	583.13	1,131.57	1,250.00	0.466507	583.13	666.87	38.891
9	1,131.57	572.53	559.04	1,350.00	0.424098	572.53	777.47	68.707
10	559.04	559.04	000.00	1,450.00	0.385543	559.04	890.96	159.374
Totals.....	31,631.81	5,668.65	.....	10,000.00	.....	5,668.65	4,331.35	*13.693

* Average.

INCOME ALLOCATION TABULATION, PREMISES II-B

Year	Capital Account			Income Allocation			Interest rate paid on outstanding capital: per cent
	Capital outstanding at beginning of year	Capital returned each year	Capital outstanding at end of year	Yearly income	Yearly interest due investor	Yearly capital return	
Column.....	A	B	C	D	F	G	H
Process.....	.....	B = G	A − B	Per notation	10% of A	D − F	F ÷ A
1	5,668.65	−16.86	5,685.51	550.00	566.86	−16.86	10
2	5,685.51	81.45	5,604.06	650.00	568.55	81.45	10
3	5,604.06	189.59	5,414.47	750.00	560.41	189.59	10
4	5,414.47	308.55	5,105.92	850.00	541.45	308.55	10
5	5,105.92	439.41	4,666.51	950.00	510.59	439.41	10
6	4,666.51	583.35	4,083.16	1,050.00	466.65	583.35	10
7	4,083.16	741.69	3,341.47	1,150.00	408.31	741.69	10
8	3,341.47	915.85	2,425.62	1,250.00	334.15	915.85	10
9	2,425.62	1,107.44	1,318.18	1,350.00	242.56	1,107.44	10
10	1,318.18	1,318.18	000.00	1,450.00	131.82	1,318.18	10
Totals.....	43,313.55	5,668.65	.....	10,000.00	4,331.35	5,668.65	*10

* Average.

INCOME ALLOCATION TABULATION, PREMISES III-B

Year	Capital Account at Beginning of Year			Income Allocation at End of Year			Sinking Fund Account During Year			Interest rate paid on outstanding capital: per cent
	Total capital	Portion of capital in sinking fund	Portion of capital in speculative enterprise	Income	Interest due investor	Contributions to sinking fund	Amount in fund at beginning of year	Interest earned during year	Amount in fund at end of year	
Column	A	B	C	D	E	F	G	I	J	K
Process	From valuation formula	B = G	A − B	Per notation	10% of A	D − E	From J preceding year	10% of G	F + G + I	E ÷ A
1	5,668.65	000.00	5,668.65	550.00	566.86	−16.86	000.00	000.00	−16.86	10
2	5,668.65	−16.86	5,685.51	650.00	566.87	83.13	−16.86	−1.69	64.58	10
3	5,668.65	64.58	5,604.07	750.00	566.86	183.14	64.58	6.46	254.18	10
4	5,668.65	254.18	5,414.47	850.00	566.87	283.13	254.18	25.42	562.73	10
5	5,668.65	562.73	5,105.92	950.00	566.86	383.14	562.73	56.27	1,002.14	10
6	5,668.65	1,002.14	4,666.51	1,050.00	566.87	483.13	1,002.14	100.22	1,585.49	10
7	5,668.65	1,585.49	4,083.16	1,150.00	566.87	583.13	1,585.49	158.55	2,327.17	10
8	5,668.65	2,327.17	3,341.48	1,250.00	566.86	683.14	2,327.17	232.72	3,243.03	10
9	5,668.65	3,243.03	2,425.62	1,350.00	566.87	783.13	3,243.03	324.30	4,350.46	10
10	5,668.65	4,350.46	1,318.19	1,450.00	566.86	883.14	4,350.46	435.05	5,668.65	10
Totals	56,686.50	.....	.....	10,000.00	5,668.65	4,331.35	.....	1,337.30	.....	*10

* Average.

INCOME ALLOCATION TABULATION, PREMISES IV-B

Year	Capital Account at Beginning of Year			Income Allocation at End of Year			Sinking Fund Account During Year			Interest rate paid on outstanding capital: per cent
	Total capital	Portion of capital in sinking fund	Portion of capital in speculative enterprise	Income	Interest due investor	Contributions to sinking fund	Amount in fund at beginning of year	Interest earned during year	Amount in fund at end of year	
Column	A	B	C	D	E	F	G	I	J	K
Process	From valuation formula	B = G	A − B	Per notation	10% of A	D − E	From J preceding year	4% of G	F + G + I	E ÷ A
1	5,279.73	000.00	5,279.73	550.00	527.97	22.03	000.00	000.00	22.03	10
2	5,279.73	22.03	5,257.70	650.00	527.97	122.03	22.03	0.88	144.94	10
3	5,279.73	144.94	5,134.79	750.00	527.98	222.02	144.94	5.80	372.76	10
4	5,279.73	372.76	4,906.97	850.00	527.97	322.03	372.76	14.91	709.70	10
5	5,279.73	709.70	4,570.03	950.00	527.98	422.02	709.70	28.39	1,160.11	10
6	5,279.73	1,160.11	4,119.62	1,050.00	527.97	522.03	1,160.11	46.40	1,728.54	10
7	5,279.73	1,728.54	3,551.19	1,150.00	527.97	622.03	1,728.54	69.14	2,419.71	10
8	5,279.73	2,419.71	2,860.02	1,250.00	527.97	722.03	2,419.71	96.79	3,238.53	10
9	5,279.73	3,238.53	2,041.20	1,350.00	527.97	822.03	3,238.53	129.54	4,190.10	10
10	5,279.73	4,190.10	1,089.63	1,450.00	527.98	922.02	4,190.10	167.61	5,279.73	10
Totals	52,797.30	.....	.....	10,000.00	5,279.73	4,720.27	.....	559.46	.....	*10

* Average.

Divisions	Valuation formulas perpetual income	Arithmetical solutions
I, II, III, V	$V = \frac{Ar \pm U}{r^2}$	$V = 13{,}000$
IV	$V = \frac{Ae \pm U}{er}$	$V = 17{,}500$
VI	$V = \frac{Aj \pm U}{jr}$	$V = 15{,}000$

In the above, the value (V) is negative when the value of U is negative and greater than the other term in the numerator, Ar, Ae, or Aj, as the case may be.

Considering Division II premises, the interest requirement at the end of the first year is \$1,300 of which \$1,000 is available from income. According to the premises, the balance of unpaid interest in the amount of \$300 must be added to capital. The \$30 added to income in the second year is just sufficient to pay interest on this added capital, and there is another interest deficiency of \$300 to be added to capital at the end of the second year, and so on, ad infinitum.

Under Division IV premises the income at the end of the first year will be \$750 less than the amount required to pay interest on outstanding capital, and this amount becomes a negative contribution to the sinking fund. The investor receives 4% interest on negative accumulations in the sinking fund, and the \$30 additional income per year is just sufficient to pay 4% interest on the yearly additions to the negative amount in the sinking fund.

Under Division VI premises, the following tabulation shows the processes involved:

PERPETUAL INCOME ALLOCATION TABULATION, PREMISES VI-B

Year	Capital Account			Income Allocation					Sinking Fund Account		
	Portion of capital in speculative enterprise at beginning of year	Portion of capital added to sinking fund	Portion of capital in speculative enterprise at end of year	Income for year	Interest Paid to Investor on Capital in			Contributions to sinking fund	Amount in fund at beginning of year	Interest earned during year	Amount in fund at end of year
					Sinking fund	Speculative enterprise	Total investment				
Column	A	B	C	D	E	F	G	H	I	J	K
Process	From valuation formula and col. C for preceding year	B = H + J	A + B	Per notation	8% of I	10% of A	E + F	D − G	From K preceding year	4% of I	H + I + J
1	15,000	−500	15,500	1,000	000	1,500	1,500	−500	000	000	− 500
2	15,500	−500	16,000	1,030	− 40	1,550	1,510	−480	− 500	−20	−1,000
3	16,000	−500	16,500	1,060	− 80	1,600	1,520	−460	−1,000	−40	−1,500
4	16,500	−500	17,000	1,090	−120	1,650	1,530	−440	−1,500	−60	−2,000
etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.

Subdivision C

Valuation Formulas for Income Increasing or Decreasing by Equal Annual Percentages.

Notation in addition to that given in Chapter II:

A = the amount of the income for the first year of the investment period.

w = the uniform percentage of the yearly increase or decrease from A.

$W = 1 \pm w$, the plus sign being required for increasing income and the minus sign for decreasing income.

Substitutions in the general formula of Chapter II:

$$A = A_1;\ AW = A_2;\ AW^2 = A_3;\ AW^3 = A_4;\ \cdots\ AW^{n-1} = A_n$$

Numerical amounts used in the arithmetical solutions will differ with the different applications and with successive simplifications of the valuation formulas. They will, therefore, be stated immediately preceding the arithmetical solutions of formulas to which they apply.

The group of formulas given first may be used under any conditions for the valuation of terminable income; but they are much more complex than those shown subsequently, to which certain limitations apply. The first group of formulas given must be used when the numerical value of W approaches equality, yet is not equal to the numerical value of R in Divisions I, II, III, and V, or of E in Division IV, or of J in Division VI. Much simpler formulas than the first group are stated thereafter, which may be used when the numerical value of W differs substantially from the numerical value of R, E, or J, as the case may be, and still simpler formulas apply when W becomes equal to R, E, or J.

Numerical amounts used in the arithmetical illustrations accompanying the first group of formulas are:

$$A = 569.842;\ w = 0.12;\ W = 1.12$$

Division	Expanded forms of valuation formulas terminable income	Arithmetical solutions
I, II, III, V	$V = \frac{A}{R^n}(R^{n-1} + WR^{n-2} + W^2R^{n-3} + \cdots W^{n-1})$	$V = 5{,}625.448$
IV	$V = \frac{Ae}{rE^n - r + e}(E^{n-1} + WE^{n-2} + W^2E^{n-3} + \cdots W^{n-1})$	$V = 5{,}261.817$
VI	$V = \frac{Aj}{rJ^n - r + j}(J^{n-1} + WJ^{n-2} + W^2J^{n-3} + \cdots W^{n-1})$	$V = 5{,}387.664$

INCOME ALLOCATION TABULATION, PREMISES I-C
$w = 0.12$; $W = 1.12$

Year	Capital Account			Income Allocation				Interest rate paid on outstanding capital: per cent
	Capital outstanding at beginning of year	Capital returned each year	Capital outstanding at end of year	Yearly income	Discount factor	Yearly capital return	Yearly interest paid to investor	
Column	A	B	C	D	F	G	H	I
Process	.....	B = G	A − B	Per notation	Table 2 App. B	D × F	D − G	H ÷ A
1	5,625.45	518.04	5,107.41	569.84	0.909091	518.04	51.80	0.921
2	5,107.41	527.46	4,579.95	638.22	0.826446	527.46	110.76	2.169
3	4,579.95	537.05	4,042.90	714.81	0.751315	537.05	177.76	3.881
4	4,042.90	546.81	3,496.09	800.58	0.683013	546.81	253.77	6.277
5	3,496.09	556.75	2,939.34	896.66	0.620921	556.75	339.91	9.723
6	2,939.34	566.88	2,372.46	1,004.26	0.564474	566.88	437.38	14.880
7	2,372.46	577.18	1,795.28	1,124.77	0.513158	577.18	547.59	23.081
8	1,795.28	587.68	1,207.60	1,259.74	0.466507	587.68	672.06	37.435
9	1,207.60	598.36	609.24	1,410.91	0.424098	598.36	812.55	67.286
10	609.24	609.24	000.00	1,580.21	0.385543	609.24	970.97	159.374
Totals	31,775.72	5,625.45	.....	10,000.00	.....	5,625.45	4,374.55	*13.767

* Average.

INCOME ALLOCATION TABULATION, PREMISES II-C
$w = 0.12$; $W = 1.12$

Year	Capital Account			Income Allocation			Interest rate paid on outstanding capital: per cent
	Capital outstanding at beginning of year	Capital returned each year	Capital outstanding at end of year	Yearly income	Yearly interest due investor	Yearly capital return	
Column	A	B	C	D	F	G	H
Process	.....	B = G	A − B	Per notation	10% of A	D − F	F ÷ A
1	5,625.45	7.30	5,618.15	569.84	562.54	7.30	10
2	5,618.15	76.41	5,541.74	638.22	561.81	76.41	10
3	5,541.74	160.63	5,381.11	714.81	554.18	160.63	10
4	5,381.11	262.48	5,118.63	800.58	538.10	262.48	10
5	5,118.63	384.79	4,733.84	896.66	511.87	384.79	10
6	4,733.84	530.87	4,202.97	1,004.26	473.39	530.87	10
7	4,202.97	704.47	3,498.50	1,124.77	420.30	704.47	10
8	3,498.50	909.89	2,588.61	1,259.74	349.85	909.89	10
9	2,588.61	1,152.05	1,436.56	1,410.91	258.86	1,152.05	10
10	1,436.56	1,436.56	000.00	1,580.21	143.65	1,436.56	10
Totals	43,745.56	5,625.45	.....	10,000.00	4,374.55	5,625.45	*10

* Average.

INCOME ALLOCATION TABULATION, PREMISES III-C

$w = 0.12$; $W = 1.12$

Year	Capital Account at Beginning of Year			Income Allocation at End of Year			Sinking Fund Account During Year			Interest rate paid on outstanding capital: per cent
	Total capital	Portion of capital in sinking fund	Portion of capital in speculative enterprise	Income	Interest due investor	Contributions to sinking fund	Amount in fund at beginning of year	Interest earned during year	Amount in fund at end of year	
Column	A	B	C	D	E	F	G	I	J	K
Process	From valuation formula	B = G	A − B	Per notation	10% of A	D − E	From J preceding year	10% of G	F + G + I	E ÷ A
1	5,625.45	000.00	5,625.45	569.84	562.54	7.30	000.00	000.00	7.30	10
2	5,625.45	7.30	5,618.15	638.22	562.55	75.67	7.30	0.73	83.70	10
3	5,625.45	83.70	5,541.75	714.81	562.54	152.27	83.70	8.37	244.34	10
4	5,625.45	244.34	5,381.11	800.58	562.54	238.04	244.34	24.43	506.81	10
5	5,625.45	506.81	5,118.64	896.66	562.54	334.12	506.81	50.68	891.61	10
6	5,625.45	891.61	4,733.84	1,004.26	562.55	441.71	891.61	89.16	1,422.48	10
7	5,625.45	1,422.48	4,202.97	1,124.77	562.55	562.22	1,422.48	142.25	2,126.95	10
8	5,625.45	2,126.95	3,498.50	1,259.74	562.55	697.19	2,126.95	212.70	3,036.84	10
9	5,625.45	3,036.84	2,588.61	1,410.91	562.55	848.36	3,036.84	303.68	4,188.88	10
10	5,625.45	4,188.88	1,436.57	1,580.21	562.54	1,017.67	4,188.88	418.90	5,625.45	10
Totals	56,254.50	.....	.....	10,000.00	5,625.45	4,374.55	.....	1,250.90	.....	*10

* Average.

INCOME ALLOCATION TABULATION, PREMISES IV-C

$w = 0.12$; $W = 1.12$

Year	Capital Account at Beginning of Year			Income Allocation at End of Year			Sinking Fund Account During Year			Interest rate paid on outstanding capital: per cent
	Total capital	Portion of capital in sinking fund	Portion of capital in speculative enterprise	Income	Interest due investor	Contributions to sinking fund	Amount in fund at beginning of year	Interest earned during year	Amount in fund at end of year	
Column	A	B	C	D	E	F	G	I	J	K
Process	From valuation formula	B = G	A − B	Per notation	10% of A	D − E	From J preceding year	4% of G	F + G + I	E ÷ A
1	5,261.82	000.00	5,261.82	569.84	526.18	43.66	000.00	000.00	43.66	10
2	5,261.82	43.66	5,218.16	638.22	526.18	112.04	43.66	1.75	157.45	10
3	5,261.82	157.45	5,104.37	714.81	526.19	188.62	157.45	6.30	352.37	10
4	5,261.82	352.37	4,909.45	800.58	526.18	274.40	352.37	14.10	640.87	10
5	5,261.82	640.87	4,620.95	896.66	526.18	370.48	640.87	25.63	1,036.98	10
6	5,261.82	1,036.98	4,224.84	1,004.26	526.18	478.08	1,036.98	41.48	1,556.54	10
7	5,261.82	1,556.54	3,705.28	1,124.77	526.19	598.58	1,556.54	62.26	2,217.38	10
8	5,261.82	2,217.38	3,044.44	1,259.74	526.18	733.56	2,217.38	88.69	3,039.63	10
9	5,261.82	3,039.63	2,222.19	1,410.91	526.18	884.73	3,039.63	121.58	4,045.94	10
10	5,261.82	4,045.94	1,215.88	1,580.21	526.18	1,054.03	4,045.94	161.85	5,261.82	10
Totals	52,618.20	.....	.....	10,000.00	5,261.82	4,738.18	.....	523.64	.....	*10

* Average.

Excepting the conditions that W is approaching equality with, or is equal to, R, E, and J, the next group of formulas of much simpler form may be used in place of the first group shown.

Numerical amounts used in the arithmetical illustration accompanying the next group of formulas are:

$$A = 1{,}663.204;\ w = -0.12;\ W = 0.88$$

Divisions	Valuation formulas terminable income	Arithmetical solutions
I, II, III, V	$V = \frac{A}{R^n}\frac{R^n - W^n}{R - W}$	$V = 6{,}748.267$
IV	$V = Ae\frac{E^n - W^n}{(E - W)(rE^n - r + e)}$	$V = 5{,}676.675$
VI	$V = Aj\frac{J^n - W^n}{(J - W)(rJ^n - r + j)}$	$V = 6{,}028.316$

INCOME ALLOCATION TABULATION, PREMISES I-C

$w = -0.12;\ W = 0.88$

Year	Capital Account			Income Allocation				Interest rate paid on outstanding capital: per cent
	Capital outstanding at beginning of year	Capital returned each year	Capital outstanding at end of year	Yearly income	Discount factor	Yearly capital return	Yearly interest paid to investor	
Column.....	A	B	C	D	F	G	H	I
Process.....	.....	B = G	A − B	Per notation	Table 2 App. B	D × F	D − G	H ÷ A
1	6,748.27	1,512.01	5,236.26	1,663.20	0.909091	1,512.01	151.19	2.240
2	5,236.26	1,209.60	4,026.66	1,463.62	0.826446	1,209.60	254.02	4.851
3	4,026.66	967.68	3,058.98	1,287.98	0.751315	967.68	320.30	7.954
4	3,058.98	774.15	2,284.83	1,133.43	0.683013	774.15	359.28	11.745
5	2,284.83	619.32	1,665.51	997.42	0.620921	619.32	378.10	16.548
6	1,665.51	495.45	1,170.06	877.73	0.564474	495.45	382.28	22.953
7	1,170.06	396.36	773.70	772.40	0.513158	396.36	376.04	32.139
8	773.70	317.09	456.61	679.71	0.466507	317.09	362.62	46.868
9	456.61	253.67	202.94	598.14	0.424098	253.67	344.47	75.441
10	202.94	202.94	000.00	526.37	0.385543	202.94	323.43	159.374
Totals.....	25,623.82	6,748.27	.....	10,000.00	.....	6,748.27	3,251.73	*12.690

* Average.

INCOME ALLOCATION TABULATION, PREMISES II-C

$w = -0.12$; $W = 0.88$

Year	Capital Account			Income Allocation			Interest rate paid on outstanding capital: per cent
	Capital outstanding at beginning of year	Capital returned each year	Capital outstanding at end of year	Yearly income	Yearly interest due investor	Yearly capital return	
Column	A	B	C	D	F	G	H
Process	.....	B = G	A − B	Per notation	10 % of A	D − F	F ÷ A
1	6,748.27	988.38	5,759.89	1,663.20	674.82	988.38	10
2	5,759.89	887.63	4,872.26	1,463.62	575.99	887.63	10
3	4,872.26	800.76	4,071.50	1,287.98	487.22	800.76	10
4	4,071.50	726.28	3,345.22	1,133.43	407.15	726.28	10
5	3,345.22	662.89	2,682.33	997.42	334.53	662.89	10
6	2,682.33	609.49	2,072.84	877.73	268.24	609.49	10
7	2,072.84	565.11	1,507.73	772.40	207.29	565.11	10
8	1,507.73	528.94	978.79	679.71	150.77	528.94	10
9	978.79	500.27	478.52	598.14	97.87	500.27	10
10	478.52	478.52	000.00	526.37	47.85	478.52	10
Totals	32,517.35	6,748.27	.....	10,000.00	3,251.73	6,748.27	*10

* Average.

INCOME ALLOCATION TABULATION, PREMISES III-C

$w = -0.12$; $W = 0.88$

Year	Capital Account at Beginning of Year			Income Allocation at End of Year			Sinking Fund Account During Year			Interest rate paid on outstanding capital: per cent
	Total capital	Portion of capital in sinking fund	Portion of capital in speculative enterprise	Income	Interest due investor	Contributions to sinking fund	Amount in fund at beginning of year	Interest earned during year	Amount in fund at end of year	
Column	A	B	C	D	E	F	G	I	J	K
Process	From valuation formula	B = G	A − B	Per notation	10 % of A	D − E	From J preceding year	10 % of G	F + G + I	E ÷ A
1	6,748.27	000.00	6,748.27	1,663.20	674.82	988.38	000.00	000.00	988.38	10
2	6,748.27	988.38	5,759.89	1,463.62	674.83	788.79	988.38	98.84	1,876.01	10
3	6,748.27	1,876.01	4,872.26	1,287.98	674.82	613.16	1,876.01	187.60	2,676.77	10
4	6,748.27	2,676.77	4,071.50	1,133.43	674.83	458.60	2,676.77	267.67	3,403.04	10
5	6,748.27	3,403.04	3,345.23	997.42	674.83	322.59	3,403.04	340.31	4,065.94	10
6	6,748.27	4,065.94	2,682.33	877.73	674.83	202.90	4,065.94	406.59	4,675.43	10
7	6,748.27	4,675.43	2,072.84	772.40	674.83	97.57	4,675.43	467.54	5,240.54	10
8	6,748.27	5,240.54	1,507.73	679.71	674.83	4.88	5,240.54	524.06	5,769.48	10
9	6,748.27	5,769.48	978.79	598.14	674.82	−76.68	5,769.48	576.95	6,269.75	10
10	6,748.27	6,269.75	478.52	526.37	674.83	−148.46	6,269.75	626.98	6,748.27	10
Totals	67,482.70	.....	.....	10,000.00	6,748.27	3,251.73	.....	3,496.54	.....	*10

* Average.

INCOME ALLOCATION TABULATION, PREMISES IV-C

$w = -0.12$; $W = 0.88$

Year	Capital Account at Beginning of Year			Income Allocation at End of Year			Sinking Fund Account During Year			Interest rate paid on outstanding capital: per cent
	Total capital	Portion of capital in sinking fund	Portion of capital in speculative enterprise	Income	Interest due investor	Contributions to sinking fund	Amount in fund at beginning of year	Interest earned during year	Amount in fund at end of year	
Column	A	B	C	D	E	F	G	I	J	K
Process	From valuation formula	B = G	A − B	Per notation	10 % of A	D − E	From J preceding year	4 % of G	F + G + I	E ÷ A
1	5,676.67	000.00	5,676.67	1,663.20	567.66	1,095.54	000.00	000.00	1,095.54	10
2	5,676.67	1,095.54	4,581.13	1,463.62	567.67	895.95	1,095.54	43.82	2,035.31	10
3	5,676.67	2,035.31	3,641.36	1,287.98	567.66	720.32	2,035.31	81.41	2,837.04	10
4	5,676.67	2,837.04	2,839.63	1,133.43	567.67	565.76	2,837.04	113.48	3,516.28	10
5	5,676.67	3,516.28	2,160.39	997.42	567.67	429.75	3,516.28	140.65	4,086.68	10
6	5,676.67	4,086.68	1,589.99	877.73	567.67	310.06	4,086.68	163.47	4,560.21	10
7	5,676.67	4,560.21	1,116.46	772.40	567.67	204.73	4,560.21	182.41	4,947.35	10
8	5,676.67	4,947.35	729.32	679.71	567.67	112.04	4,947.35	197.89	5,257.28	10
9	5,676.67	5,257.28	419.39	598.14	567.66	30.48	5,257.28	210.29	5,498.05	10
10	5,676.67	5,498.05	178.62	526.37	567.67	−41.30	5,498.05	219.92	5,676.67	10
Totals	56,766.70	.....	.....	10,000.00	5,676.67	4,323.33	.....	1,353.34	.....	*10

* Average.

The following perpetuity formulas are applicable when W is less than R, E, or J, as the case may be. When W is equal to, or greater than, these terms, the values for V become infinite, or negative.

Numerical values used for arithmetical solutions of this group of formulas are:

$A = 1{,}000;\ w = 0.02;\ W = 1.02$

Divisions	Valuation formulas perpetual income	Arithmetical solutions
I, II, III, V	$V = \frac{A}{R - W}$	$V = 12{,}500$
IV	$V = \frac{Ae}{r(E - W)}$	$V = 20{,}000$
VI	$V = \frac{Aj}{r(J - W)}$	$V = 15{,}000$

According to the premises of Division II, capital is increased, or decreased, annually by an amount upon which the additions to, or subtractions from, income will just cover interest at the speculative rate.

Discussion of the sinking fund premises will be omitted, since processes similar to those previously described will be operative.

SOLUTION FOR n WHEN V IS KNOWN

Divisions	Formulas
I, II, III, V	$n = \frac{\log A - \log [A - V(R - W)]}{\log R - \log W}$
*IV	$n = \frac{\log F + \log A - \log[FA - V(E - W)]}{\log E - \log W}$
VI	Cannot be solved for n

* Algebraic solution for n is possible only when the safety factor (F) is both known and constant, irrespective of the duration of the period n.

If (in the formulas of the first groups hown under Subdivision C) R and W are made equal, if E and W are made equal, and if J and W are made equal, they reduce to simplified forms of determinate type, as shown below, whereas, if these equalities are introduced into the formulas of the

second group shown, they, in turn, reduce to an indeterminate form.

In the arithmetical solutions of the simplified formulas referred to, the numerical values of A and W will differ with the different premises of valuation as follows:

For Divisions I, II, III, and V, $A = 627.454$; $w = 0.10$; $W = 1.10$
For Division IV $A = 832.909$; $w = 0.04$; $W = 1.04$
For Division VI $A = 758.680$; $w = 0.06$; $W = 1.06$

The numerical values for A are computed so that the total income for the 10-year period is 10,000 in each case, and the numerical values for W are respectively equal to the numerical values used for R, E, and J.

Divisions	Valuation formulas terminable income	Arithmetical solutions
I, II, III, V	$V = \frac{An}{R}$	$V = 5{,}704.127$
IV	$V = \frac{AneE^{n-1}}{rE^n - r + e}$	$V = 5{,}387.095$
VI	$V = \frac{AnjJ^{n-1}}{rJ^n - r + j}$	$V = 5{,}529.462$

The above three formulas cannot be made subject to perpetual income, for the reason that the value (V) becomes infinite when n is infinite.

INCOME ALLOCATION TABULATION, PREMISES I-C

$W = R = 1.10$

Year	Capital Account			Income Allocation				Interest rate paid on outstanding capital: per cent
	Capital outstanding at beginning of year	Capital returned each year	Capital outstanding at end of year	Yearly income	Discount factor	Yearly capital return	Yearly interest paid to investor	
Column.....	A	B	C	D	F	G	H	I
Process.....	.....	B = G	A − B	Per notation	Table 2 App. B	D × F	D − G	H ÷ A
1	5,704.13	570.41	5,133.72	627.45	0.909091	570.41	57.04	1.000
2	5,133.72	570.41	4,563.31	690.20	0.826446	570.41	119.79	2.333
3	4,563.31	570.42	3,992.89	759.22	0.751315	570.42	188.80	4.137
4	3,992.89	570.41	3,422.48	835.14	0.683013	570.41	264.73	6.630
5	3,422.48	570.41	2,852.07	918.66	0.620921	570.41	348.25	10.175
6	2,852.07	570.42	2,281.65	1,010.52	0.564474	570.42	440.10	15.431
7	2,281.65	570.41	1,711.24	1,111.57	0.513158	570.41	541.16	23.718
8	1,711.24	570.41	1,140.83	1,222.73	0.466507	570.41	652.32	38.120
9	1,140.83	570.42	570.41	1,345.00	0.424098	570.42	774.58	67.896
10	570.41	570.41	000.00	1,479.51	0.385543	570.41	909.10	159.374
Totals.....	31,372.73	5,704.13	.....	10,000.00	.....	5,704.13	4,295.87	*13.693

* Average.

INCOME ALLOCATION TABULATION, PREMISES II-C

$W = R = 1.10$

Year	Capital Account			Income Allocation			Interest rate paid on outstanding capital: per cent
	Capital outstanding at beginning of year	Capital returned each year	Capital outstanding at end of year	Yearly income	Yearly interest due investor	Yearly capital return	
Column.....	A	B	C	D	F	G	H
Process.....	.....	B = G	A − B	Per notation	10 % of A	D − F	F ÷ A
1	5,704.13	57.04	5,647.09	627.45	570.41	57.04	10
2	5,647.09	125.49	5,521.60	690.20	564.71	125.49	10
3	5,521.60	207.06	5,314.54	759.22	552.16	207.06	10
4	5,314.54	303.69	5,010.85	835.14	531.45	303.69	10
5	5,010.85	417.57	4,593.28	918.66	501.09	417.57	10
6	4,593.28	551.19	4,042.09	1,010.52	459.33	551.19	10
7	4,042.09	707.37	3,334.72	1,111.57	404.20	707.37	10
8	3,334.72	889.26	2,445.46	1,222.73	333.47	889.26	10
9	2,445.46	1,100.46	1,345.00	1,345.00	244.54	1,100.46	10
10	1,345.00	1,345.00	000.00	1,479.51	134.51	1,345.00	10
Totals.....	42,958.76	5,704.13	.....	10,000.00	4,295.87	5,704.13	*10

* Average.

INCOME ALLOCATION TABULATION, PREMISES III-C

$W = R = 1.10$

Year	Capital Account at Beginning of Year			Income Allocation at End of Year			Sinking Fund Account During Year			Interest rate paid on outstanding capital: per cent
	Total capital	Portion of capital in sinking fund	Portion of capital in speculative enterprise	Income	Interest due investor	Contributions to sinking fund	Amount in fund at beginning of year	Interest earned during year	Amount in fund at end of year	
Column	A	B	C	D	E	F	G	I	J	K
Process	From valuation formula	B = G	A − B	Per notation	10 % of A	D − E	From J preceding year	10 % of G	F + G + I	E ÷ A
1	5,704.13	000.00	5,704.13	627.45	570.41	57.04	000.00	000.00	57.04	10
2	5,704.13	57.04	5,647.09	690.20	570.41	119.79	57.04	5.70	182.53	10
3	5,704.13	182.53	5,521.60	759.22	570.41	188.81	182.53	18.25	389.59	10
4	5,704.13	389.59	5,314.54	835.14	570.41	264.73	389.59	38.96	693.28	10
5	5,704.13	693.28	5,010.85	918.66	570.42	348.24	693.28	69.33	1,110.85	10
6	5,704.13	1,110.85	4,593.28	1,010.52	570.41	440.11	1,110.85	111.08	1,662.04	10
7	5,704.13	1,662.04	4,042.09	1,111.57	570.41	541.16	1,662.04	166.21	2,369.41	10
8	5,704.13	2,369.41	3,334.72	1,222.73	570.41	652.32	2,369.41	236.94	3,258.67	10
9	5,704.13	3,258.67	2,445.46	1,345.00	570.42	774.58	3,258.67	325.87	4,359.12	10
10	5,704.13	4,359.12	1,345.01	1,479.51	570.42	909.09	4,359.12	435.92	5,704.13	10
Totals	57,041.30	.....	.....	10,000.00	5,704.13	4,295.87	.....	1,408.26	.....	*10

* Average.

INCOME ALLOCATION TABULATION, PREMISES IV-C

$W = E = 1.04$

Year	Capital Account at Beginning of Year			Income Allocation at End of Year			Sinking Fund Account During Year			Interest rate paid on outstanding capital: per cent
	Total capital	Portion of capital in sinking fund	Portion of capital in speculative enterprise	Income	Interest due investor	Contributions to sinking fund	Amount in fund at beginning of year	Interest earned during year	Amount in fund at end of year	
Column	A	B	C	D	E	F	G	I	J	K
Process	From valuation formula	B = G	A − B	Per notation	10 % of A	D − E	From J preceding year	4 % of G	F + G + I	E ÷ A
1	5,387.09	000.00	5,387.09	832.91	538.71	294.20	000.00	000.00	294.20	10
2	5,387.09	294.20	5,092.89	866.23	538.71	327.52	294.20	11.76	633.48	10
3	5,387.09	633.48	4,753.61	900.87	538.70	362.17	633.48	25.34	1,020.99	10
4	5,387.09	1,020.99	4,366.10	936.91	538.71	398.20	1,020.99	40.84	1,460.03	10
5	5,387.09	1,460.03	3,927.06	974.39	538.71	435.68	1,460.03	58.40	1,954.11	10
6	5,387.09	1,954.11	3,432.98	1,013.36	538.71	474.65	1,954.11	78.16	2,506.92	10
7	5,387.09	2,506.92	2,880.17	1,053.90	538.71	515.19	2,506.92	100.28	3,122.39	10
8	5,387.09	3,122.39	2,264.70	1,096.05	538.71	557.34	3,122.39	124.89	3,804.62	10
9	5,387.09	3,804.62	1,582.47	1,139.89	538.71	601.18	3,804.62	152.19	4,557.99	10
10	5,387.09	4,557.99	829.10	1,185.49	538.71	646.78	4,557.99	182.32	5,387.09	10
Totals	53,870.90	.....	.....	10,000.00	5,387.09	4,612.91	.....	774.18	.....	*10

* Average.

SOLUTIONS FOR n WHEN V IS KNOWN

Divisions	Formulas
I, II, III, V	$n = \frac{VR}{A}$
*IV	$n = \frac{VE}{FA}$
VI	Cannot be solved for n

* Algebraic solution is possible only when the factor of safety (F) is both known and constant, irrespective of the duration of the period (n).

SUBDIVISION D

Valuation Formulas for Income Subject to Intermittent Changes from One Amount of Equal Annual Income for One Period to Another Amount of Equal Annual Income for a Successive Period.

It would be equally feasible to develop formulas for differing amounts and percentage rates of income; yearly changes within successive periods of years, such as income increasing at 2% per annum for the first period, at 4% per annum for the second period, and decreasing at 10% per annum for the third period, etc.; but formulas of that type would be rarely needed. On the other hand, it frequently happens that an evaluator is able to predict the amount of income changes from one level to another as additions to plant facilities are completed. The following formulas are for two changes in the level of the income:

Notation in addition to that given in Chapter II:

m and k = specified successive periods of years after date of investment, both within the total period n, that is, $m + k$ is less than n.

Notation changed from that used in Chapter II:

A_1 = the equal annual income for the first period of m years.

A_2 = the equal annual income for the succeeding period of k years.

A_3 = the equal annual income for the remaining period of $n - m - k$ years.

Numerical values used for arithmetical solutions of this group of formulas are:

$$A_1 = 500;\ A_2 = 1{,}000;\ A_3 = 1{,}200;\ m = 2;\ k = 3$$

Divisions	Valuation formulas terminable income	Arithmetical solutions
I, II, III, V	$V = A_1\frac{R^m - 1}{rR^m} + A_2\frac{R^k - 1}{rR^{m+k}} + A_3\frac{R^{n-m-k} - 1}{rR^n}$	$V = 5{,}747.555$
IV	$V = \frac{1}{rE^n - r + c}\left[A_1(E^n - E^{n-m}) + A_2(E^{n-m} - E^{n-m-k}) + A_3(E^{n-m-k} - 1)\right]$	$V = 5{,}313.721$
VI	$V = \frac{1}{rJ^n - r + j}\left[A_1(J^n - J^{n-m}) + A_2(J^{n-m} - J^{n-m-k}) + A_3(J^{n-m-k} - 1)\right]$	$V = 5{,}464.243$

From the formulas shown above, it should be possible for the evaluator to write other formulas for any number of changes in the level of income receipt, with successive periods of years.

INCOME ALLOCATION TABULATION, PREMISES II-D

Year	Capital Account			Income Allocation			Interest rate paid on outstanding capital: per cent
	Capital outstanding at beginning of year	Capital returned each year	Capital outstanding at end of year	Yearly income	Yearly interest due investor	Yearly capital return	
Column.....	A	B	C	D	F	G	H
Process.....	.....	B = G	A − B	Per notation	10 % of A	D − F	F ÷ A
1	5,747.55	−74.76	5,822.31	500.00	574.76	−74.76	10
2	5,822.31	−82.23	5,904.54	500.00	582.23	−82.23	10
3	5,904.54	409.55	5,494.99	1,000.00	590.45	409.55	10
4	5,494.99	450.50	5,044.49	1,000.00	549.50	450.50	10
5	5,044.49	495.55	4,548.94	1,000.00	504.45	495.55	10
6	4,548.94	745.10	3,803.84	1,200.00	454.90	745.10	10
7	3,803.84	819.62	2,984.22	1,200.00	380.38	819.62	10
8	2,984.22	901.58	2,082.64	1,200.00	298.42	901.58	10
9	2,082.64	991.73	1,090.91	1,200.00	208.27	991.73	10
10	1,090.91	1,090.91	000.00	1,200.00	109.09	1,090.91	10
Totals.....	42,524.43	5,747.55	.....	10,000.00	4,252.45	5,747.55	*10

* Average.

INCOME ALLOCATION TABULATION, PREMISES IV-D

Year	Capital Account at Beginning of Year			Income Allocation at End of Year			Sinking Fund Account During Year			Interest rate paid on outstanding capital: per cent
	Tota. capital	Portion of capital in sinking fund	Portion of capital in speculative enterprise	Income	Interest due investor	Contributions to sinking fund	Amount in fund at beginning of year	Interest earned during year	Amount in fund at end of year	
Column	A	B	C	D	E	F	G	I	J	K
Process	From valuation formula	B = G	A − B	Per notation	10 % of A	D − E	From J preceding year	4 % of G	F + G + I	E ÷ A
1	5,313.72	000.00	5,313.72	500.00	531.37	−31.37	000.00	000.00	−31.37	10
2	5,313.72	−31.37	5,345.09	500.00	531.37	−31.37	−31.37	−1.26	−64.00	10
3	5,313.72	−64.00	5,377.72	1,000.00	531.37	468.63	−64.00	−2.56	402.07	10
4	5,313.72	402.07	4,911.65	1,000.00	531.37	468.63	402.07	16.08	886.78	10
5	5,313.72	886.78	4,426.94	1,000.00	531.37	468.63	886.78	35.47	1,390.88	10
6	5,313.72	1,390.88	3,922.84	1,200.00	531.38	668.62	1,390.88	55.64	2,115.14	10
7	5,313.72	2,115.14	3,198.58	1,200.00	531.37	668.63	2,115.14	84.61	2,868.38	10
8	5,313.72	2,868.38	2,445.34	1,200.00	531.37	668.63	2,868.38	114.73	3,651.74	10
9	5,313.72	3,651.74	1,661.98	1,200.00	531.37	668.63	3,651.74	146.07	4,466.44	10
10	5,313.72	4,466.44	847.28	1,200.00	531.38	668.62	4,466.44	178.66	5,313.72	10
Totals	53,137.20	.....	.....	10,000.00	5,313.72	4,686.28	.....	627.44	.....	*10

* Average.

Subdivision E

Deferment Valuation Formulas: No Income during the First d Years of the Investment Period, Followed by n-d Years of Equal Annual Income.

Deferment formulas are stated only for deferred equal annual income, since the infrequent commercial necessity for the other types of deferment formulas does not justify their development. When any other type of deferred income is valued, the evaluator may use the general formula of Chapter II or derive from that formula the one desired.

Notation in addition to that given in Chapter II:

A = the amount of equal annual income for each year after the dth year.
d = the number of years of deferment, or years of no income.

Numerical values used for arithmetical solutions of this group of formulas are:

$$A = 1{,}250;\ d = 2$$

Divisions	Valuation formulas terminable income	Arithmetical solutions
I, II, III, V	$V = \frac{A}{R^d} \cdot \frac{R^{n-d} - 1}{rR^{n-d}} = A\frac{R^{n-d} - 1}{rR^n}$	$V = 5{,}511.287$
IV	$V = A\frac{E^{n-d} - 1}{rE^n - r + e} = \frac{eE^n}{rE^n - r + e} \cdot A\frac{E^{n-d} - 1}{eE^n}$	$V = 5{,}233.903$
VI	$V = A\frac{J^{n-d} - 1}{rJ^n - r + j} = \frac{jJ^n}{rJ^n - r + j} \cdot A\frac{J^{n-d} - 1}{jJ^n}$	$V = 5{,}337.106$

When the premises of Division IV are modified to provide that interest shall be paid to the investor at the speculative rate upon negative sinking fund accumulations of the deferment period, which is the method used by Hoskold for valuing deferred income of equal annual amounts, the valuation formula for such deferred income may be obtained from the general formulas under Division H of Chapter II, or by direct derivation as shown in that chapter, or it may be derived from the following series, showing by years sinking fund contributions plus interest accretions.

1st year: $O - Vr = S_1$
2nd year: $O - VrR = S_2$
3rd year: $O - VrR^2 = S_3$
4th year: $O - VrR^3 = S_4$
dth year: $O - VrR^{d-1} = S_d$
$(d + 1)$th year: $A - VrR^d = S_{d+1}$
$(d + 2)$th year: $(A - VrR^d)E = S_{d+2}$
$(d + 3)$th year: $(A - VrR^d)E^2 = S_{d+3}$
$(d + 4)$th year: $(A - VrR^d)E^3 = S_{d+4}$
nth year: $(A - VrR^d)E^{n-d-1} = S_n$

The sum of the terms of the above series gives the formula derived in Chapter II, or:

$$V = A\frac{E^{n-d} - 1}{rE^{n-d} - r + c} \cdot \frac{1}{R^d}$$ (Arithmetical solution, $V = 4{,}539.588$, where $A = 1666.667$ and $d = 4$)

INCOME ALLOCATION TABULATION, PREMISES I-E

Year	Capital Account			Income Allocation				Interest rate paid on outstanding capital: per cent
	Capital outstanding at beginning of year	Capital returned each year	Capital outstanding at end of year	Yearly income	Discount factor	Yearly capital return	Yearly interest paid to investor	
Column.....	A	B	C	D	F	G	H	I
Process.....	.....	B = G	A − B	Per notation	Table 2 App. B	D × F	D − G	H ÷ A
1	5,511.29	000.00	5,511.29	000.00	0.909091	000.00	000.00	0.000
2	5,511.29	000.00	5,511.29	000.00	0.826446	000.00	000.00	0.000
3	5,511.29	939.14	4,572.15	1,250.00	0.751315	939.14	310.86	5.640
4	4,572.15	853.77	3,718.38	1,250.00	0.683013	853.77	396.23	8.666
5	3,718.38	776.15	2,942.23	1,250.00	0.620921	776.15	473.85	12.743
6	2,942.23	705.59	2,236.64	1,250.00	0.564474	705.59	544.41	18.503
7	2,236.64	641.45	1,595.19	1,250.00	0.513158	641.45	608.55	27.208
8	1,595.19	583.14	1,012.05	1,250.00	0.466507	583.14	666.86	41.804
9	1,012.05	530.12	481.93	1,250.00	0.424098	530.12	719.88	71.131
10	481.93	481.93	000.00	1,250.00	0.385543	481.93	768.07	159.374
Totals.....	33,092.44	5,511.29	.....	10,000.00	.....	5,511.29	4,488.71	*13.564

* Average.

INCOME ALLOCATION TABULATION, PREMISES II-E

Year	Capital Account			Income Allocation			Interest rate paid on outstanding capital: per cent
	Capital outstanding at beginning of year	Capital returned each year	Capital outstanding at end of year	Yearly income	Yearly interest due investor	Yearly capital return	
Column.....	A	B	C	D	F	G	H
Process.....	.....	B = G	A − B	Per notation	10 % of A	D − F	F ÷ A
1	5,511.29	−551.13	6,062.42	000.00	551.13	−551.13	10
2	6,062.42	−606.24	6,668.66	000.00	606.24	−606.24	10
3	6,668.66	583.14	6,085.52	1,250.00	666.86	583.14	10
4	6,085.52	641.45	5,444.07	1,250.00	608.55	641.45	10
5	5,444.07	705.59	4,738.48	1,250.00	544.41	705.59	10
6	4,738.48	776.15	3,962.33	1,250.00	473.85	776.15	10
7	3,962.33	853.77	3,108.56	1,250.00	396.23	853.77	10
8	3,108.56	939.14	2,169.42	1,250.00	310.86	939.14	10
9	2,169.42	1,033.06	1,136.36	1,250.00	216.94	1,033.06	10
10	1,136.36	1,136.36	000.00	1,250.00	113.64	1,136.36	10
Totals.....	44,887.11	5,511.29	.....	10,000.00	4,488.71	5,511.29	*10

* Average.

INCOME ALLOCATION TABULATION, PREMISES III-E

Year	Capital Account at Beginning of Year			Income Allocation at End of Year			Sinking Fund Account During Year			Interest rate paid on outstanding capital: per cent
	Total capital	Portion of capital in sinking fund	Portion of capital in speculative enterprise	Income	Interest due investor	Contributions to sinking fund	Amount in fund at beginning of year	Interest earned during year	Amount in fund at end of year	
Column	A	B	C	D	E	F	G	I	J	K
Process	From valuation formula	B = G	A − B	Per notation	10 % of A	D − E	From J preceding year	10 % of G	F + G + I	E ÷ A
1	5,511.29	000.00	5,511.29	000.00	551.13	−551.13	000.00	000.00	−551.13	10
2	5,511.29	− 551.13	6,062.42	000.00	551.13	−551.13	− 551.13	− 55.11	−1,157.37	10
3	5,511.29	−1,157.37	6,668.66	1,250.00	551.13	698.87	−1,157.37	−115.74	− 574.24	10
4	5,511.29	− 574.24	6,085.53	1,250.00	551.13	698.87	− 574.24	− 57.42	67.21	10
5	5,511.29	67.21	5,444.08	1,250.00	551.13	698.87	67.21	6.72	772.80	10
6	5,511.29	772.80	4,738.49	1,250.00	551.12	698.88	772.80	77.28	1,548.96	10
7	5,511.29	1,548.96	3,962.33	1,250.00	551.13	698.87	1,548.96	154.89	2,402.72	10
8	5,511.29	2,402.72	3,108.57	1,250.00	551.13	698.87	2,402.72	240.28	3,341.87	10
9	5,511.29	3,341.87	2,169.42	1,250.00	551.13	698.87	3,341.87	334.19	4,374.92	10
10	5,511.29	4,374.92	1,136.37	1,250.00	551.13	698.87	4,374.92	437.49	5,511.29	10
Totals	55,112.90	.....	.....	10,000.00	5,511.29	4,488.71	.....	1,022.58	.....	*10

* Average.

INCOME ALLOCATION TABULATION, PREMISES IV-E

Year	Capital Account at Beginning of Year			Income Allocation at End of Year			Sinking Fund Account During Year			Interest rate paid on outstanding capital: per cent
	Total capital	Portion of capital in sinking fund	Portion of capital in speculative enterprise	Income	Interest due investor	Contributions to sinking fund	Amount in fund at beginning of year	Interest earned during year	Amount in fund at end of year	
Column	A	B	C	D	E	F	G	I	J	K
Process	From valuation formula	B = G	A − B	Per notation	10 % of A	D − E	From J preceding year	4 % of G	F + G + I	E ÷ A
1	5,233.90	000.00	5,233.90	000.00	523.39	−523.39	000.00	000.00	− 523.39	10
2	5,233.90	− 523.39	5,757.29	000.00	523.39	−523.39	− 523.39	−20.94	−1,067.72	10
3	5,233.90	−1,067.72	6,301.62	1,250.00	523.39	726.61	−1,067.72	−42.71	− 383.82	10
4	5,233.90	− 383.82	5,617.72	1,250.00	523.39	726.61	− 383.82	−15.35	327.44	10
5	5,233.90	327.44	4,906.46	1,250.00	523.39	726.61	327.44	13.10	1,067.15	10
6	5,233.90	1,067.15	4,166.75	1,250.00	523.39	726.61	1,067.15	42.69	1,836.45	10
7	5,233.90	1,836.45	3,397.45	1,250.00	523.39	726.61	1,836.45	73.45	2,636.51	10
8	5,233.90	2,636.51	2,597.39	1,250.00	523.39	726.61	2,636.51	105.46	3,468.58	10
9	5,233.90	3,468.58	1,765.32	1,250.00	523.39	726.61	3,468.58	138.74	4,333.93	10
10	5,233.90	4,333.93	899.97	1,250.00	523.39	726.61	4,333.93	173.36	5,233.90	10
Totals	52,339.00	.....	.....	10,000.00	5,233.90	4,766.10	.....	467.80	.....	*10

* Average.

INCOME ALLOCATION TABULATION, PREMISES IV-H
HERE, (*d*) IS CHANGED TO 4 AND EQUAL ANNUAL INCOME TO 1666.67

Year	Capital Account			Income Allocation at End of Year			Sinking Fund Account During Year			Sinking fund deficiencies added to capital	Interest rate paid on outstanding capital: per cent
	Capital outstanding at beginning of year	Added to capital at end of year	Capital outstanding at end of year	Income	Interest due investor	Contributions to sinking fund	Amount in fund at beginning of year	Interest earned during year	Amount in fund at end of year		
Column	A	B	C	D	E	F	G	I	J	K	L
Process	From val. formula and C preceding yr.	From K same yr.	A + B	Per notation	10 % of A	D − E	Positive amts. from J preceding year	4 % of G	F + G + I	Negative amounts in column J	E ÷ A
1	4,539.59	453.96	4,993.55	000.00	453.96	−453.96	000.00	000.00	−453.96	453.96	10
2	4,993.55	499.35	5,492.90	000.00	499.35	−499.35	000.00	000.00	−499.35	499.35	10
3	5,492.90	549.29	6,042.19	000.00	549.29	−549.29	000.00	000.00	−549.29	549.29	10
4	6,042.19	604.22	6,646.41	000.00	604.22	−604.22	000.00	000.00	−604.22	604.22	10
5	6,646.41	000.00	6,646.41	1,666.67	664.64	1,002.03	000.00	000.00	1,002.03	000.00	10
6	6,646.41	000.00	6,646.41	1,666.66	664.64	1,002.02	1,002.03	40.08	2,044.13	000.00	10
7	6,646.41	000.00	6,646.41	1,666.66	664.64	1,002.02	2,044.13	81.77	3,127.92	000.00	10
8	6,646.41	000.00	6,646.41	1,666.67	664.64	1,002.03	3,127.92	125.12	4,255.07	000.00	10
9	6,646.41	000.00	6,646.41	1,666.67	664.65	1,002.02	4,255.07	170.20	5,427.29	000.00	10
10	6,646.41	000.00	6,646.41	1,666.67	664.64	1,002.03	5,427.29	217.09	6,646.41	000.00	10
Totals	60,946.69	2,106.82	63,053.51	10,000.00	6,094.67	3,905.33	.....	634.26	.....	2,106.82	*10

* Average.

Numerical values used in the following arithmetical solutions are:

$A = 1{,}000; d = 10$

Divisions	Valuation formulas deferred perpetual income	Arithmetical solutions
I, II, III, V	$V = \frac{A}{rR^d}$	$V = 3{,}855.434$
IV	$V = \frac{A}{rE^d}$	$V = 6{,}755.643$
VI	$V = \frac{A}{rJ^d}$	$V = 5{,}583.947$

Note.—When the investor receives a speculative rate of interest upon negative accumulations in the sinking fund, the above Division IV Formula becomes:

$$V = \frac{A}{rR^d}$$

as in Divisions I, II, III, and V.

Under the premises of Division II, the initial value of 3,855.434 increased by 10% interest compounded annually for 10 years, becomes 10,000 at the end of the tenth year, and the yearly income of 1,000 will just pay the stipulated rate of interest of 10% upon that value.

Under the premises of Division IV, the value of 6,755.643 increased by 4% interest, compounded annually, for 10 years, amounts to 10,000 at the end of the tenth year, and the yearly income of 1,000 will just pay the stipulated rate of 10% upon this amount.

SOLUTIONS FOR n WHEN V IS KNOWN

Divisions	Formulas
I, II, III, V	$n = d + \frac{\log A - \log (A - VrR^d)}{\log R}$
IV	$n = d + \frac{\log [A - V(r - e)] - \log (A - VrE^d)}{\log E}$
VI	$n = d + \frac{\log [A - V(r - j)] - \log (A - VrJ^d)}{\log J}$

Subdivision F

Valuation Formulas for Income Deferred for a Period of Years and Requiring Equal Annual Capital Expenditures during Deferment Period in Order That Income May Be Realized.

Formulas for this type are stated only for equal annual income following the deferment period. The treatment of capital additions during the deferment period will be identical, regardless of the type of income which follows such period, and formulas for any other time distribution of income may be derived from the general formula as occasion for their use arises.

Notation in addition to that given in Chapter II:

A = the amount of equal annual income, the receipt of which begins after d years of no income, or deferment.
d = the number of years which will elapse before the receipt of income begins.
Q = the total of the capital expenditures which is spread uniformly over the first d years after the date of investment, that is, $\frac{Q}{d}$ is the yearly capital investment considered as negative income for that period.

Numerical values used for the following arithmetical solutions are:

$A = 1{,}500;\ d = 2;\ Q = 2{,}000$

Divisions	Valuation formulas terminable income	Arithmetical solutions
I, II, III, V	$V = A\frac{R^{n-d} - 1}{rR^n} - \frac{Q}{d}\frac{R^d - 1}{rR^d}$	$V = 4{,}878.008$
IV	$V = A\frac{E^{n-d} - 1}{rE^n - r + e} - \frac{Q}{d}\frac{E^n - E^{n-d}}{rE^n - r + e}$	$V = 5{,}011.999$
VI	$V = A\frac{J^{n-d} - 1}{rJ^n - r + j} = \frac{Q}{d}\frac{J^n - J^{n-d}}{rJ^n - r + j}$	$V = 4{,}988.127$

INCOME ALLOCATION TABULATION, PREMISES II-F

Year	Capital Account			Income Allocation and Capital Expenditures				Interest rate paid on outstanding capital: per cent
	Capital outstanding at beginning of year	Capital returned each year	Capital outstanding at end of year	Yearly income	Yearly capital expenditure	Yearly interest due investor	Yearly capital return	
Column	A	B	C	D	E	F	G	H
Process	.....	B = G	A − B	Per notation	Per notation	10 % of A	D − E − F	F ÷ A
1	4,878.01	−1,487.80	6,365.81	000.00	1,000.00	487.80	−1,487.80	10
2	6,365.81	−1,636.58	8,002.39	000.00	1,000.00	636.58	−1,636.58	10
3	8,002.39	699.76	7,302.63	1,500.00	000.00	800.24	699.76	10
4	7,302.63	769.74	6,532.89	1,500.00	000.00	730.26	769.74	10
5	6,532.89	846.71	5,686.18	1,500.00	000.00	653.29	846.71	10
6	5,686.18	931.38	4,754.80	1,500.00	000.00	568.62	931.38	10
7	4,754.80	1,024.52	3,730.28	1,500.00	000.00	475.48	1,024.52	10
8	3,730.28	1,126.97	2,603.31	1,500.00	000.00	373.03	1,126.97	10
9	2,603.31	1,239.67	1,363.64	1,500.00	000.00	260.33	1,239.67	10
10	1,363.64	1,363.64	000.00	1,500.00	000.00	136.36	1,363.64	10
Totals	51,219.94	4,878.01	.....	12,000.00	2,000.00	5,121.99	4,878.01	*10
	Less capital expenditures.....			2,000.00				
	Balance, net income valued....			10,000.00				

* Average.

INCOME ALLOCATION TABULATION, PREMISES IV-F

Year	Capital Account at Beginning of Year			Income Allocation at End of Year			Sinking Fund Account During Year				Interest rate paid on outstanding capital: per cent
	Total capital	Portion of capital in sinking fund	Portion of capital in speculative enterprise	Income	Interest due investor	Contributions to sinking fund	Amount in fund at beginning of year	Yearly capital expenditures	Interest earned during year	Amount in fund at end of year	
Column	A	B	C	D	E	F	G	H	I	J	K
Process	From valuation formula	B = G	A − B	Per notation	10 % of A	D − E	From J preceding year	Per notation	4 % of G	F + G + I − H	E ÷ A
1	5,012.00	000.00	5,012.00	000.00	501.20	−501.20	000.00	1,000.00	000.00	−1,501.20	10
2	5,012.00	−1,501.20	6,513.20	000.00	501.20	−501.20	−1,501.20	1,000.00	− 60.05	−3,062.45	10
3	5,012.00	−3,062.45	8,074.45	1,500.00	501.20	998.80	−3,062.45	000.00	−122.50	−2,186.15	10
4	5,012.00	−2,186.15	7,198.15	1,500.00	501.20	998.80	−2,186.15	000.00	− 87.44	−1,274.79	10
5	5,012.00	−1,274.79	6,286.79	1,500.00	501.20	998.80	−1,274.79	000.00	− 50.99	− 326.98	10
6	5,012.00	− 326.98	5,338.98	1,500.00	501.20	998.80	− 326.98	000.00	− 13.08	658.74	10
7	5,012.00	658.74	4,353.26	1,500.00	501.20	998.80	658.74	000.00	26.35	1,683.89	10
8	5,012.00	1,683.89	3,328.11	1,500.00	501.20	998.80	1,683.89	000.00	67.35	2,750.04	10
9	5,012.00	2,750.04	2,261.96	1,500.00	501.20	998.80	2,750.04	000.00	110.00	3,858.84	10
10	5,012.00	3,858.84	1,153.16	1,500.00	501.20	998.80	3,858.84	000.00	154.36	5,012.00	10
Totals	50,120.00	.....	.....	12,000.00	5,012.00	6,988.00	.....	2,000.00	24.00	.....	*10
	Less capital expenditures.....			2,000.00							
	Balance, net income valued.....			10,000.00							

* Average.

Numerical values used for the following arithmetical solutions are:

$A = 1{,}000;\ d = 10;\ Q = 5{,}000$

Divisions	Valuation formulas perpetual income	Arithmetical solutions
I, II, III, V	$V = \frac{A}{rR^d} - \frac{Q}{d} \cdot \frac{R^d - 1}{rR^d}$	$V = 783.150$
IV	$V = \frac{A}{rE^d} - \frac{Q}{d} \cdot \frac{E^d - 1}{rE^d}$	$V = 5{,}133.464$
VI	$V = \frac{A}{rJ^d} - \frac{Q}{d} \cdot \frac{J^d - 1}{rJ^d}$	$V = 3{,}375.920$

The value of 783.150 determined under Premises II plus interest at the rate of 10% compounded annually for 10 years, will amount to 2,031.29, and capital additions of 500 per year at the end of each of the first 10 years, also with 10% interest, compounded annually, to the end of the tenth year, will amount to 7,968.71. Upon the sum of these amounts, or 10,000, the yearly income of 1,000 will just pay 10% interest to perpetuity.

With Premises IV the value of 5,133.464 is increased at the rate of 4% interest, compounded annually, for 10 years, and the capital additions are likewise increased at the rate of 4% interest, compounded annually, to the end of the tenth year. At this time the initial capital has been increased from 5,133.464 to 7,598.779, upon which annual interest at the rate of 10% (or 759.88) is thereafter paid; and capital additions of 5,000 and interest amount to 6,003.05, upon which interest is thereafter paid at the rate of 4% (or 240.12) per annum. The sum of these amounts of interest just equals the yearly income of 1,000 which, under the hypothesis, is available to perpetuity.

SOLUTIONS FOR n WHEN V IS KNOWN

Divisions	Formulas
I, II, III, V	$n = d + \frac{\log A - \log\left[A - VrR^d - \frac{Q}{d}(R^d - 1)\right]}{\log R}$
IV	$n = d + \frac{\log [A - V(r - e)] - \log\left[A - VrE^d - \frac{Q}{d}(E^d - 1)\right]}{\log E}$
VI	$n = d + \frac{\log [A - V(r - j)] - \log\left[A - VrJ^d - \frac{Q}{d}(J^d - 1)\right]}{\log J}$

SUBDIVISION G

Formulas for Determining Reductions of Value Attributable to Necessity of Making Irregular Future Capital Expenditures in Order That Income May Be Maintained.

The following may be regarded as valuation formulas for capital additions. Previously developed formulas for the valuation of income of several types have been stated. The income valuation formulas, minus the formulas for valuing future capital expenditures as negative income, give the formulas for valuing income subject to such expenditures.

Notation in addition to that given in Chapter II:

$Q_1, Q_2, Q_3, Q_4, \ldots Q_n$ = the irregular future capital expenditures required in each of the years indicated by the subscripts to the letter Q.

Divisions	Formulas for valuing future expenditures of capital
I, II, III, V	$V = \frac{Q_1}{R} + \frac{Q_2}{R^2} + \frac{Q_3}{R^3} + \frac{Q_4}{R^4} + \ldots \frac{Q_n}{R^n}$
IV	$V = \frac{eE^n}{rE^n - r + e}\left\{\frac{Q_1}{E} + \frac{Q_2}{E^2} + \frac{Q_3}{E^3} + \frac{Q_4}{E^4} + \ldots \frac{Q_n}{E^n}\right\}$
VI	$V = \frac{jJ^n}{rJ^n - r + j}\left\{\frac{Q_1}{J} + \frac{Q_2}{J^2} + \frac{Q_3}{J^3} + \frac{Q_4}{J^4} + \ldots \frac{Q_n}{J^n}\right\}$

The above formulas are identical with those for valuing income, except that the symbol Q, denoting capital expenditure, is used in place of the symbol A, denoting income receipt.

The formulas which have been stated in this chapter cover the types of income receipt which are most frequently valued. The only advantage of formulas for valuing specific types of income is the time saving resulting from the use of a formula simpler than the general formulas of Chapter II. As more complex specific types of income are considered, the formulas adapted to the valuation of such specific varieties of income become so complex that there is no advantage to be derived from their use. For types of income not mentioned in this chapter, the general formulas of Chapter II are, therefore, recommended.

Chapter IV

VALUATION OF INCOME SUBJECT TO AN INCOME TAX

The imposition of an income tax serves, not only to reduce the amount of future income which the investor retains from the yield of the income producing property, but also to reduce the net interest earnings of sinking funds set aside to insure the return of the initial investments. When the tax is imposed on gross income the problem is simple, as the income to be valued is that which will remain after the income tax has been paid. But when depreciation, or other deductions in the nature of a capital return, operate to reduce the amount of income subject to tax, and the price which an investor is willing to pay for a property is determined on the basis of a future net yield after the payment of the tax on income and proper provision for the return of capital, the problem is much more complex, as the value sought depends upon the amount of future tax, which tax depends upon future deductions for depreciation, which, in turn, depend upon the value to be determined for the property or the price to be paid for it by the investor. Thus there are three unknowns, each of which is affected by one of the others.

Effects of a tax upon interest earned by the sinking fund.—When the investor uses a sinking fund valuation formula of any type to determine a fair price for an income producing property, an item for consideration is that the entire interest earnings of the sinking fund are subject to tax. The effective rate of interest earned by the sinking fund is found, (1) by deducting from 100% the percentage rate of income tax, and (2) by multiplying the stipulated sinking fund earning rate subject to tax by the percentage so determined. In other words, if the sinking fund is expected to earn 4% interest and this interest is subject to a tax of 10%, the effective rate of interest retained by the sinking fund to provide for the retirement of capital is $(1-0.10)(0.04)$ or 3.6%.

The mathematical proof of this statement is not required for Premises I and II, which involve no sinking funds, and may be presented for all sinking fund premises by an analysis applicable to the premises of Division VI, since all other sinking fund valuation formulas may be derived from the general formula of that type.

Additional Notation.

i = the percentage rate of income tax on sinking fund earnings expressed as a decimal; that is, $i = 0.10$ for a 10% income tax.

$j_t = r - r_1 + e - ei = j - ei$

$J_t = 1 + j_t = J - ei$

$r_t = r - ri$

$R_t = 1 + r_t$

$e_t = e - ei$

$E_t = 1 + e_t$

S_1, S_2, S_3, etc., to S_n = the net amounts added to the sinking fund during years indicated by the subscript to the letter S; that is, the addition to the sinking fund includes the amount contributed from income for the year, plus interest earned during the year by the amount in the sinking fund at the beginning of the year, less tax paid on such interest earnings.

All numerical values used in the arithmetical solution and tabulation are the same as those substituted for algebraic symbols in the arithmetical illustrations of Chapter II, with the exception of the additional numerical value, $i = 0.10$.

The yearly terms of the algebraic series which show, for Premises VI of Chapter II, the yearly contributions to the sinking fund and their yearly interest accretions subject to tax, will be as follows:

1st year: $A_1 - Vr = S_1$

2nd year: $A_2 - Vr + S_1r - S_1r_1 + S_1e - S_1ei = S_2$

3rd year: $A_3 - Vr + (S_1 + S_2)r - (S_1 + S_2)r_1 + (S_1 + S_2)e - (S_1 + S_2)ei = S_3$

4th year: $A_4 - Vr + (S_1 + S_2 + S_3)r - (S_1 + S_2 + S_3)r_1 + (S_1 + S_2 + S_3)e - (S_1 + S_2 + S_3)ei = S_4$

nth year: $A_n - Vr + (S_1 + S_2 + S_3 + \cdots S_{n-1})r - (S_1 + S_2 + S_3 + \cdots S_{n-1})r_1 + (S_1 + S_2 + S_3 + \cdots S_{n-1})e - (S_1 + S_2 + S_3 + \cdots S_{n-1})ei = S_n$

The sum of the terms of the above series will, if made equal to V, give the formula for Premises VI, from the expanded

form of which the formulas for other sinking fund premises may be derived. The formulas thus derived and shown below are the general formulas of Chapter II, adjusted so as to provide for the payment of a tax on sinking fund earnings.

Premises	General formulas
I and II	No sinking fund involved, hence no adjustment
III	$V = r_t \dfrac{A_1 R_t^{n-1} + A_2 R_t^{n-2} + A_3 R_t^{n-3} + \cdots A_n}{r R_t^n - r + r_t}$
IV	$V = e_t \dfrac{A_1 E_t^{n-1} + A_2 E_t^{n-2} + A_3 E_t^{n-3} + \cdots A_n}{r E_t^n - r + e_t}$
V	$V = (r - ei) \dfrac{A_1(1 + r - ei)^{n-1} + A_2(1 + r - ei)^{n-2} + A_3(1 + r - ei)^{n-3} + \cdots A^n}{r(1 + r - ei)^n - ei}$
VI	$V = j_t \dfrac{A_1 J_t^{n-1} + A_2 J_t^{n-2} + A_3 J_t^{n-3} + \cdots A_n}{r J_t^n - r + j_t}$

These formulas will all reduce to corresponding general valuation formulas of the same premises, as given in Chapter II, when j_t is allowed to equal j with Premises VI; when i = zero, under Premises V; when e_t is allowed to equal e with Premises IV; and when r_t is allowed to equal r with Premises III. The formulas of this chapter for the several sinking fund premises were all derived from the formula applying to Premises VI, by processes identical with those used in Chapter II for similar derivations.

Formulas similar to those of Chapter III for valuing specific types of income may be readily derived from preceding general formulas by the following processes: Take the Premises VI formula of Chapter III and write J_t for J and j_t for j. Substitute the expanded forms of j_t and J_t, that is, $j_t = r - r_1 + e - ei$ and $J_t = 1 + r - r_1 + e - ei$. Then, to derive the formula desired, let r_1 and e both equal r for Premises III, let $r_1 = r$ for Premises IV, and let $r_1 = e$ for Premises V. Or, if desired, the formulas can be derived directly from the general formulas of this chapter by summation of terms after proper substitutions of notation for yearly income have been made.

The arithmetical solution and income allocation tabulation for the Hoskold sinking fund premises of Division IV will be sufficient illustration of the differences between formulas of this chapter and those of Chapter II. The tabulation follows:

INCOME ALLOCATION TABULATION

Year	Total capital	Income Allocation at End of Year			Sinking Fund Account During Year				
		Income	Interest due investor	Contributions to sinking fund	Amount in fund at beginning of year	Interest earned during year	Less tax on interest earned by sinking fund	Contributions to sinking fund at end of year	Amount in fund at end of year
Column	A	B	C	D	E	F	G	H	I
Process	From valuation formula	Per notation	10 % of A	B − C	From I preceding year	4 % of E	10 % of F	H = D	E + F − G + H
1	5,404.34	1,000.00	540.43	459.57	000.00	000.00	000.00	459.57	459.57
2	5,404.34	1,200.00	540.43	659.57	459.57	18.38	1.84	659.57	1,135.68
3	5,404.34	500.00	540.43	−40.43	1,135.68	45.42	4.54	−40.43	1,136.13
4	5,404.34	800.00	540.44	259.56	1,136.13	45.45	4.55	259.56	1,436.59
5	5,404.34	1,600.00	540.43	1,059.57	1,436.59	57.46	5.74	1,059.57	2,547.88
6	5,404.34	1,300.00	540.43	759.57	2,547.88	101.91	10.19	759.57	3,399.17
7	5,404.34	900.00	540.44	359.56	3,399.17	135.97	13.60	359.56	3,881.10
8	5,404.34	600.00	540.43	59.57	3,881.10	155.24	15.52	59.57	4,080.39
9	5,404.34	700.00	540.44	159.56	4,080.39	163.22	16.32	159.56	4,386.85
10	5,404.34	1,400.00	540.43	859.57	4,386.85	175.47	17.55	859.57	5,404.34
Totals	54,043.40	10,000.00	5,404.34	4,595.67	.....	898.52	89.85	4,595.67	.....

Formulas Involving Nonallowable Deductions from Income, for Income Tax Purposes.

The total future gross income is composed of two parts: (1) the part for which the purchaser will pay, because it is retained by him; and (2) the part for which he will not pay, because it must be expended and cannot be retained. The latter element of income includes all operating costs and income taxes as an especial item of such costs. Income taxes differ from other costs, because they cannot be deducted from gross receipts in determining the net amount of profit which is subject to the income tax. The purchaser of future income will pay only for the net expected profits which he will retain after income taxes are deducted, and the present worth of these retainable profits at the date of purchase will determine the purchase price to be paid. Under the Federal Revenue Acts of the United States of America, the purchase price of assets subject to depreciation or depletion becomes a tax-free deduction from income, which applies to the return of capital, thus reducing the amount of income subject to tax in the case of income producing properties of a limited life. The three independent factors are therefore as follows:

1. The gross profits before income taxes are deducted.
2. The income tax which must be paid on the gross profits less tax-free deductions for the return of capital.
3. The value of the operating profits remaining after payment of income taxes.

Another item of operating costs which frequently must be considered in determining the net profits to be valued is interest on the amount of working capital which it is necessary to retain in the business, either in inventories or as a cash reserve, in order to carry on the business. This cash reserve must be provided as an initial outlay and maintained presumably as a constant amount throughout the life of the enterprise. Under this assumption the same amount is left intact at the end of the operations when it is withdrawn with no loss of capital, but the gross profits from the investment must be reduced by interest on this working capital at a suitable rate. The amount of this interest on working capital would be deducted from the gross profits to determine the net profits to be valued, but such interest is subject to taxes

imposed upon income and the effect thereof on the value of the expected profits must be considered.

Two formulas will be derived, one with income taxes alone considered, and another with both income taxes and interest on working capital considered.

Notation and Derivations.

Let:

A = total operating profits before income taxes, interest on working capital, and tax-exempt income applying to the return of capital values are deducted:

x = total operating profits less total expected income taxes:

y = total operating profits less total expected income taxes and less total interest on working capital:

F = present worth factor, determined by life, interest rates, and type of present worth formula:

i = income tax rate as a percentage of net taxable income, that is, of the net profits less tax-free deductions for the return of capital:

I = total interest on working capital:

Then:

Fx = present worth of total profits after income taxes are deducted:

Fy = present worth of total profits after income taxes and interest on working capital are deducted:

$A - x$ = total expected income taxes, where interest on working capital is not considered:

$A - y - I$ = total expected income taxes, where interest on working capital is considered:

$A - Fx$ or $A - Fy$ = total taxable income, as Federal taxes and interest on working capital are not deductible in computing net taxable income, and as Fx and Fy represent total deductions for depletion and depreciation.

Therefore:

$(A - Fx)i$ or $(A - Fy)i$ = total expected income taxes.

Hence, where income taxes alone are considered:

$$(A - Fx)i = A - x$$

Solve the preceding for x and obtain,

$$x = \frac{A(1 - i)}{1 - Fi}$$

which is the desired general formula, considering income tax on net earnings, but not considering interest on working capital. This formula applies only to the condition that the tax-free deductions from yearly income vary proportionately to the amounts of yearly income.

Where both income taxes and interest on working capital are considered, then:

$$(A - Fy)i = A - y - I$$

Solve the above for y and obtain,

$$y = \frac{A(1 - i) - I}{1 - Fi}$$

which is the desired general formula, considering both the income tax on net earnings and that on interest credited to working capital. This formula applies only in the event that both the tax-free deductions from income and the interest on working capital vary from year to year proportionately to the variations in the amounts of income.

Similar formulas might be derived for variable rates of tax during different portions of the period of the receipt of income, and for excess profits or surtaxes in addition to income taxes. This has not been done, as excess profits taxes are no longer a feature of the Revenue Acts of the United States of America, and as the legislation with respect to surtaxes is changing so constantly and involves a large number of rates of taxation in a determination of a single amount of tax.

Computations as to the expected amount of tax to be paid can also be made by the so-called "cut and try" method. This method involves a determination of the value of the operating profits without consideration of income and excess profits taxes or surtaxes, a deduction of that value from the operating profits to determine the amount of taxable income, a computation of the tax on that amount of taxable income, and a deduction of this amount of tax from the operating profits leaving a remainder. This remainder is valued as the operating profits were first valued, and the process just

described is repeated until the amount of tax determined in two successive trials is equal or approximately equal.

The "cut and try" method is recommended as the simplest method for use with the premises of Divisions IV and VI in the valuation of income changing from year to year at one rate of variation, while the tax-free deductions from income or the charges for interest on working capital, or both of these amounts, remain constant, or have a rate of variation differing from that of the income.

It is very simple with the premises of Divisions I, II, III, and V to make adjustments for differing rates of variation for income, for tax-free deductions from income, and for interest on working capital. The interest on working capital and the taxes on income, if their amounts by years are known, may each be valued separately by means of their own compound interest valuation factors, and the sum of these values may then be deducted from the value of the gross income (without deduction for taxes or interest on working capital) to determine the value of the net income in excess of taxes and interest on working capital. The value of the net income from the property is its worth as an investment.

The formulas just derived are, therefore, applicable more especially to periodic changes of income, as it would be reasonable to assume in a valuation of regularly changing income that the tax-free deductions would vary proportionately to the amount of income and that the working capital required would also be proportionate to the activity of the business as measured by the variable income.

Arithmetical Illustration for Valuation of Income Subject to a Tax of 10% upon Net Income and upon Sinking Fund Interest, without Consideration of Working Capital

The value determined for the income subject to a 10% income tax upon sinking fund earnings, as determined in the previous arithmetical illustration of this chapter, was 5,404.34, and this amount divided by 10,000, the amount of the income, gives 0.540434 as the numerical value of the algebraic symbol F to be substituted in arithmetical solutions of the formulas just derived. The other numerical substitutions are: $A = 10{,}000$; and $i = 0.10 = 10\%$.

Solving the equation, $x = \frac{A(1-i)}{1-iF}$, gives 9,514.18 as the amount of net income to be valued on a tax-free basis, and the balance of the gross income of 10,000, or 485.82, will be the amount of income taxes paid during the investment period. The value of the net income of 9,514.18 is determined by multiplying by the valuation factor of 0.540434, the result being 5,141.79. This amount is the value for investment purposes, which value will also be returned to the investor in tax-free deductions during the term of investment. The taxable income will be 10,000 less 5,141.79, or 4,858.21, and 10% of this amount gives the total tax of 485.82 as previously determined. This solution depends upon the assumption that the tax-free deduction from income is a constant percentage of the income for each year, in this case 51.4179% of the yearly income. The income allocation tabulation follows:

INCOME ALLOCATION TABULATION

Year	Total capital	Income Allocation at End of Year						Sinking Fund Account During Year				
		Income	Tax free deduction from income for capital return	Taxable income	Tax at 10 % of taxable income	Interest due investor	Contributions to sinking fund	Amount in fund at beginning of year	Interest earned during year	Less tax on interest earned by sinking fund	Contributions to sinking fund at end of year	Amount in fund at end of year
Column	A	B	C	D	E	F	G	H	I	J	K	L
Process	From valuation formula	Per notation	*	B − C	10 % of D	10 % of A	B − E − F	From L preceding year	4 % of H	10 % of I	K = G	H + I − J + K
1	5,141.79	1,000.00	514.18	485.82	48.58	514.18	437.24	000.00	000.00	000.00	437.24	437.24
2	5,141.79	1,200.00	617.01	582.99	58.30	514.18	627.52	437.24	17.49	1.75	627.52	1,080.50
3	5,141.79	500.00	257.09	242.91	24.29	514.18	−38.47	1,080.50	43.22	4.32	−38.47	1,080.93
4	5,141.79	800.00	411.34	388.66	38.87	514.18	246.95	1,080.93	43.24	4.32	246.95	1,366.90
5	5,141.79	1,600.00	822.69	777.31	77.73	514.18	1,008.09	1,366.90	54.68	5.47	1,008.09	2,424.10
6	5,141.79	1,300.00	668.43	631.57	63.16	514.18	722.66	2,424.10	96.97	9.70	722.66	3,234.03
7	5,141.79	900.00	462.76	437.24	43.72	514.18	342.10	3,234.03	129.36	12.93	342.10	3,692.56
8	5,141.79	600.00	308.51	291.49	29.15	514.18	56.67	3,692.56	147.70	14.77	56.67	3,882.16
9	5,141.79	700.00	359.93	340.07	34.01	514.18	151.81	3,882.16	155.29	15.53	151.81	4,173.73
10	5,141.79	1,400.00	719.85	680.15	68.01	514.18	817.81	4,173.73	166.95	16.69	817.80	5,141.79
Totals	51,417.90	10,000.00	5,141.79	4,858.21	485.82	5,141.80	4,372.38	.....	854.86	85.49	4,372.27	.....

* Value divided by total income = $\frac{5,141.79}{10,000}$ = 0.514179, which is the factor by which the income for any year is multiplied to determine the amount of that year's income, which is a tax-free deduction for the return of capital.

Arithmetical Illustration of the Valuation of Income Subject to a 10% Tax, and with Provision for Working Capital.

In this illustration equal annual income of 1,000 a year for 10 years will be valued according to Premises II, which are not sinking fund premises, with interest of 10% upon the investment, with the return of the investment tax-free during the 10-year period, and with a tax of 10% upon the balance of the income subject to tax. Working capital of 1,000 is assumed with a 10% annual interest payment to the investor upon working capital during each year of the investment period, and the return of the working capital to the investor at the end of 10 years. Numerical equivalents of the algebraic symbols will then be:

$$F = .614457;\ A = 10{,}000;\ i = 0.10;\ \text{and}\ I = 1{,}000$$

Substituting for algebraic symbols and solving,

$$y = \frac{A(1 - i) - I}{1 - Fi} = 8{,}523.75$$

which is the portion of the 10,000 gross income which is valuable to the investor. This amount of 8,523.75 might be called the net income to be valued. It is multiplied by the same valuation factor ($F = .614457$) to determine the value of 5,237.48 for the gross income of 10,000 under the conditions imposed. The gross income of 10,000, less 1,000 interest on working capital and 8,523.75 net income, gives the total amount of taxes, or 476.25. The gross income of 10,000 less 5,237.48 tax-free deduction for the return of capital gives the amount of taxable income, or 4,762.52, upon which a 10% tax would be 476.25 as previously determined. A tabulation follows which shows income allocation year by year under these conditions.

INCOME ALLOCATION TABULATION

Year	Capital Account During Year			Income Account						
	Capital outstanding at beginning of year	Capital returned each year	Capital outstanding at end of year	Yearly income	Yearly tax-free return of capital	Yearly taxable income	Yearly tax on income	Yearly interest on 1,000 working capital	Yearly interest due investor	Yearly capital return
Column	A	B	C	D	E	F	G	H	I	J
Process	1st year from formula after from C preceding year	B = J	A − B	Per notation	One tenth of 5,237.48	D − E	10 % of F	10 % of 1,000	10 % of A	D − G − H − I
1	5,237.48	328.63	4,908.85	1,000.00	523.75	476.25	47.62	100.00	523.75	328.63
2	4,908.85	361.49	4,547.36	1,000.00	523.75	476.25	47.63	100.00	490.88	361.49
3	4,547.36	397.64	4,149.72	1,000.00	523.75	476.25	47.62	100.00	454.74	397.64
4	4,149.72	437.40	3,712.32	1,000.00	523.75	476.25	47.63	100.00	414.97	437.40
5	3,712.32	481.14	3,231.18	1,000.00	523.74	476.26	47.63	100.00	371.23	481.14
6	3,231.18	529.26	2,701.92	1,000.00	523.75	476.25	47.62	100.00	323.12	529.26
7	2,701.92	582.19	2,119.73	1,000.00	523.75	476.25	47.62	100.00	270.19	582.19
8	2,119.73	640.40	1,479.33	1,000.00	523.74	476.26	47.63	100.00	211.97	640.40
9	1,479.33	704.44	774.89	1,000.00	523.75	476.25	47.63	100.00	147.93	704.44
10	774.89	774.89	000.00	1,000.00	523.75	476.25	47.62	100.00	77.49	774.89
Totals	32,862.78	5,237.48	.....	10,000.00	5,237.48	4,762.52	476.25	1,000.00	3,286.27	5,237.48

The preceding formula may be used to value income varying uniformly from year to year, if both the tax-free return of capital and the amount of working capital may be expected to show the same degree of yearly variation as does the income. The formula is not applicable to the valuation of irregular annual income, nor to the valuation of varying income with constant interest on a uniform amount of working capital, for the reason that one valuation factor would be required for valuing income and a different valuation factor would be needed to value deductions from income.

Chapter V

INTEREST RATES

Books and articles on the valuation of anticipated income frequently include a statement that amounts to be received at future times should be discounted to present worth at date of valuation by use of the current or going rate of interest. If the current rate of interest is also the anticipated future rate of interest during the period over which the receipt of income is expected, such a statement is correct, in so far as it goes, but leaves much to be desired under other conditions. The question also arises in the mind of the evaluator as to which of several types of current rates of interest to use.

The yields from different types of investments in the same locality will vary chiefly according to the attendant risk of loss, but numerous other elements are worthy of consideration. Marketability is a very important factor in determining the yield of an investment, principally because risk of loss decreases as marketability increases. Anything which tends to increase the marketability of an asset also tends to reduce the yield which an investor expects. Organized markets, such as stock exchanges, and increases in the current amount of trading with respect to particular types of property serve to increase marketability, speculation being one form of increased trading. Thus, the investor expects a higher return from a stock which is not listed on a stock exchange than from one which is listed, and a higher yield from a stock with a restricted local market than from one which has a nation-wide market. Similarly the sale of evidences of participating ownership in the form of stocks and bonds of small denominations creates a market of greater scope than could possibly exist for the sale of the assets which are represented by those securities, for the simple reason that the smaller the amount of an investment the greater is the number of prospective investors.

Funds for investment are supplied from some localities and demanded at others. The rate of interest is a price paid

for the use of money and the price of capital follows the law of supply and demand in much the same manner as do other prices. Interest rates in financial centers are, therefore, lower than at other localities, while interest rates in rapidly developing regions or industries are comparatively higher, according to the intensity of the demand for imported capital, together with the lack of safe and cheap facilities both for transference of capital from place of supply to place of demand and supervision of its investment at place of demand.

Uncertainty is an element of major importance. There are numerous varieties of uncertainty, some based upon the current trend of laws and legal decisions in geographical units with respect to financial transactions, others upon the management of enterprises seeking capital, and so on, almost indefinitely. While uncertainty may or may not increase the risk of loss, it always increases the reluctance of an investor to place his funds, and this reluctance must be overcome by an increase of the inducement offered. The nearer the stipulated date for the repayment of the investment, the smaller is the uncertainty that current conditions will be maintained, and this serves frequently to reduce the yield of short term investments and loans in comparison with long term financing of the same type. The number of investors who are willing to venture into the field of uncertainty is small in comparison with the total number of investors, hence the increase of uncertainty decreases marketability. As the distance between the place of investment and the source of capital increases, the investor's ability to have personal supervision of his funds decreases, and the resultant uncertainty creates a demand for a higher yield which is quite apart from the further cost of paid supervision and commissions to financial middlemen. New enterprises involve more uncertainty than established enterprises of the same general type, and new types of investment are subject to greater uncertainty than those for which years or even generations of experience are available. It is not unusual to see the stock of a new unit of an established industry double in value between the inception of the enterprise and a time when experience proves its ability to withstand economic disturbances and produce the results which were expected when capital was sought; and the profits arising from successful investment in new indus-

tries are so extensively advertised by promoters that their citation would be superfluous. Similar increases in value occur when a badly managed enterprise is taken over by capable hands, but these increases are chiefly effected when sufficient experience is attained to prove that expectations of future good management have been correct. Thus, uncertainty disappears as predications are founded on history rather than upon estimate or surmise, and elimination of elements of uncertainty is reflected both by increases of value and in the willingness of investors to accept a lower return for capital.

Yields of most investments comprehend a payment for the use of capital, a further payment for the supervision which every investor must accord to his investments either personally or through an agent, and a number of other recompenses which may include insurance against risk of capital loss, a potential profit sufficient to attract capital to speculative ventures, a potential profit for investment before uncertainty is eliminated and marketability insured, and, with short term investments, a reimbursement for cost of investment and for loss of interest during the average period required for reinvestment. These components by no means exhaust the list of those worthy of mention, but are sufficient to indicate that the yield of an investment is almost invariably greater than the current rate of interest; and that it is a complex aggregate payment of which pure interest is but a part. But, in common commercial parlance, the terms "yield" and "rate of interest" are used synonymously at times, and, on other occasions, yield means the percentage of return at the current market price, and rate of interest means the percentage of return at a price equivalent to a fixed price, such as cost or par value. The two terms are used as synonyms in the tabulations and charts of this chapter.

The interest yield of governmental securities is usually the lowest of all interest rates in a geographic area of stable political condition. The factors of very ready marketability, absence of risk, and minimum requirement as to management all contribute to the low yield. This type of investment probably shows interest, unaffected by other elements, more accurately than do other types of investment. The rate of interest paid to depositors by savings banks is also some reflection of a pure rate of interest, but is less sensitive as an

indicator of changing economic conditions than is the yield of governmental securities.

This work is concerned principally with the valuation of expected income from comparatively long term investments, and the subjects of interest rate trends and of normal rates of interest have been considered to be of sufficient importance to justify prolonged statistical study, the results of which are herewith presented. That study has been confined to the lower yields which may be derived from essentially safe investments in ownership equities protected by marginal equities sufficient to absorb any normal loss without impairment of the value of the equity from which the yield is derived. The classes of investments studied include governmental securities, first mortgage loans, the investments of savings banks, the investments of life insurance companies, collateral loans, and commercial paper. It is perhaps unnecessary to emphasize the statement that these are yields from very well protected equities in property; and that, as the extent of the equity approaches that of an unencumbered fee ownership with its attendant risks of loss and burdens of management, investors demand and receive higher yields. It follows that still higher yields are expected from those marginal equities which absorb the risk of loss and the cares of management with respect to properties encumbered by prior lien obligations. Thus, higher yields are expected from fee ownerships than from lessor equities in property and the lessee equity must promise a greater return than that which would be attractive to a fee owner of the same property. Within the United States farm mortgage rates of interest are customarily higher than rates on improved city real estate and rates of interest in small towns are generally higher than those in the large cities. In addition, there are wider geographic variations ascribable to several causes, such as relations between the supply of, and demand for, capital; the dependability of climatic conditions in communities depending chiefly upon agriculture for their existence; facilities existing for the safe transfer of capital and supervision of its investment; and the local financial history of types of investment, districts, and political units. These causes are by no means a complete list, but are among the more important of those effecting variations of interest, within a nation, for investments of the same

general type. Accurate statistics with respect to rates of interest realized by life insurance companies in the several states are available for a number of years, and it is apparently the policy of the Association of Life Insurance Presidents to continue the publication of similar data in its Proceedings. Such statistics show the broader but not the local geographic variations of interest rates.

Few generalizations are possible with respect to speculative rates of interest. The preceding statements for essentially safe rates of interest will serve as some guide to the trends of speculative rates. Investors obtaining safe rates of interest are usually protected by a first lien on property valued at approximately double the amount of the investment. As the margin of security lessens, higher yields are demanded by investors; the common stockholder, as fee owner of a stable, conservative, and well managed industrial enterprise demanding approximately twice the rate—in combined dividends and increases of surplus—that is required by a well protected bondholder, although dividends alone need be but slightly higher than the rate of interest paid to bondholders.[1] The minimum earning rate of a competitive industrial enterprise that is sufficient to attract capital is approximately 8%; and the earning rates increase from that minimum with risk, uncertainty, irregularity of earnings, and other conditions that add to the element of speculation and to the possibility of loss of capital. A proper speculative yield from an individual investment may be stated only for particular times and localities by thoroughly informed persons of sound judgment; but such speculative yield will rarely be less than double the yield of a safe investment in the same locality.

The accompanying charts (pages 126–132), it is believed, will serve to illustrate general principles with respect to rates of interest which may be enunciated as follows:

1. The rate of interest varies with world wide economic conditions which govern its general trend in all nations, but which are modified for individual nations by local influences, such as wars and panics.

2. The rate of interest for any nation tends to vary directly with, and somewhat proportionately to, commodity

[1] For detailed discussion see the "Valuation of Industrial Securities," by Ralph Eastman Badger, published by Prentice-Hall, Inc., 1925.

prices. This is strikingly illustrated by charts showing interest rate and commodity price indices for the United States, and the same relation has been noted with respect to interest rates and commodity prices in Great Britain. Irving Fisher has also shown by statistics that rates of interest increase or decrease when prices of commodities increase or decrease, but that interest rate changes are insufficient to balance changes in the purchasing power of money in terms of commodities.

3. There are important differences in the rates of interest prevailing at the financial centers of important nations, and such differences, partially, at least, may be ascribed to relations between the supply of, and demand for, capital in the furtherance of industrial development within each nation. Thus England, France, Belgium, and Holland have long been exporters of capital to a greater extent than other nations and have enjoyed lower domestic rates of interest than those prevailing in other nations. Germany has required practically all its capital for home consumption in connection with rapid industrial development, has exported a smaller proportion of its capital, and has had appreciably higher domestic rates of interest than those of the previously named countries. The United States has, until recently, imported large amounts of foreign capital for the rapid development of its national resources and industries and has paid very much higher rates of interest in order to attract capital. These comparisons are limited to countries in which the financial and political structures have been of recognized stability in order that the effects of influences other than the supply of, and demand for, capital might be eliminated to the greatest possible extent. Barriers to the free transfer of capital created by national boundaries and the inertia of capital, which prevents it from moving to the field of greatest earning, are also indicated by the differences in rates of interest among nations.

4. The effects of risk during periods of national stress are very apparent in the great increases affecting rates of interest for individual nations, in distinction to changes affecting many or all nations, when either the continued existence of such nation or its ability and desire to repay its debts are in doubt.

5. The sympathetic increase of interest rates in noncombatant countries during periods of war is not clearly shown

in the accompanying charts; but it is a prevalent condition, which owes its existence to, and is proportionate to, the diversion of capital to the combatant nations.

6. The increases of the rate of interest with the duration of loans, other things being equal, are shown in the progressive increase of rates of interest on collateral time loans of longer duration in New York, these increases being the result of progressively increasing uncertainty with respect to the maintenance of current conditions, as the length of time for which the loan is made increases. It is worthy of note that the difference between rates of interest on short and long term loans decreases as the general rate of interest increases, and that at times of very high general rates of interest the rates for very short term loans may exceed those for longer term loans. Similar differences for loans of varying duration and identical safety, that is, between floating money and discounts on three months' bank bills, and between discounts on three and six months' bank bills in London, are noted by F. G. Peake in the *Bankers Magazine of London*, Volume 3, page 654, the first difference being 0.61%, and the second, 0.78%, both for the period 1882 to 1913, inclusive.

Description of Accompanying Statistical Data on Rates of Interest.

(Figures from which the charts of this chapter are derived will be found in Appendix A)

Governmental securities.—British Consols and French Rentes are securities of England and France, respectively, which have no dates of maturity, and which can be retired only by purchase on the open market. Data for British Consols were obtained from page 289, Preliminary Volume 1, *Harvard Review of Economic Statistics*, for the period 1790 to 1820; and from pages 32 and 33 of the *Bankers Magazine of London*, for January, 1923, covering the period 1820 to 1922. These sources of information were checked against other authorities. Data for French Rentes were obtained from a book entitled "French Public Finance," published by the Bankers Trust Company of New York. Their average annual yields, which cover the longest periods obtainable for individual securities, are shown on the chart on page 129.

The approximate annual average yield to the investor from U. S. A. governmental loans purchased at current market prices and held to date of redemption was determined from prices at which the various securities were quoted in New York City at or about the first of each month, considering the arithmetical average of the available monthly prices to be the yearly average price, and determining yields for each security from the average annual price to an accuracy of one-tenth of 1%, or less, and, finally, by weighting the yields of the several loans for each year by the amounts of the loans outstanding at the beginning of the calendar or fiscal year to determine the approximate weighted average yield of all issues. The figures obtained for the 126-year period covered, including two years during which there was no interest-bearing debt of the United States, comprise yields for the following percentages of the total interest-bearing debt:

Number of Years	*Per Cent of Total Debt*
2	60 to 65
3	70 to 75
3	75 to 80
12	80 to 85
16	85 to 90
11	90 to 95
77	95 to 100
2	No interest-bearing debt
126 years	= the total period covered

Prices were obtained and yields were computed for greater proportions of the outstanding debt for a number of years, but the yields of certain securities were not included in the average yield figures for various reasons. At times the yields discarded were for short term loans convertible into long term funded indebtedness, and the prices of such securities were materially influenced by the values of the conversion privileges. Again, prices for the last year or two prior to the maturity of some issues would represent scattered and negligible amounts of sales, and reflected yields so disproportionate to the yields of other loans that they were discarded. During the Greenback period following the Civil War, the prices of issues redeemable in gold in the immediate future were controlled by the speculative value of gold in currency at

future dates, and could not be included in the averages. During the period 1837 to 1842, following two years of no indebtedness, there were no long term obligations of the United States, and Treasury Notes of from a few months' to two years' maturity, and paying rates of interest varying from one-tenth of 1% to 6% comprised essentially the entire debt. As the amounts issued and reissued at each rate of interest could not be ascertained, and as price quotations were scattered and uncertain, either the maximum interest rate or the average interest rate on new note issues during the year was substituted for average yield figures. Annual yields as stated for the period 1837 to 1842 are, therefore, known to be higher than the average yields for governmental securities during this period; but the amount of the difference cannot be ascertained from available data. All governmental securities outstanding were available for deposit by national banks to secure the circulation of bank notes from the date of passage of the National Bank Act in 1869 to the date of issue of the 3% Panama Canal Loan in 1911. Examination of the average annual yield curve gives indication that the price of governmental securities to about 1900 was controlled by their investment value, and that thereafter the value of the circulation privilege governed the prices of securities issued prior to 1911. From 1911 to 1925 the yields have been separated into classes: the first class includes securities with circulation or similar valuable privileges; the second, is composed of the 3% Panama Canal Loan of 1911 to 1961 and the $3\frac{1}{2}$% Liberty Loan, both of which have important tax exempt features; and a third class, which comprises other government financing from 1917 to 1925, and is most representative of a true rate of interest. The average yield of all governmental securities is used on the charts for the sake of continuity and because it is but slightly lower than the yield of the third class of securities above described.

Sources of data for computing the approximate yearly average yields of governmental securities of the United States are numerous and vary with different bond issues. Yields computed by the government actuary were obtained for nearly all issues from 1880 to date, and from 1860 to 1920 the prices for which it was necessary to compute yields were

averages of the monthly high and low quotations published both in annual reports of the Comptroller of the Currency and in the *Financial Review*. Prior to 1860 the arithmetical average of prices at or about the first of each month was the basis for the determination of yearly average prices and the principal source of price data was the "New York Shipping and Commercial List and Price Current," with subsidiary data from the *New York Commercial* and from various daily papers quoting sales of governmental securities on the New York Stock Exchange. When actual sales were not quoted the arithmetical average of bid and asked quotations was substituted. When some of the monthly quotations were not obtained, the arithmetical average of available quotations was considered to be the yearly average price. The data with respect to amounts of each issue outstanding at the first of each calendar or fiscal year of the period 1790 to 1885 were obtained from Executive Document No. 116 furnished by the Secretary of the Treasury to the House of Representatives of the First Session of the Forty-ninth Congress, and data for later years were obtained from annual reports either of the Comptroller of the Currency or of the Secretary of the Treasury.

Approximate yields of investments of New York mutual savings banks.—These approximate yields have been determined upon the basis of certain assumptions. Sources of information are Volume II of the "History of Savings Banks in the United States," by Emerson W. Keyes, and reports of the Superintendent of the Banking Department, State of New York. Amounts due to depositors at the beginning and end of each year are averaged, and the interest paid to depositors during each year is divided by this average amount of deposits to determine average annual rate of interest paid to depositors. The annual expenses of the banks and the annual increases or decreases in surplus, computed on the basis of market prices of securities held, are both divided by the average yearly amount due to depositors to determine, respectively, the annual costs of investment and the yearly profits or losses from appreciation or depreciation in values. Assumption is made that the "paid in capital" of the banks may be ignored both with respect to amounts of dividends paid to stockholders and amounts of savings in expenses to

depositors in rentals on banking properties and facilities. Assumptions are also made that the percentage of invested to total assets is constant, and that the difference between invested and total assets is approximately equivalent to capital and surplus. While these assumptions have the probable effect of slightly understating the actual annual yields of investments by the banks, the relative yields over periods of time should be representative of the trends of interest rates.

Average yield of mean invested assets of life insurance companies.—Sources of information for this series of yields are Zartman's "Investments of Life Insurance Companies" for the period 1850 to 1871 and the "Insurance Year Book" for 1872 and later years. Yields are given by Zartman for those individual companies which had total assets in excess of $5,000,000 in 1904. The arithmetical average of the yields of individual companies is assumed to be the average annual yield for life insurance companies as a class. That this assumption is nearly correct is indicated by the close approximation of the average annual yield thus determined to the later weighted figures for all companies as quoted from the "Insurance Year Book," comparative yields by the two methods being available from 1872 to 1904.

Average yields from mortgage loan investments by life insurance companies.—Sources of this information are arithmetical averages of yields for individual companies from Zartman's "Investments of Life Insurance Companies" for the period 1860 to 1904; weighted yields computed by the authors for the period 1905 to 1914 from "Best's Insurance Reports" for companies having mortgage loan investments in excess of $1,000,000 for the period 1911 to 1914; and yields for all life insurance companies for 1915 and later years as determined by the "Institute for Research in Land Economics and Public Utilities." While yields for earlier years are determined in a less accurate manner than those for later years, the correspondence between yields determined for mean invested assets by both methods would indicate that no great error will be found in the yields computed in the less accurate manner.

Average annual rates of interest on collateral time loans and commercial paper in New York.—The sources of informa-

tion for interest rates of these descriptions are varied and will be stated separately. Other sources of information available with respect to each of these interest rate series, and which have been used to check the substantial accuracy of the data appearing in following tabulations, will be given occasional mention. Call money rates from 1849 to 1865 are from the "Rate of Interest," a book by Professor Irving Fisher; and for 1865 and later years the annual average is the arithmetical average of monthly average rates published in the *Harvard Review of Economic Statistics*. Data for all other collateral time loans, except over year loans, were obtained from the *Financial Review* for the period 1890 to 1920, and from the Bank and Quotation Section of the *Commercial and Financial Chronicle* for later years, the yearly average figures being averages of the weekly high and low quotations. When quotations for not more than three successive weeks were lacking, interpolated figures were substituted for missing quotations, such figures being compared with available quotations for other collateral loans of the most comparable type. When weekly quotations for any type of collateral loan were lacking in large numbers, the average weekly difference was determined between available quotations for that loan and quotations of the same weeks for the most comparable type of collateral loan for which complete quotations were available. The difference between average annual quotations was assumed to be the average difference between comparable weekly quotations. It was necessary to follow this procedure only with respect to 30-day and 7-month collateral loans. The quotations given for over year collateral loans were obtained by averaging weekly high and low quotations given in Senate Document No. 262 of the 2d Session of the Sixty-sixth Congress, these figures being supplied by the Federal Reserve Board. The earlier rates on 60 to 90 day commercial paper are those given by Professor Irving Fisher in his "Rate of Interest" for 60-day commercial paper for the period 1849 to 1859 and for prime two-name, 60-day paper for the period 1860 to 1865. Annual rates for the period 1866 to 1924 are arithmetical averages of the monthly average quotations published by the *Harvard Review of Economic Statistics*, from which quotations were obtained in the same manner for the period 1890 to 1925 with respect

to 4 to 6 months' commercial paper. A continuous series for 60 to 90 day commercial paper covering the period 1831 to 1920 has been prepared by Professor Frederick McCauley and published in the form of a chart by the Federal Reserve Bank of New York. This chart, No. 120, gives maximum and minimum yearly rates as well as average annual rates, the figures for the average rates being used in the interest rate charts of this chapter.

Quotations for 30-day and 7-month collateral loans on the New York Exchange were never extensive and practically ceased in 1912, while trade acceptances had replaced 60 to 90 day commercial paper to such an extent that 60 to 90 day paper is not quoted after 1924 by the *Commercial and Financial Chronicle.*

Minimum rates of discount of European banks.—These quotations were obtained from the "Bankers Almanac and Year Book," published by Thomas Skinner & Company of London, and converted to decimal yields by the authors. Ten-year average rates of discount appear upon an accompanying chart (page 132), as the chart for yearly average rates exhibited so many crossing lines that it was impossible to make intelligent determinations either as to interest rate trends or with respect to geographical variations in such rates.

Index figures for interest rate series.—Determinations of the interest rates for the several series which correspond to an index figure of 100 are approximations, since any period may be selected for such a determination, according to individual judgment. For the purpose of the following tabulations and curves, the period, 1861 to 1900, inclusive, has been selected, for the reason that it is the longest representative period common to the several interest rate series. The selection of 1900 as a terminal date for the period arises from the fact that rates of yield on government bonds at current market prices, from a few years prior to 1900 to the year 1918, are not representative rates of interest, but are controlled by the availability of the major portion of Government bonds during this period for deposit by national banks to secure the circulation of bank notes. This circulation privilege is reflected in yields which are much lower than would have obtained on the basis of investment for interest without other inducement.

The approximate 40-year average interest rates for the period 1861 to 1900, inclusive, are:

Government Bonds	3.708
Investments of New York State Mutual Savings Banks	5.54
Total Investments of Life Insurance Companies	5.90
Mortgage Loan Investments of Life Ins. Companies	6.71
60 to 90 Day Commercial Paper in New York	5.68

For the same period the average index price of wholesale commodities is 113.56. Dividing the average rates of interest by the average index price of wholesale commodities for the 40-year period gives the following rates of interest, which are used as the rates corresponding to an index figure of 100, in the preparation of the charts of this chapter.

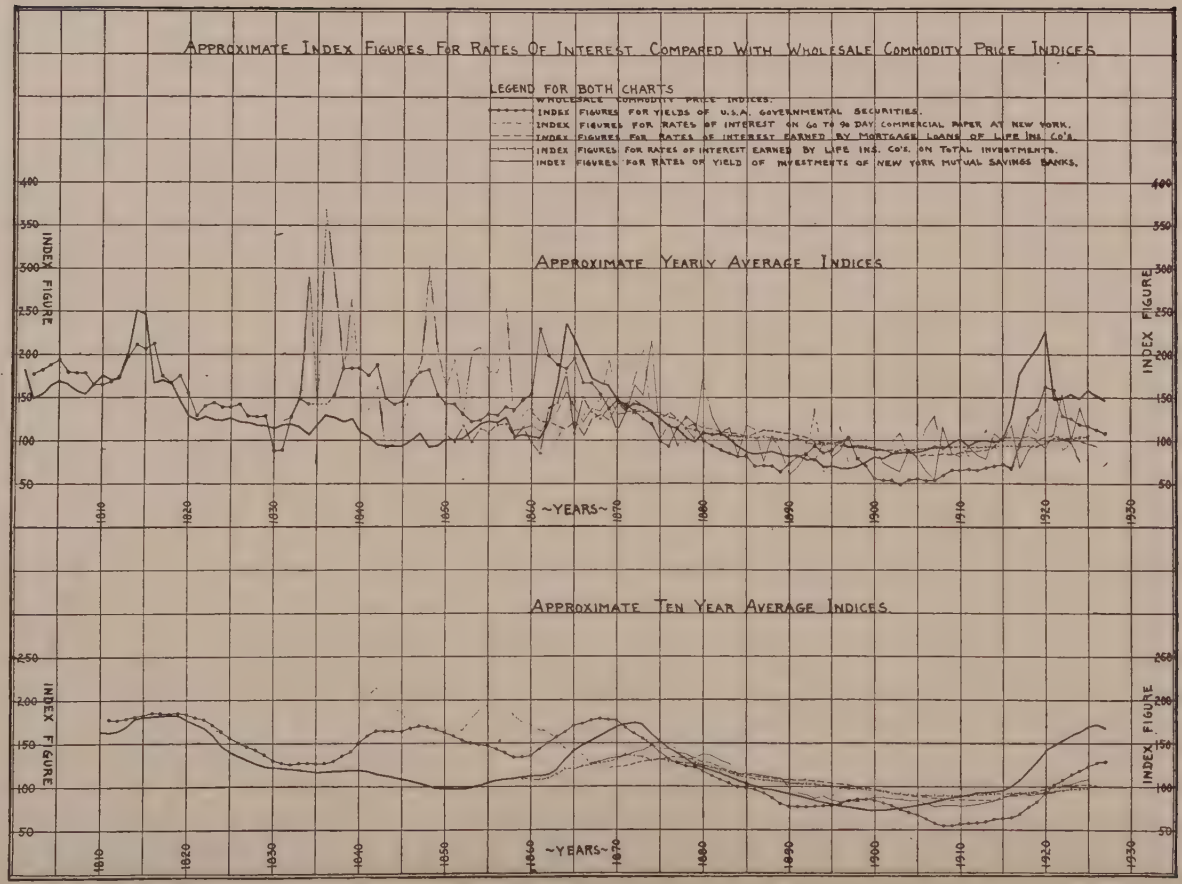
Interest rates corresponding to an index figure of 100.

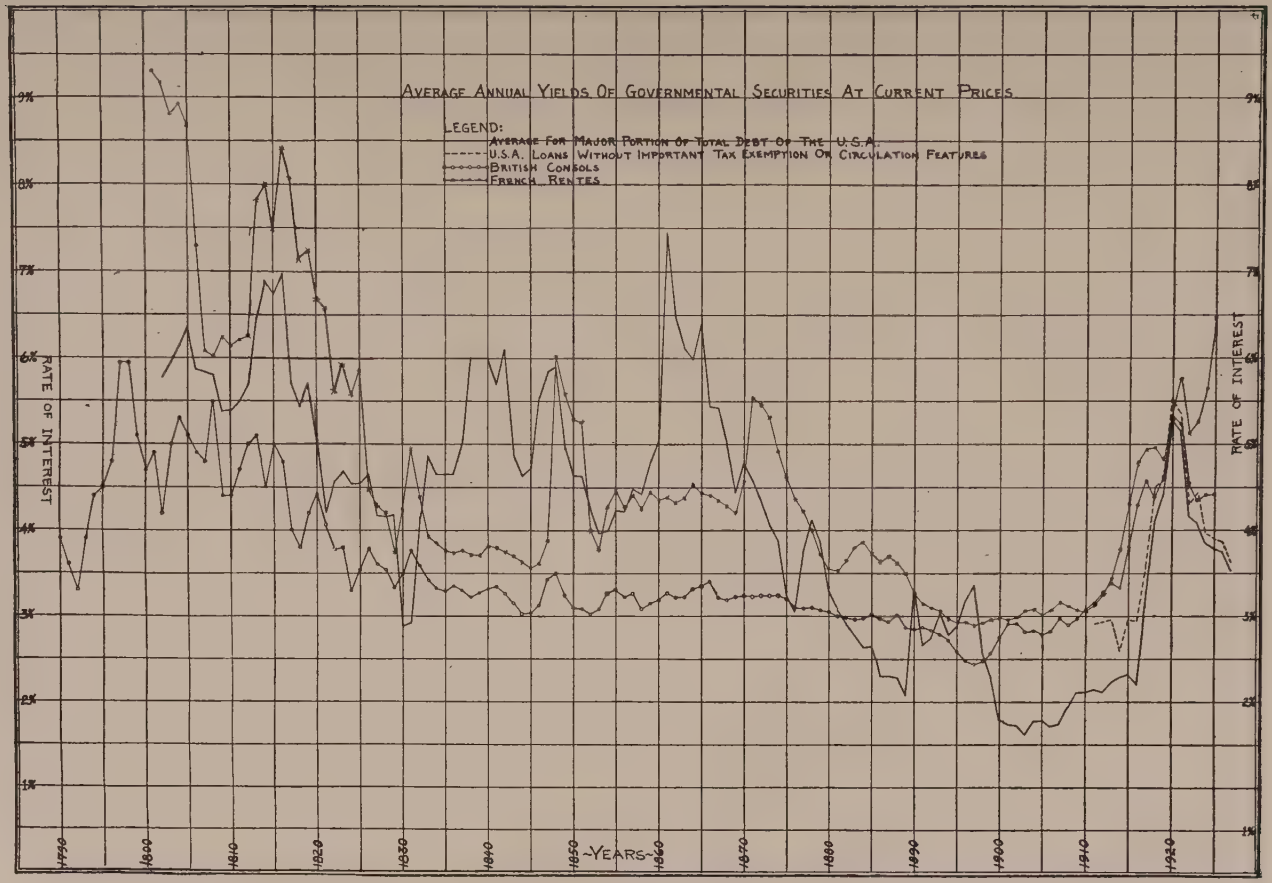
Government Bonds	3.265%
Investments of New York State Mutual Savings Banks	4.88
Total Investments of Life Insurance Companies	5.20
Mortgage Loan Investments of Life Insurance Companies	5.91
60 to 90 Day Commercial Paper in New York	5.00

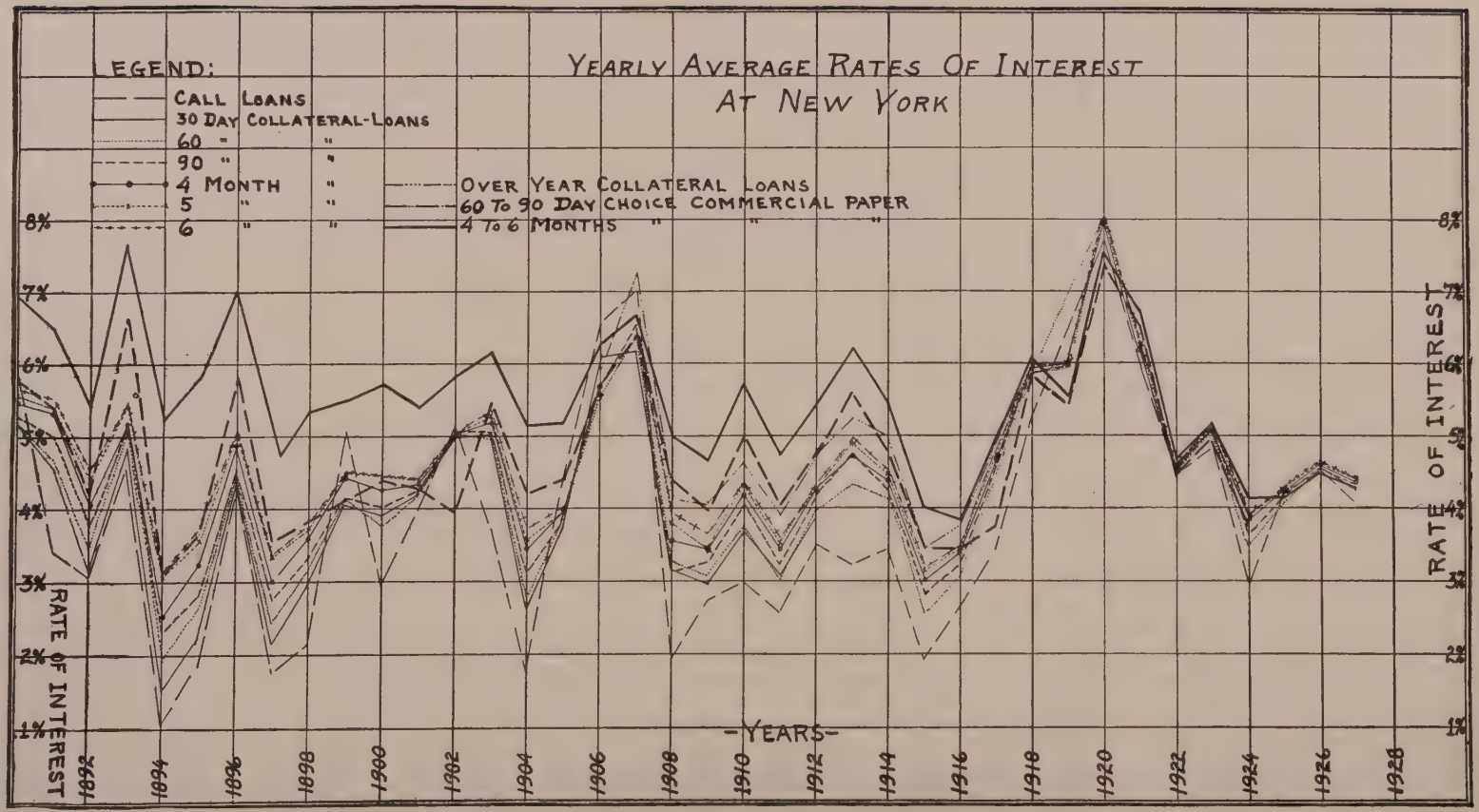
Dividing each rate of the series of yearly average rates of interest, and of ten year average rates of interest, by the rate of interest equivalent to the index figure of 100, gives series of annual and 10 year average index figures.

Approximate Annual Average Rates Of Interest
Legend:
U.S.A. Governmental Securities
60 To 90 Day Commercial Paper At New York
Investments Of New York Mutual Savings Banks
Mortgage Loans Interest Of Life Insurance Companies
Total Investments Of Life Insurance Companies
RATE OF INTEREST
-YEARS-
1810
1820
1830
1840
1850
1860
1870
1880
1890
1900
1910
1920
1930
1940
1%
2%
3%
4%
5%
6%
7%
8%
9%
10%
11%
12%
13%
14%
15%
16%
17%
18%
19%

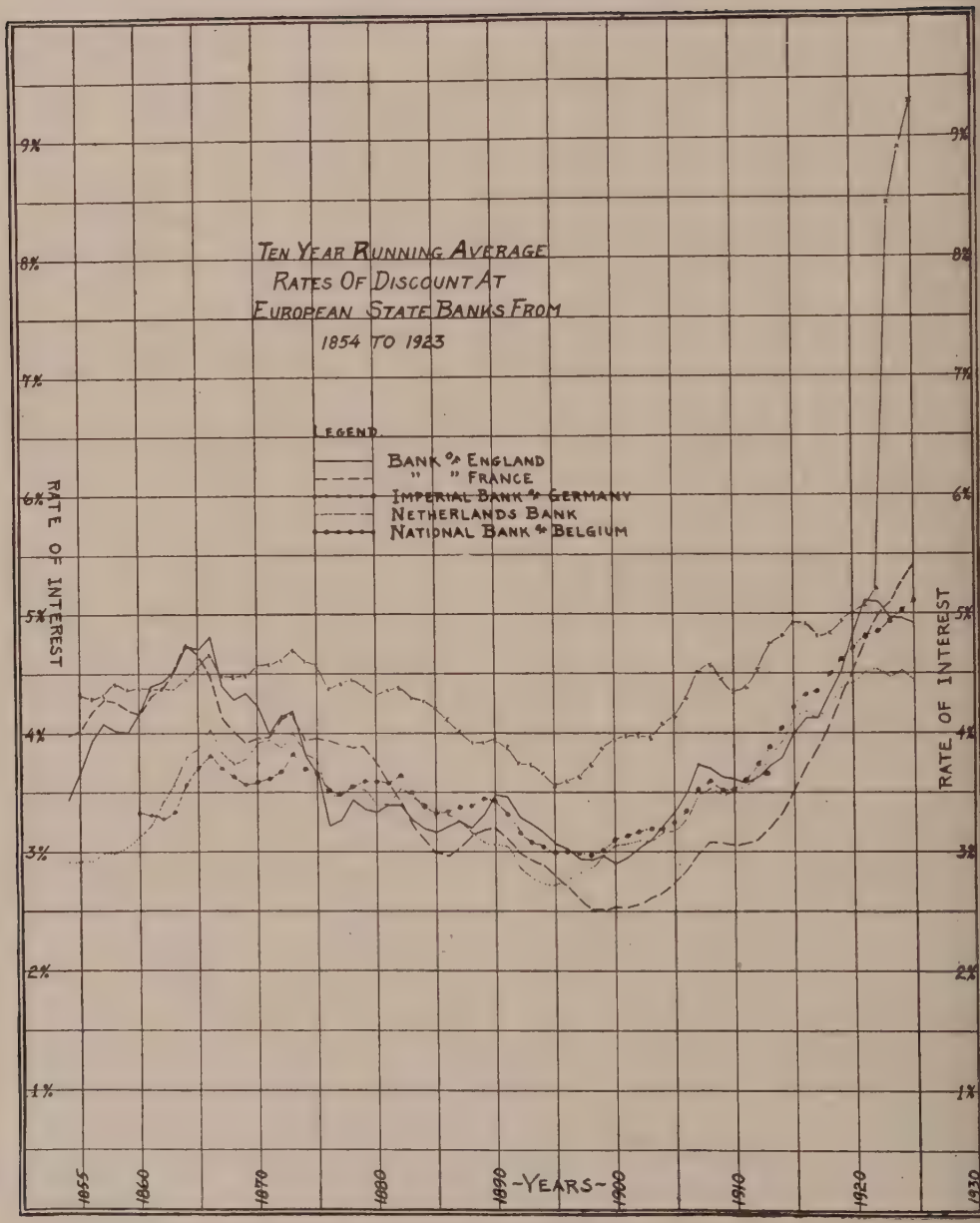
Approximate Ten Year Running Average Rates Of Interest
Legend:
U.S.A. Governmental Securities
60 To 90 Day Commercial Paper At New York
Investments By New York Mutual Savings Banks
Investments By Insurance Companies
Total Investments Of Life Insurance Companies
RATE OF INTEREST
RATE OF INTEREST
~YEARS~
1810
1820
1830
1840
1850
1860
1870
1880
1890
1900
1910
1920
1930


Approximate Index Figures For Rates Of Interest Compared With Wholesale Commodity Price Indices
LEGEND FOR BOTH CHARTS
WHOLESALE COMMODITY PRICE INDICES.
INDEX FIGURES FOR YIELDS OF U.S.A. GOVERNMENTAL SECURITIES.
INDEX FIGURES FOR RATES OF INTEREST ON 60 TO 90 DAY COMMERCIAL PAPER AT NEW YORK.
INDEX FIGURES FOR RATES OF INTEREST EARNED BY MORTGAGE LOANS OF LIFE INS. CO'S.
INDEX FIGURES FOR RATES OF INTEREST EARNED BY LIFE INS. CO'S. ON TOTAL INVESTMENTS.
INDEX FIGURES FOR RATES OR YIELD OF INVESTMENTS OF NEW YORK MUTUAL SAVINGS BANKS.
Approximate Yearly Average Indices
INDEX FIGURE
400
350
300
250
200
150
100
50
1810
1820
1830
1840
1850
1860
~YEARS~
1870
1880
1890
1900
1910
1920
1930
Approximate Ten Year Average Indices
INDEX FIGURE
250
200
150
100
50
1810
1820
1830
1840
1850
1860
~YEARS~
1870
1880
1890
1900
1910
1920
1930


AVERAGE ANNUAL YIELDS OF GOVERNMENTAL SECURITIES AT CURRENT PRICES
LEGEND:
AVERAGE FOR MAJOR PORTION OF TOTAL DEBT OF THE U.S.A.
U.S.A. LOANS WITHOUT IMPORTANT TAX EXEMPTION OR CIRCULATION FEATURES
BRITISH CONSOLS
FRENCH RENTES
RATE OF INTEREST
RATE OF INTEREST
9%
8%
7%
6%
5%
4%
3%
2%
1%
1790
1800
1810
1820
1830
1840
1850
1860
1870
1880
1890
1900
1910
1920
-YEARS-


LEGEND:
CALL LOANS
30 DAY COLLATERAL-LOANS
60 " "
90 " "
4 MONTH " "
5 " "
6 " "
OVER YEAR COLLATERAL LOANS
60 TO 90 DAY CHOICE COMMERCIAL PAPER
4 TO 6 MONTHS " "
YEARLY AVERAGE RATES OF INTEREST
AT NEW YORK
RATE OF INTEREST
8%
7%
6%
5%
4%
3%
2%
1%
-YEARS-
1892
1894
1896
1898
1900
1902
1904
1906
1908
1910
1912
1914
1916
1918
1920
1922
1924
1926
1928

TEN YEAR RUNNING AVERAGE
RATES OF INTEREST IN NEW YORK
LEGEND:
COLLATERAL LOANS
I = CALL
II = 30 DAYS
III = 60 "
IV = 90 "
V = 4 MONTHS
VI = 5 "
VII = 6 "
VIII = 7 "
IX = OVER YEAR
COMMERCIAL PAPER
X = 60 TO 90 DAYS, CHOICE.
XI = 4 TO 6 MONTHS, CHOICE.
RATE OF INTEREST
7%
6%
5%
4%
3%
2%
1%
1900
1902
1904
1906
1908
1910
1912
1914
1916
1918
1920
1922
1924
1926
1928
1930
1932
YEARS


TEN YEAR RUNNING AVERAGE
RATES OF DISCOUNT AT
EUROPEAN STATE BANKS FROM
1854 TO 1923
LEGEND
BANK OF ENGLAND
" " FRANCE
IMPERIAL BANK OF GERMANY
NETHERLANDS BANK
NATIONAL BANK OF BELGIUM
RATE OF INTEREST
RATE OF INTEREST
9%
8%
7%
6%
5%
4%
3%
2%
1%
1855
1860
1870
1880
1890
-YEARS-
1900
1910
1920
1930

Chapter VI

THE DISCOUNT FOR HAZARD METHOD OF VALUATION

This process for the determination of values for future speculative income comprises two separate discounts: the first, a determination of the safe interest value which could be attributed to assured future income of the same amounts and dates of receipt; and the second, a reduction of that safe interest value by an amount representative of the aggregate effect of a number of other influences which are more or less proportionate to estimated hazard or risk of loss. The percentage of the latter reduction of the safe interest value is termed the "discount for hazard." If 40% of the safe interest value is deducted to determine the appraised value of speculative income a 40% discount for hazard has been made. The complement of the discount for hazard is known as the "factor of safety," which would be 60% with a 40% discount for hazard. The safe interest rate discount factor multiplied by the decimal figure for a factor of safety gives a combined discount factor for interest and hazard corresponding to the discount represented by a speculative rate of interest; and for that reason charts and tables of this chapter have been prepared for factors of safety rather than on the basis of discounts for hazard. Unity less the decimal amount of the factor of safety gives the discount for hazard as a decimal, and such decimals multiplied by one hundred give the percentage figures for factor of safety and discount for hazard. This brief statement fully outlines the simple arithmetical processes involved in the application of the discount for hazard method to valuations of future speculative income, directly, by the process just described, or, indirectly, by applying the same principles to the selection of a speculative rate of interest.

Certain principles of speculative investment are almost axiomatic. The statement can be made without fear of

contradiction that the minimum yield expected from an investment involving risk of loss will be at least as great as the safe rate of interest obtainable without risk of loss. Losses of capital in some ventures are inevitable when speculation is an element of investment, and the yield promised by any class of speculative investments must, at least, be sufficient for the aggregate return in excess of a safe rate of interest from successful speculations to offset the aggregate losses from unsuccessful speculations. This additional return, which may be called insurance against loss of capital, would just permit the speculative investor to obtain the same reward for the use of his capital as would accrue to the investor in riskless enterprises. As compensation for the assumption of the risk of loss attendant upon a single venture, the yield promised by any speculative enterprise must include an attractive profit, over and above a safe rate of interest and insurance against loss, in order that the speculative may compete with the safe enterprise for capital. The cost of investment management as reflected in the yields of speculative investments is essentially dependent upon the degree of risk of loss, while loss of income during periods between investments and the cost of reinvestment are reflected in the higher yields expected from either safe or speculative enterprises when frequent reinvestment is required and no greater safety attaches to the short term in comparison with the long term investment.

A speculative yield thus comprehends an aggregate of charges, such as a safe rate of interest, cost of management, and other charges common to both safe and speculative yields, an amount sufficient to cover insurance against ultimate loss of capital in the event that a large number of speculative investments are made, and a margin of profit over and above all other charges which is sufficient to attract capital from safe to speculative enterprise. All of the algebraic formulas developed in previous chapters involve the use of speculative rates of interest in determining present worth of future income as at date of valuation. This chapter provides a basis for determinations, over varying terms of investment, of speculative yields, which reflect definite safe rates of interest combined with discounts for hazard proportionate to the estimated risk of investment. The same

principles are applicable in the determination of yields which should be expected from speculative as compared with normal business, or from new in comparison with established enterprises.

It will be obvious that a majority of the components of any speculative rate of interest are dependent upon risk independent of time, but that the safe rate of interest is entirely a function of time. Such dissimilar elements may not be combined into one discount factor reflecting with fair accuracy their combined effect, unless the evaluator has made intelligent and individual analysis of the relative importance of each component. It is particularly advisable that the effects of risk and time should be separated in studies of comparative sales of similar properties, as the same degree of risk and the same safe rate of interest combine into speculative rates of interest which vary inversely with the term of the investment. Separation of the speculative rate of interest into its constituent parts with individual consideration of each element is the basic principle of the discount for hazard method of valuation. Such separation and analysis of components may be carried to as great a degree of refinement as the evaluator desires, but should always embody independent consideration of the predominant elements of time and hazard, neither of which is in any way related to the other.

Enumeration and discussion of other components peculiar to speculative yields and of those common to both safe and speculative rates of interest might be illuminating, but those mentioned will suffice for the purposes of this discussion. A more elaborate investigation would but strengthen the evidence in support of the inevitable conclusions that: of the unrelated components of speculative yields, the most important are those dependent upon, and proportionate to, hazard, independent of the length of investment; the relative importance of the safe or investment rate of interest as a component will increase as the period of investment increases; and there are minor components which may be regarded as fixed charges quite independent of either risk or time.

It is possible to make current estimates which will approximate conditions with respect to many of the principal elements of speculative yields, but as one attempts to analyze past transactions the difficulties increase. The investment

rate of interest in the past may be determined quite accurately by statistics and it is always possible to separate a speculative yield into two parts: one, the discount required to give an investment yield; the other, a further discount representative of the aggregate effect of all other influences. Since nearly every important component other than the investment yield varies more or less proportionately with the degree of speculation, it will usually suffice if the speculative yield is separated into two components: one called a discount for interest at a safe rate; and the other an aggregate component called a discount for hazard.

For practically every industry and locality, it is possible for the evaluator to compile and use statistical data in approximations of both the safe rate of interest and the customary discount for hazard, the latter element being determined by analysis of the sales of fairly comparable properties. Even for an industry such as metal mining, in which but few sales of important properties are made and in which properties are seldom comparable, rather definite conclusions as to the customary discount for hazard may be reached by statistical analysis of available sales data. The more frequently sales take place and the more comparable are the properties of an industry, the more accurately may the customary discount for hazard be determined by statistical methods.

While always an element of a speculative yield, discounts for hazard are very frequently included and concealed in estimates of the amount of net expected yearly income, and may also be found at times in each and every estimate necessary to the valuation. The proper use of the discount for hazard method of valuation requires that each factor be estimated as accurately as possible, that the actual estimates be used in the computation of value at a safe rate of interest, and that the entire discount for hazard be applied at one time to the value so determined. This forces the evaluator to consider the actual hazard involved, rather than to make an arbitrary deduction here and there throughout the whole valuation, at last reaching a conclusion as to value without even knowing the amount of hazard for which provision has been made.

Engineering practice recognizes the frequent use of empirical formulas, and factors of safety are customarily used in

connection with such formulas. An experience formula is developed, based upon physical tests and experience combined with mathematics, and this formula is solved to obtain a result which is further reduced by application of a factor of safety to cover contingencies, variation in qualities of materials, unseen flaws, etc. The discount for hazard or factor of safety method of valuation merely applies this generally recognized engineering practice to computations or estimates of values of future income, deduction for uncertainty being the function of a factor of safety.

As one departs from the field of safe investment, one passes progressively through speculative investment, speculation, and gambling. Any investment that is not safe is more or less of a gamble. The discount for hazard method of valuation applies to speculative income, and the principles involved are simply another means of expressing the betting odds on the venture. If it is expected that four out of five comparable ventures will be as successful as estimated by the evaluator, and no profit over and above a safe rate of interest and insurance against loss is required, the betting odds are 4 for success against 1 for failure in comparison with safe investment, or the bet should pay 5 to 4 if successful. In such a case the maximum factor of safety used would be 80%, or there would be a minimum discount of 20% for hazard. These illustrations purposely omit the elements of profit to the investor, which will vary with the degree of risk and at times with the relative abilities of the investor and the seeker of capital as barterers. If the odds are such that one of two ventures will be a total loss and the other a complete success, both the maximum factor of safety and the minimum discount for hazard would be 50%, the discount for hazard being the complement of the factor of safety in all cases.

A successful speculative investment in prospective future income of long term duration becomes a safe and conservative investment in many instances. The speculator takes his profit for accepting the risk and reaps his reward as the risk disappears, which is usually within a relatively short time after it is taken. The conservative investor then steps in and places his capital in the enterprise with the expectation of a more conservative return. The investment over its entire history thus yields a quick profit to the speculator

through an appreciation in value, the anticipated amount of which is reflected in the discount for hazard used in valuation, and a later safe yield to the investor for which provision is made in the safe rate of interest used in discounting future expected income to a safe interest value as at date of valuation. The discount for hazard method of valuation thus either separates the elements of profit and interest, or assists in the selection of a speculative rate of interest which is an accurate index of their estimated combined effect.

The fact that the same speculative rate of interest does not represent the same degree of hazard when applied to investment periods of different length may be forcefully illustrated by concrete examples. In the following examples a speculative rate of interest of 10%, a safe rate of interest of 4%, and an equal annual income rate of $1,000 a year, valued under the premises of the Hoskold type of sinking fund valuation, by Formula IV-A of Chapter III, will be used. The income is valued, first, at the speculative rate of interest; and, secondly, at the safe rate of interest; the value determined at the speculative rate of interest then being divided by the value determined at the safe rate of interest to find the factor of safety represented by the speculative rate of interest.

The discount for hazard is the complement of the factor of safety, that is, one minus the factor of safety is the discount for hazard in each case. The following tabulation is self-explanatory:

Term of investment in years (n yrs.)	Values Determined		Factor of safety	Discount for hazard	Factor of safety plus discount for hazard
	With 10% speculative rate of interest	With 4% safe rate of interest			
	Col. I	Col. II	Col. III	Col. IV	Col. V
	Valuation formula	Valuation formula	I ÷ II	1 − III	III + IV
1	909.09	961.54	0.945452	0.054548	1.000000
5	3,513.37	4,451.82	0.789199	0.210801	1.000000
10	5,455.81	8,110.90	0.672652	0.327648	1.000000
20	7,486.05	13,590.33	0.550837	0.449163	1.000000
30	8,486.80	17,292.03	0.490793	0.509207	1.000000
40	9,047.85	19,792.77	0.457129	0.542871	1.000000
50	9,385.25	21,482.18	0.436885	0.563115	1.000000
Perpetual	10,000.00	25,000.00	0.400000	0.600000	1.000000

It is evident from the preceding tabulation that, as the term of investment varies from one year to perpetuity, the speculative rate of interest of 10% in comparison with a safe rate of interest of 4% represents discounts for hazard progressively increasing from 5.45% to 60% of the safe interest value.

As a further illustration it will be interesting to determine the speculative rates of interest for other periods of investment which represent the same factor of safety as is represented, let us say, by a 10% yield for a 10-year term of investment, all factors of safety being in comparison with a 4% safe rate of interest and determined in accordance with the premises of the Hoskold sinking fund method of valuation for equal annual income. The problem consists in determinations of speculative yields for varying terms of investment, which represent a factor of safety of 0.672652. The safe interest rate values determined for various periods of investment are those under Column II of the preceding tabulation and under Column I of the succeeding tabulation. Multiplying those values by 0.672652 determines a value for each term of investment which reflects that factor of safety and appears under Column II

of the following table. For each of these values under Column II, it is necessary to determine the yearly contribution to the sinking fund (accumulating at 4% interest compounded annually), which will be necessary to amortize the value at the end of the term of investment, the amounts of these contributions to sinking fund being those under Column IV of the following table. Subtracting the sinking fund contribution from the yearly income of $1,000 a year for each period of investment determines the sum available for the payment of interest to the investor. The amount available for the payment of interest divided by the value upon which interest must be paid gives the rate of interest paid, which is the result sought. The processes just described are indicated in the following tabulation:

Term of investment in years (*n* yrs.)	Value of income at a safe rate of interest of 4 %	Value of income with a factor of safety of 0.672652	Amount per year required by sinking fund to amortize 1 in *n* years at 4 % interest	Yearly amount required for sinking fund to amortize the investment given under Column II	Yearly amount available to pay interest to the investor	Rate of interest representing a factor of safety of 0.672652
	Col. I	Col. II	Col. III	Col. IV	Col. V	Col. VI
	Valuation formula	0.6725162 × I	Table 3	II × III	$1000.00 − IV	V ÷ II
1	$ 961.54	$ 646.78	$1.0000000	$646.78	$353.22	*54.61 %
5	4,451.82	2,994.52	0.1846271	552.87	447.13	14.93 %
10	8,110.90	5,455.81	0.0832909	454.42	545.58	10.00 %
20	13,590.33	9,141.56	0.0335818	306.99	693.01	7.58 %
30	17,292.03	11,631.51	0.0178301	207.39	792.61	6.81 %
40	19,792.77	13,313.64	0.0105235	140.11	859.89	6.46 %
50	21,482.18	14,450.02	0.0065502	94.65	905.35	6.27 %
Perpetual	25,000.00	16,816.29	0.000000	0.000000	1,000.00	5.95 %

* As an exaggerated illustration it is interesting to note that for a period of investment of one day, compounding interest annually, a speculative yield of 17,769 % would be required to represent a factor of safety of 0.672652 in comparison with a safe interest rate of 4 %.

The preceding discussion and computations inevitably lead to the conclusion that in a valuation of future speculative income, interest and hazard should be considered separately, either by means of separate discounts or through the selection of a speculative rate of interest which accurately reflects their combined influence. One exception to this conclusion may be

noted, no separation for risk and hazard being required when the speculative rate of interest selected for discounting to present worth is one reflected in current sales of future income to be received over identical periods of time and subject to identical risk.

The fundamental conception that a speculative yield comprises the unrelated elements of risk and interest is more generally recognized in commercial practice than it is in mathematical treatises on the principles of finance, possibly because the discount for hazard is the simpler of the two methods for valuing terminable speculative income. The practice of assuming perpetual speculative income for the purpose of valuation is common both in commercial practice and in treatises on valuation, although the assumption of perpetuity is logical only in the event that the income is assured rather than speculative. With assumption that perpetuity is the term of investment, the necessity of providing by deductions from income for return of capital vanishes, and division of the safe rate of interest by the speculative yield determines the factor of safety. This practice is correct in the valuation of income from investments which also assure the return of capital, such as bonds and mortgages; but it is incorrect when applied to income from which deduction must be provided for the return of capital. Some authorities on valuation recognize this principle and state that amounts required by the sinking fund should be deducted from gross income in determining net income to be valued.

Having outlined the principles involved and the necessity for separate consideration of risk and hazard both in the analysis of past transactions and in the valuation of future terminable income, we will next proceed to the development of methods which will assist in the application of this process of valuation, first considering the factors of safety embodied in speculative rates of interest and subsequently those which may be included and concealed in other estimates and places.

Factors of safety reflected in speculative rates of interest used for discounting to present worth.—The numerical value of the factor of safety, represented by any speculative rate of interest, for any term of investment, with any time distribution of income, and under any of the several premises of valuation, is found by dividing the value determined by

means of the speculative rate of interest by the value found by discounting at a safe rate of interest, both values being the present worth of future income as at date of investment or valuation. The equivalent algebraic process is a division of the valuation formula properly selected for use in discounting to present worth at a speculative rate of interest, by a formula identical in every respect, except that the selected safe rate of interest has been consistently substituted for the speculative, or for both the speculative and the semispeculative, rate of interest appearing in the formula.

With the Hoskold Type of Sinking Fund Formulas, Division IV, the factor of safety represented by any speculative rate of interest in comparison with any safe rate of interest varies only with the duration of the period of investment, being constant irrespective of the time distribution of income over an identical term of investment. With any other premises of valuation, that is, the premises of Divisions I, II, III, V, and VI (or combinations of the premises of Division IV when the sinking fund accumulation is positive in amount, with premises of Division II when the sinking fund is negative in amount), the factor of safety also varies with the time distribution of income within a fixed term of investment. This attribute of the premises of Division IV permits the direct computation of factors of safety from the algebraic formula,

$$F = \frac{eE^n}{rE^n - r + e}$$

in which, F is the factor of safety, e is the decimal equivalent of the safe rate of interest, r is the decimal equivalent of the speculative rate of interest, n is the term or period of the investment in years, and E is unity plus e. It likewise permits of a solution of the preceding formula for the speculative rate of interest equivalent to any factor of safety, such solution giving the formula:

$$r = \frac{e(E^n - F)}{F(E^n - 1)}$$

As algebraic proof of these statements, take the general formula of Premise IV for determination of values by both the safe and the speculative rates of interest, and divide the value

under the speculative rate of interest by the value under the safe rate of interest as follows:

$$\text{Speculative } V = e\frac{A_1E^{n-1} + A_2E^{n-2} + A_3E^{n-3} + \cdots A_n}{rE^n - r + e}$$

$$\text{Safe } V = e\frac{A_1E^{n-1} + A_2E^{n-2} + A_3E^{n-3} + \cdots A_n}{eE^n - e + e}$$

$$\frac{\text{Speculative } V}{\text{Safe } V} = F = \frac{eE^n}{rE^n - r + e}$$

Similarly, take the general formula common to Premises I, II, III, and V, and repeat the above process as follows:

$$\text{Speculative } V = \frac{A_1R^{n-1} + A_2R^{n-2} + A_3R^{n-3} + \cdots A_n}{R^n}$$

$$\text{Safe } V = \frac{A_1E^{n-1} + A_2E^{n-2} + A_3E^{n-3} + \cdots A_n}{E^n}$$

$$\frac{\text{Speculative } V}{\text{Safe } V} = F = \frac{E^n(A_1R^{n-1} + A_2R^{n-2} + A_3R^{n-3} + \cdots A_n)}{R^n(A_1E^{n-1} + A_2E^{n-2} + A_3E^{n-3} + \cdots A_n)}$$

Similarly, take the general formula of Premises VI and repeat the process shown above as follows:

$$\text{Speculative } V = j\frac{A_1J^{n-1} + A_2J^{n-2} + A_3J^{n-3} + \cdots A_n}{rJ^n - r + j}$$

$$\text{Safe } V = e\frac{A_1E^{n-1} + A_2E^{n-2} + A_3E^{n-3} + \cdots A_n}{eE^n - e + e}$$

$$\frac{\text{Speculative } V}{\text{Safe } V} = F = \frac{jE^n(A_1J^{n-1} + A_2J^{n-2} + A_3J^{n-3} + \cdots A_n)}{(rJ^n - r + j)(A_1E^{n-1} + A_2E^{n-2} + A_3E^{n-3} + \cdots A_n)}$$

Since the relations just shown hold for the general formulas of each division, they will hold for all other derived formulas representing the various time distribution types of income. It is thus shown for Premises IV that the formula for the safety factor (F) is the same for all types of income distribution if negative sinking fund accumulations are permitted, since the income terms, which are confined to the numerators of the fractions representing the speculative and the safe values, are eliminated by the division process resulting in the safety factor formula; and that, with the formulas of all other Premises, the safety factor varies with the variations of income types, for the reason that the income terms in the numerators of the fractions representing the speculative and safe values are not the same and, therefore, appear in the

safety factor formula. The reason for this difference between the premises of Division IV, Hoskold type of sinking fund valuation formulas, and those of other divisions is found in the condition imposed under the premises of Division IV that interest is never compounded at the speculative rate. The amount of capital upon which interest is paid at the speculative rate does not change during the entire term of investment, and interest is paid on this capital, yearly, either from income or from the sinking fund. The other premises of valuation are all based upon the payment of a speculative rate of interest to the investor upon an amount of capital amount which varies from year to year, excepting the premises of Division III, in accordance with which, it will be recalled, the varying amounts in the sinking fund are presumed to earn a speculative rate of interest, and this causes the variation of the factor of safety as the time distribution of income changes within the investment period. Since the product of a constant and a variable is always a variable, valuation by means of a combination of the premises of Divisions II and IV causes the factor of safety to vary with changes in the time distribution of income, when such changes relate to income within the period to which Division II premises apply, which will be the years in which negative accumulations in the sinking fund are added to the speculative investment. Thus, with Division II premises applied to a deferment period during which no income is received and Division IV premises applied to the entire period of income receipt, there will be a constant factor of safety regardless of time distribution of income within the period of income receipt; but, with Division II premises applied to negative total amounts in the sinking fund, after the receipt of income has started, the factor of safety will vary with changing distributon of income over the period of its receipt.

Tables will be found in Appendix B for factors of safety determined for equal annual income valued according to the premises of Divisions I to V, inclusive, and for deferment at compound interest. The factor of safety for a period of deferment multiplied by the factor of safety for a succeeding period of equal annual income gives a factor of safety for the entire period of investment, it being understood that when a

deferment factor of safety is thus used the premises of compound interest are used during the deferment period and the premises used in the valuation of the equal annual income apply during the period of income receipt. The factors of safety determined for the Hoskold type of valuation formulas for any term of investment apply to any time distribution of income, either equal annual amounts or variable annual amounts, which may be positive or negative in quantity for any year. The tables have been computed to show the factors of safety reflected by each speculative rate of interest up to 20%, in comparison with safe interest rates of 3%, 4%, 5%, and 6%, for terms of investment from 1 to 50 years, for each yearly increment of the 50-year period, and for a range of speculative yields giving a variation in the factor of safety from 1, safety, to as low as 0.10, or one chance out of ten for success. The lower factor certainly exceeds the limit that any evaluator would consider proper in the application of appraisal methods to the valuation of speculative income, as, with such small chance of success, the gambling element is so predominant that no consideration of the element of investment is required. Factors of safety for periods of less than one year have not been computed; but, with interest payable and compounded at yearly intervals, beginning with the end of the first year, such factors of safety may be determined according to any of the premises of valuation by one formula which is:

$$F = \frac{1 + en}{1 + rn}$$

in which n is the fractional part of a year for which the factor of safety is desired; e, the safe rate of interest; r, the speculative rate of interest; and F, the factor of safety.

The factors of safety appearing in the tables of Appendix B were plotted on curves showing factors of safety for each rate of interest over a term of investment varying from 1 to 50 years, or less than 50 years if the factor of safety had been reduced to less than 0.10 at some prior year. By scaling points from these curves representing life in years equivalent to a constant factor of safety for varying rates of interest and plotting such points as the basis for a second set of curves, the charts on pages 148–159 showing rates of interest equivalent to a fixed factor of safety for any term of investment

between the limits of 1 and 50 years were obtained. Solution of the formula, $F = \frac{1 + en}{1 + rn}$, for n as a fractional year gives a formula from which points on the curves of any of the charts may be plotted directly for periods of less than one year, the solution being $n = \frac{1 - F}{r - 2}$. The set of twelve charts is limited to a maximum speculative yield of 20% in comparison with safe rates of interest of 3%, 4%, 5%, and 6%.

Charts designated as those for factors of safety in connection with the compound interest deferment formula are based on the factor of safety formula, $F = \frac{E^n}{R^n}$, which is equivalent to $\frac{\frac{1}{R^n}}{\frac{1}{E^n}}$, the value at the speculative rate of interest divided by the value at the safe rate of interest.

Charts designated as those for factors of safety in connection with the compound interest *annuity* valuation formulas apply to the valuation of equal annual income under the premises of Divisions I, II, III, or V, and are based on the formula,

$$F = \frac{eE^n(R^n - 1)}{rR^n(E^n - 1)},$$

which is equivalent to the value at the speculative rate of interest divided by the value at the safe rate of interest or:

$$\frac{\frac{R^n - 1}{rR^n}}{\frac{E^n - 1}{eE^n}}$$

Charts designated as those for factors of safety in connection with Hoskold type valuation formulas apply to any type of income under Division IV, whether such income is equal or variable in annual amount, and are computed directly from the formula,

$$F = \frac{eE^n}{rE^n - r + e}$$

which has been shown previously to be the expression by which the value at a safe rate of interest of any time distribution of income may be multiplied to obtain the value of

that income when discounted to present worth by means of a speculative rate of interest.

No factors of safety have been computed, and, therefore, no charts appear with respect to the valuation of equal annual income under the premises of Division VI.

Increments of 0.05 (5%) have been selected as differences between the numerical values of factors of safety for which curves have been plotted, as it is believed that no evaluator will be able to estimate hazard to a much closer degree, and that estimating a factor of safety to the nearest 5% will compare very favorably as to accuracy with the other estimates required in any valuation of terminable speculative income.

The charts may be used to determine a speculative interest rate corresponding to any factor of safety reflected by the speculative yield in comparison with safe rates of interest of 3%, 4%, 5%, and 6%, for the valuation of terminable equal annual income according to any of the premises of valuation using one or two rates of interest. The factor of safety is followed to the horizontal line indicating the term of years for which income is being valued, and the vertical line, passing through this intersection, gives the speculative rate of interest equivalent to the selected factor of safety for that period of investment.

Chart 1.—Single Payment Compound Interest Deferment Valuation Formula. Factors of safety for speculative rates of interest to 20% and for terms of investment from one to fifty years on the basis of a safe rate of interest of 3%. Each curve represents a factor of safety of the numerical value shown by the figure on the curve.

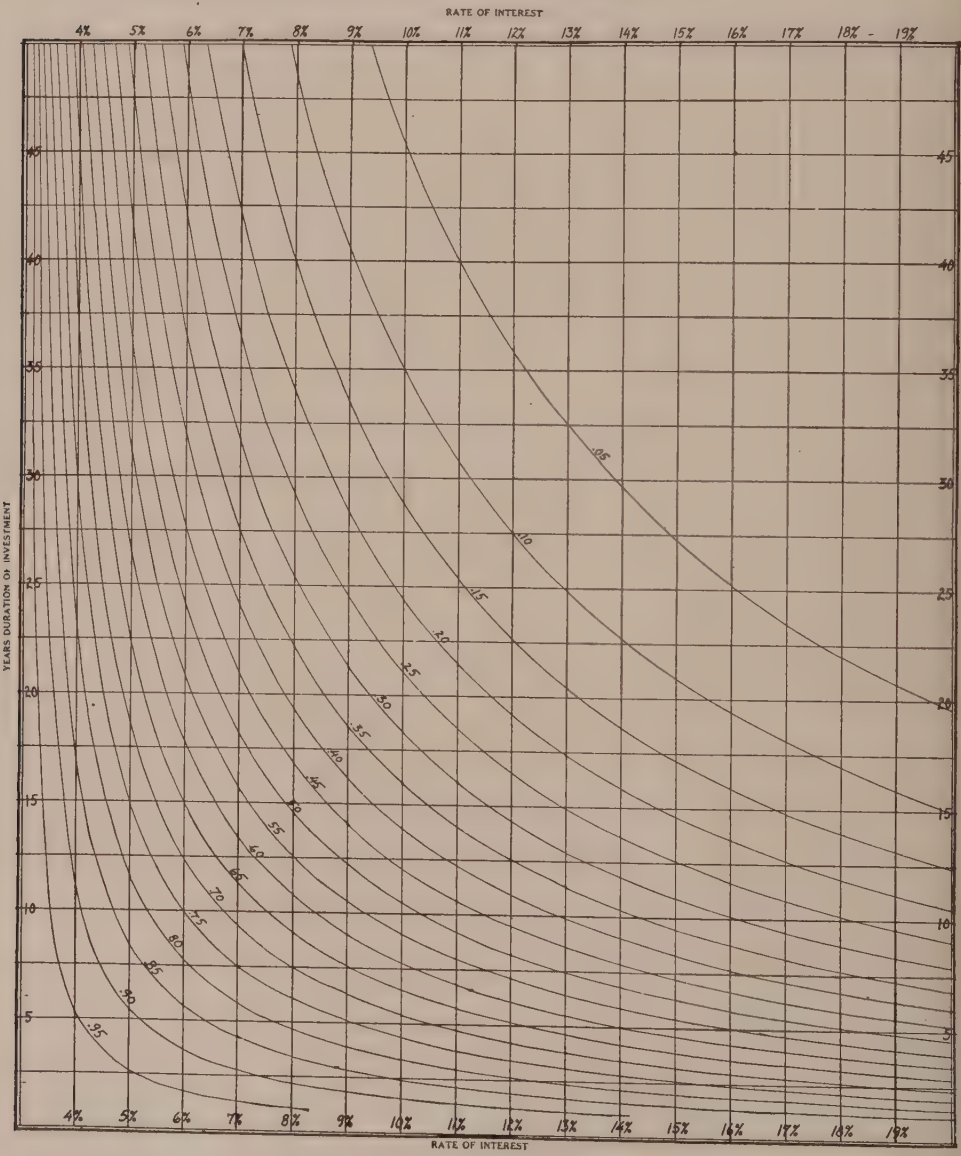


Chart 2.—Single Payment Compound Interest Deferment Valuation Formula. Factors of safety for speculative rates of interest to 20% and for terms of investment from one to fifty years on the basis of a safe rate of interest of 4%. Each curve represents a factor of safety of the numerical value shown by the figure on the curve.

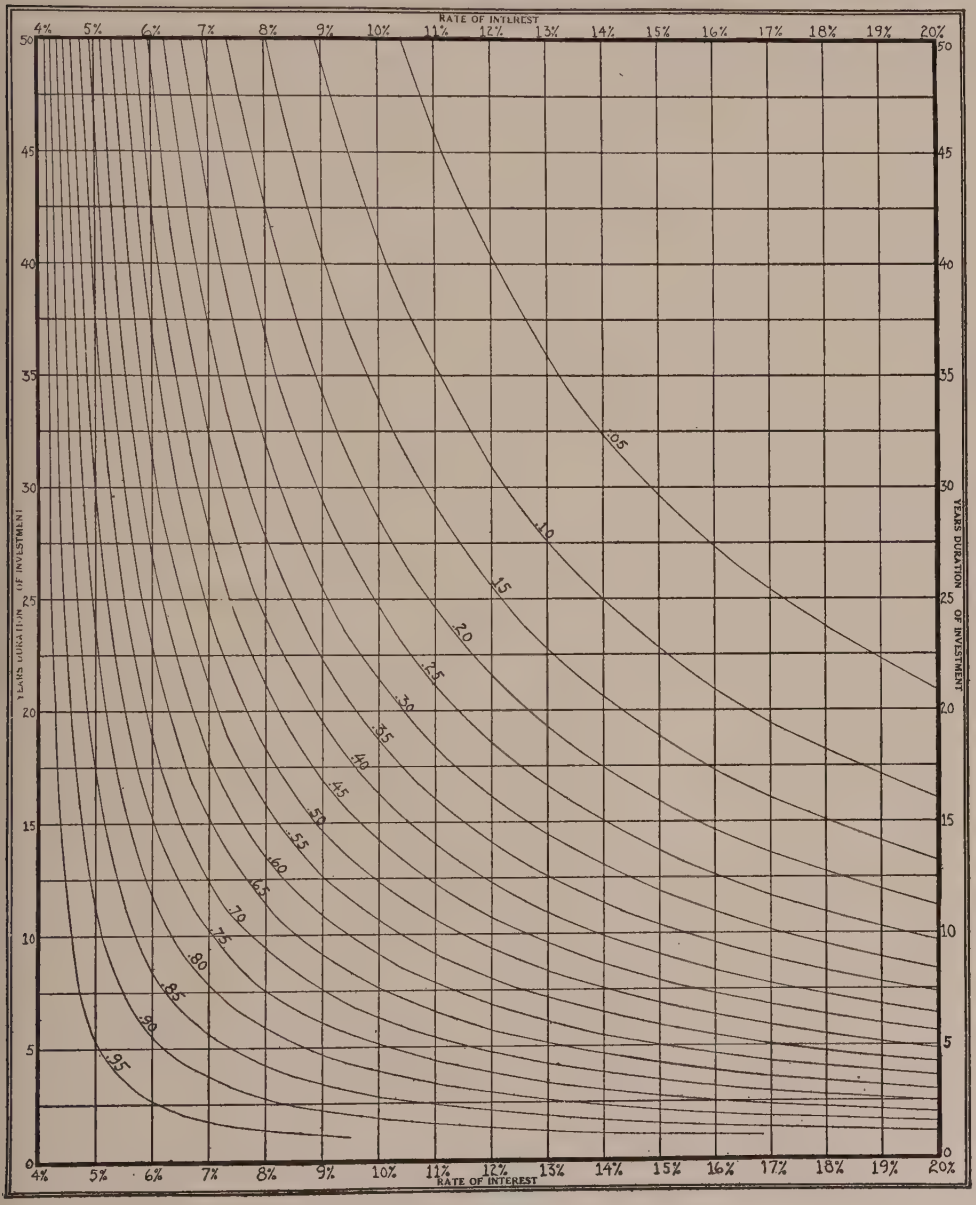


Chart 3.—Single Payment Compound Interest Deferment Valuation Formula. Factors of safety for speculative rates of interest to 20% and for terms of investment from one to fifty years on the basis of a safe rate of interest of 5%. Each curve represents a factor of safety of the numerical value shown by the figure on the curve.

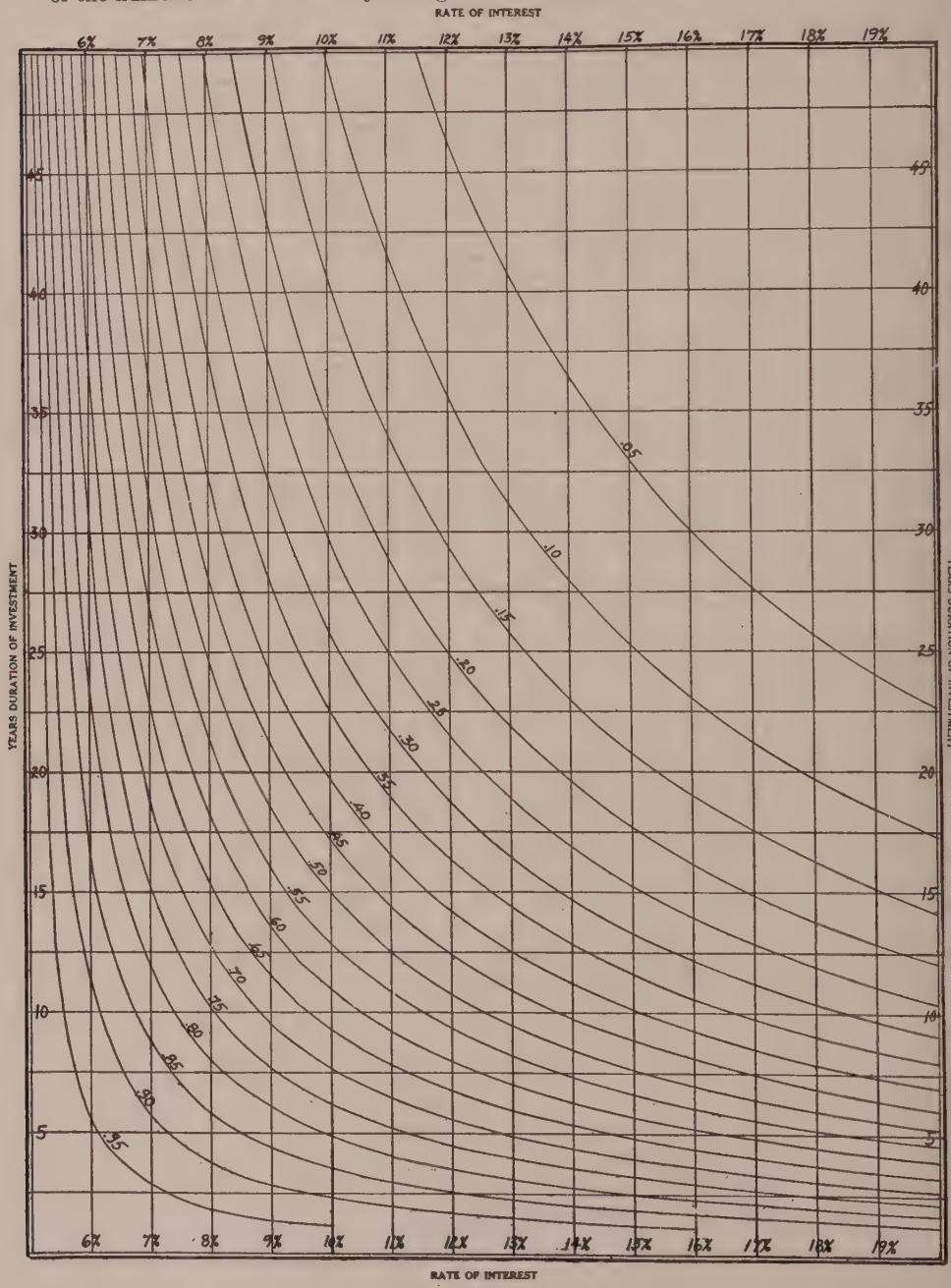


Chart 4.—Single Payment Compound Interest Deferment Valuation Formula. Factors of safety for speculative rates of interest to 20% and for terms of investment from one to fifty years on the basis of a safe rate of interest of 6%. Each curve represents a factor of safety of the numerical value shown by the figure on the curve.

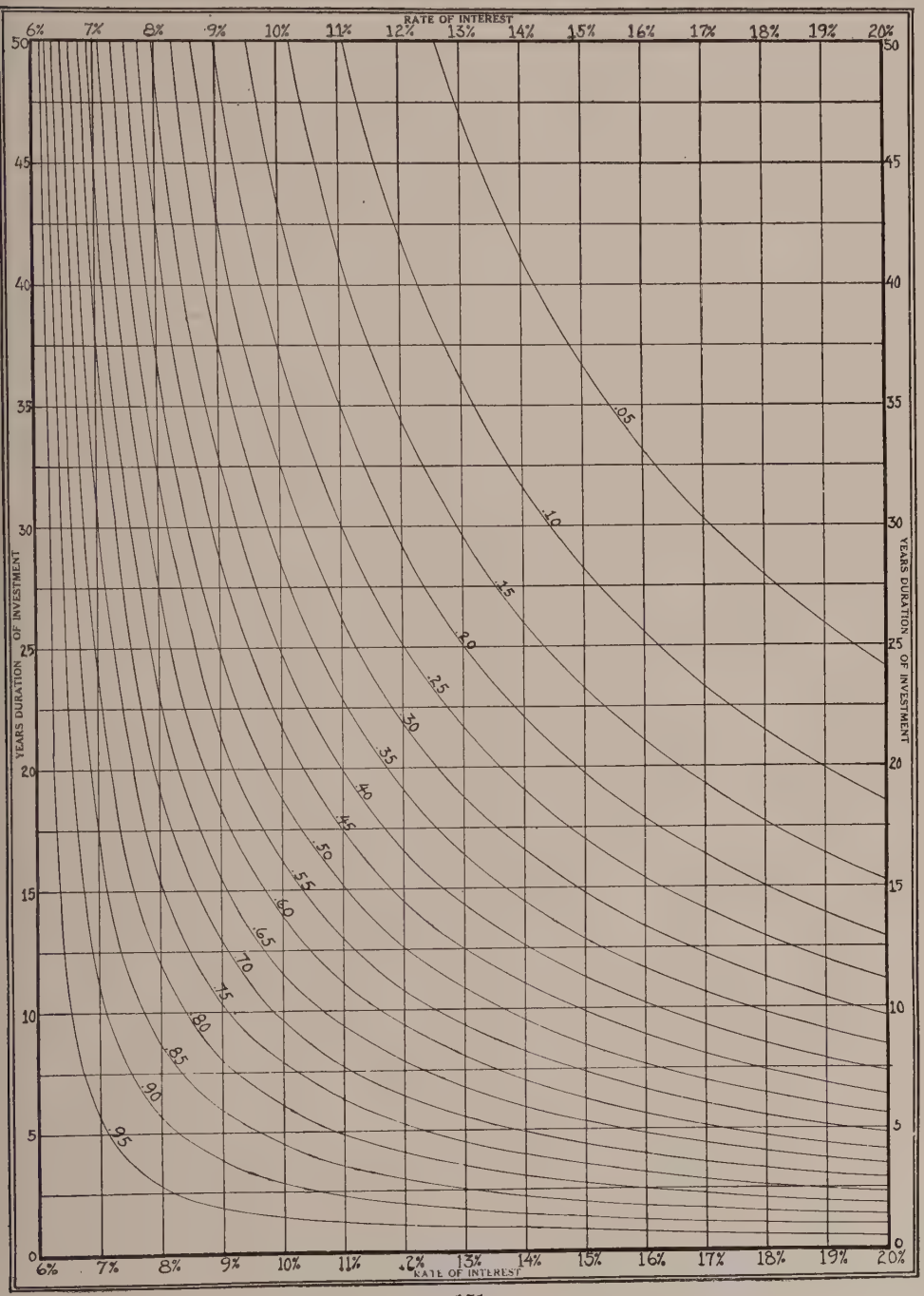


Chart 5.—Compound Interest Annuity Valuation Formula. Factors of safety for speculative rates of interest to 20% and for terms of investment from one to fifty years on the basis of a safe rate of interest of 3%. Each curve represents a factor of safety of the numerical value shown by the figure on the curve.

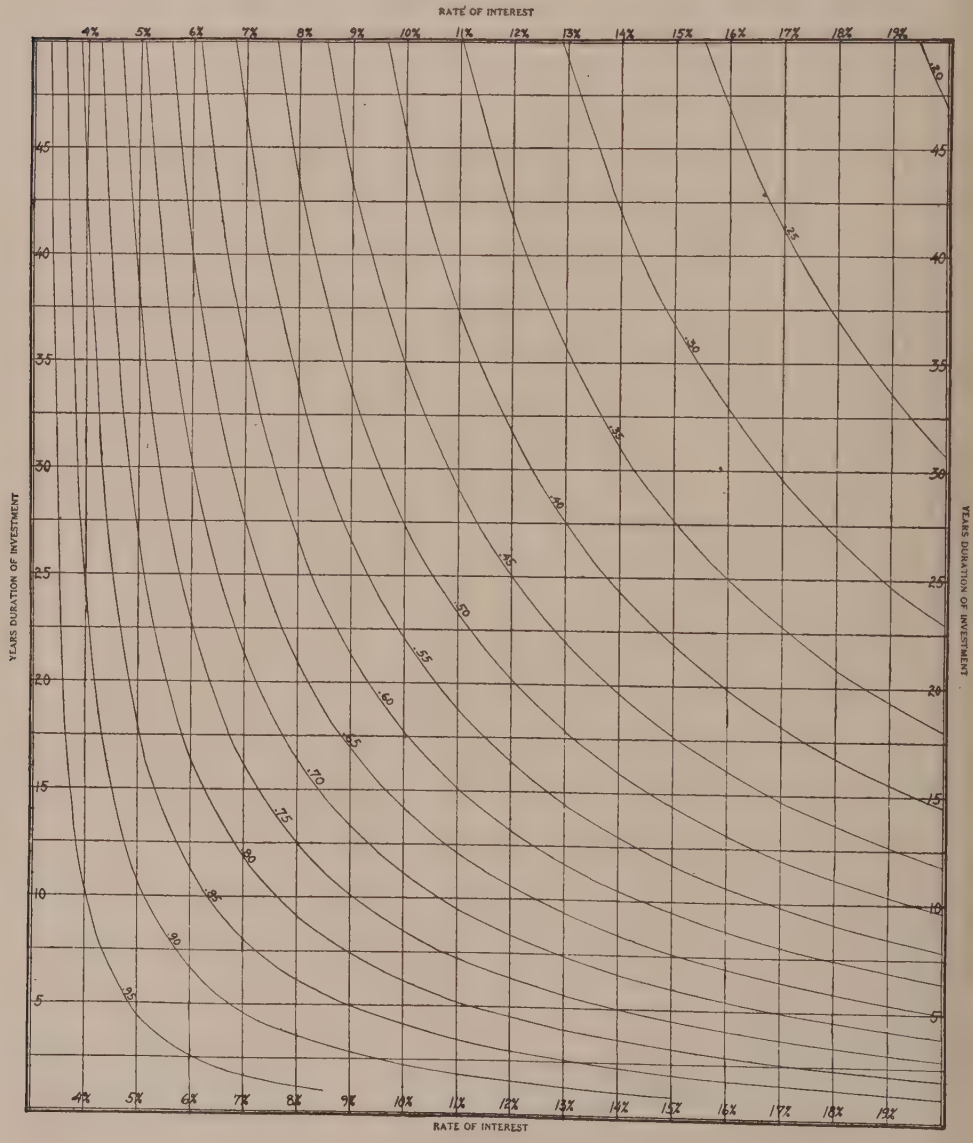

RATE OF INTEREST
4% 5% 6% 7% 8% 9% 10% 11% 12% 13% 14% 15% 16% 17% 18% 19%
YEARS DURATION OF INVESTMENT
45 40 35 30 25 20 15 10 5
.99 .25 .30 .35 .40 .45 .50 .55 .60 .65 .70 .75 .80 .85 .90 .95
RATE OF INTEREST
YEARS DURATION OF INVESTMENT

Chart 6.—Compound Interest Annuity Valuation Formula. Factors of safety for speculative rates of interest to 20% and for terms of investment from one to fifty years on the basis of a safe rate of interest of 4%. Each curve represents a factor of safety of the numerical value shown by the figure on the curve.

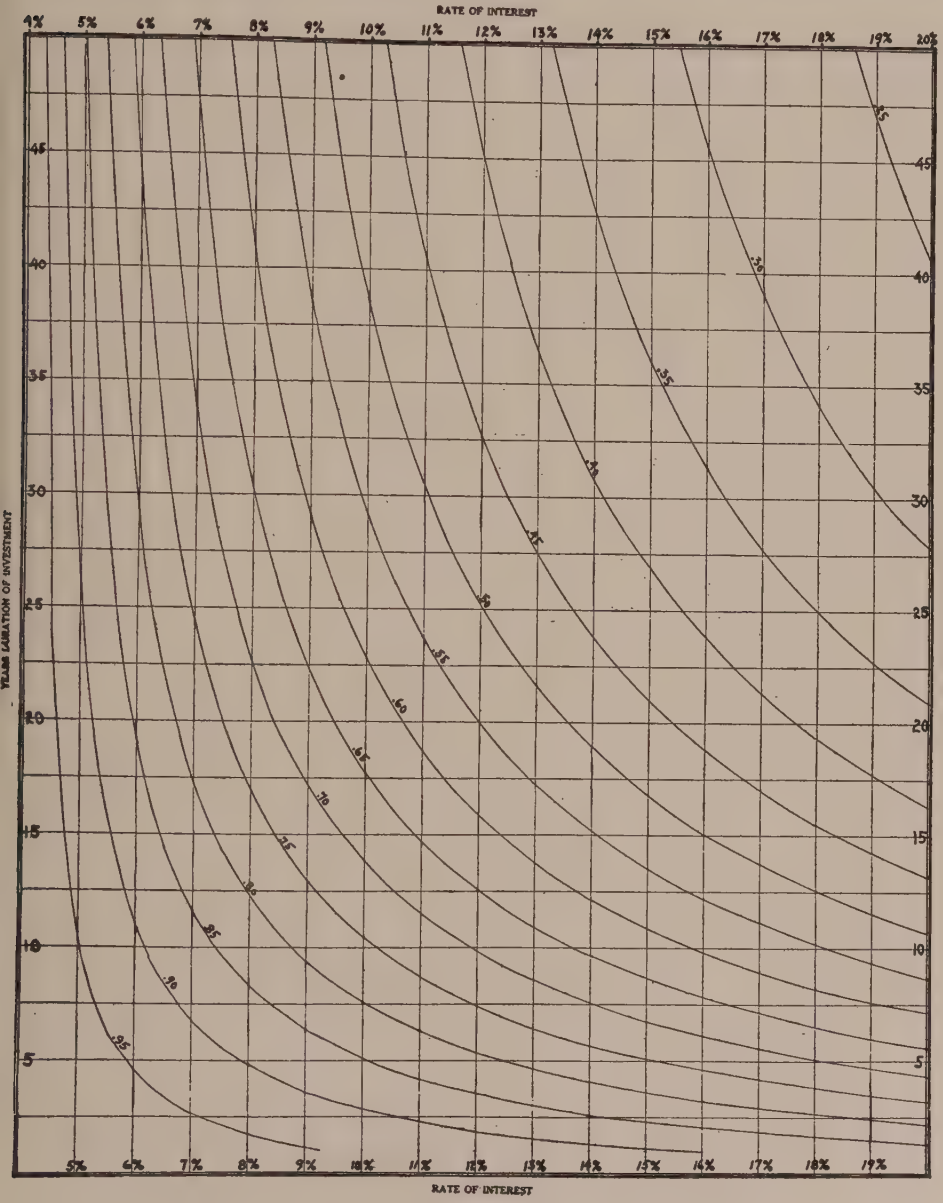


Chart 7.—Compound Interest Annuity Valuation Formula. Factors of safety for speculative rates of interest to 20% and for terms of investment from one to fifty years on the basis of a safe rate of interest of 5%. Each curve represents a factor of safety of the numerical value shown by the figure on the curve.

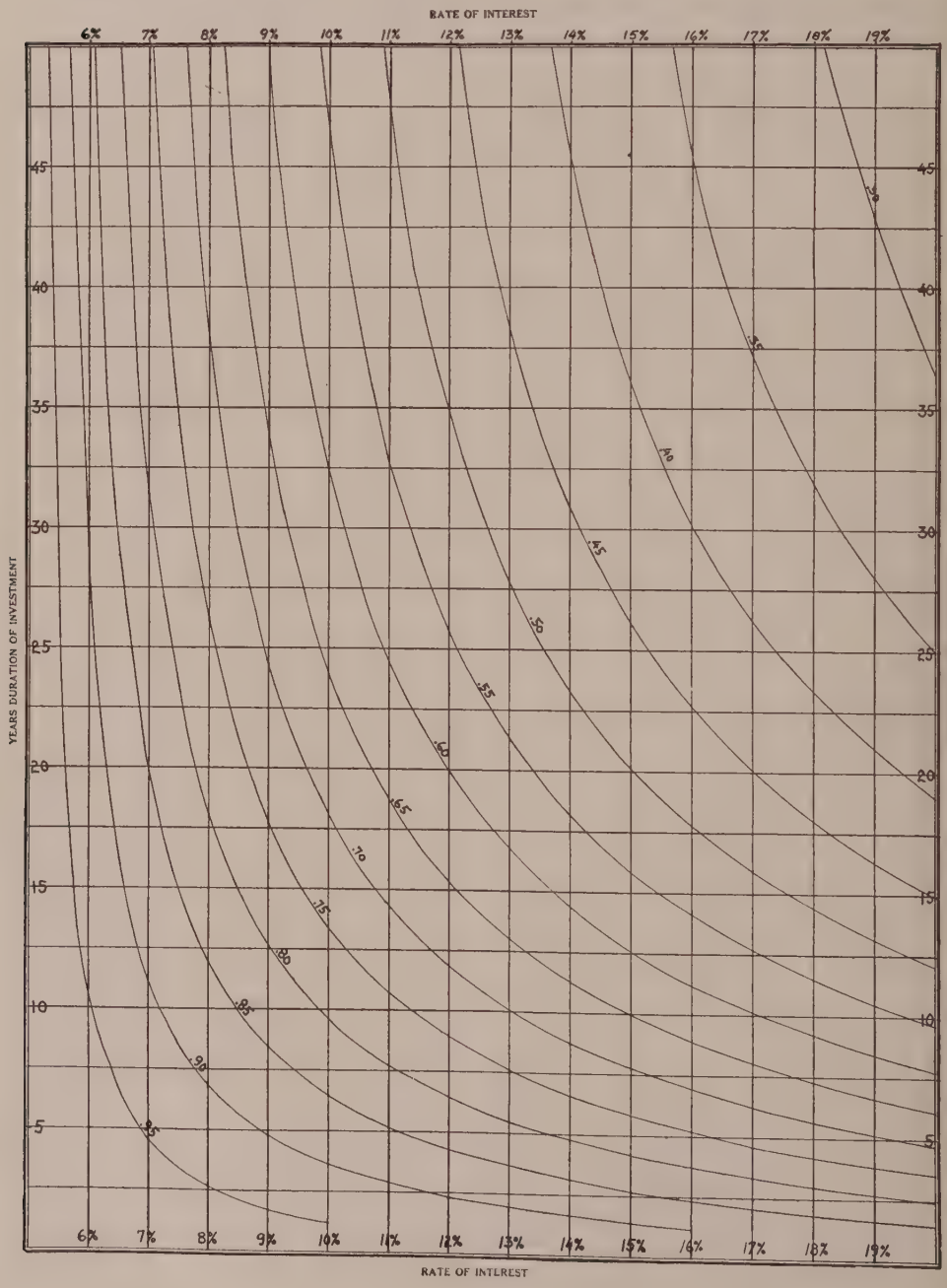

RATE OF INTEREST
6% 7% 8% 9% 10% 11% 12% 13% 14% 15% 16% 17% 18% 19%
YEARS DURATION OF INVESTMENT
45 40 35 30 25 20 15 10 5
.95 .90 .85 .80 .75 .70 .65 .60 .55 .50 .45 .40 .35 .30
RATE OF INTEREST
YEARS DURATION OF INVESTMENT

Chart 8.—Compound Interest Annuity Valuation Formula. Factors of safety for speculative rates of interest to 20% and for terms of investment from one to fifty years on the basis of a safe rate of interest of 6%. Each curve represents a factor of safety of the numerical value shown by the figure on its curve.

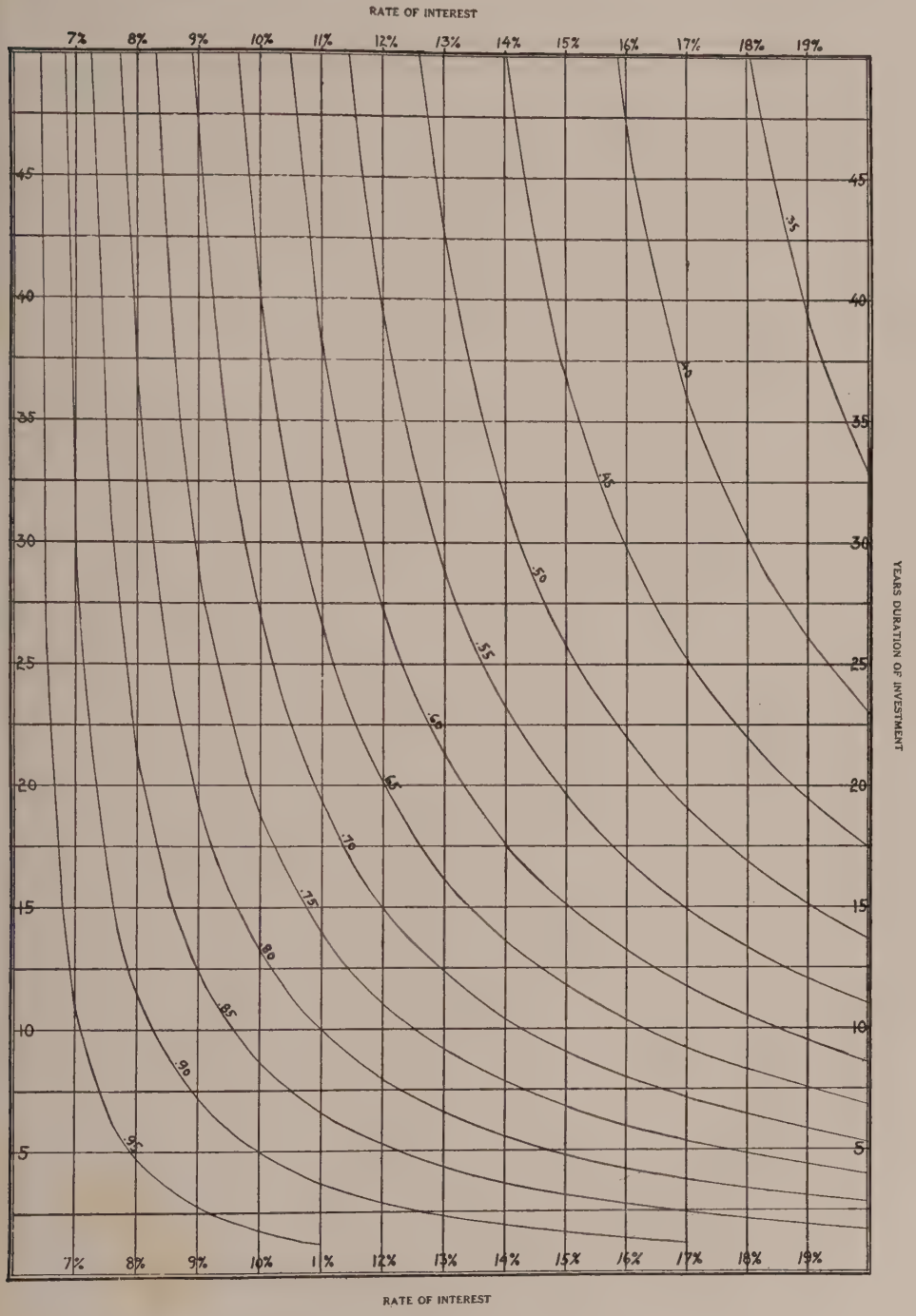

RATE OF INTEREST
7% 8% 9% 10% 11% 12% 13% 14% 15% 16% 17% 18% 19%
45 40 35 30 25 20 15 10 5
.35 .40 .45 .50 .55 .60 .65 .70 .75 .80 .85 .90 .95
YEARS DURATION OF INVESTMENT
YEARS DURATION OF INVESTMENT
RATE OF INTEREST

Chart 9.—Hoskold Type Valuation Formula. Factors of safety for speculative rates of interest to 20% and for terms of investment from one to fifty years on the basis of a safe rate of interest of 3%. Each curve represents a factor of safety of the numerical value shown by the figure on the curve.

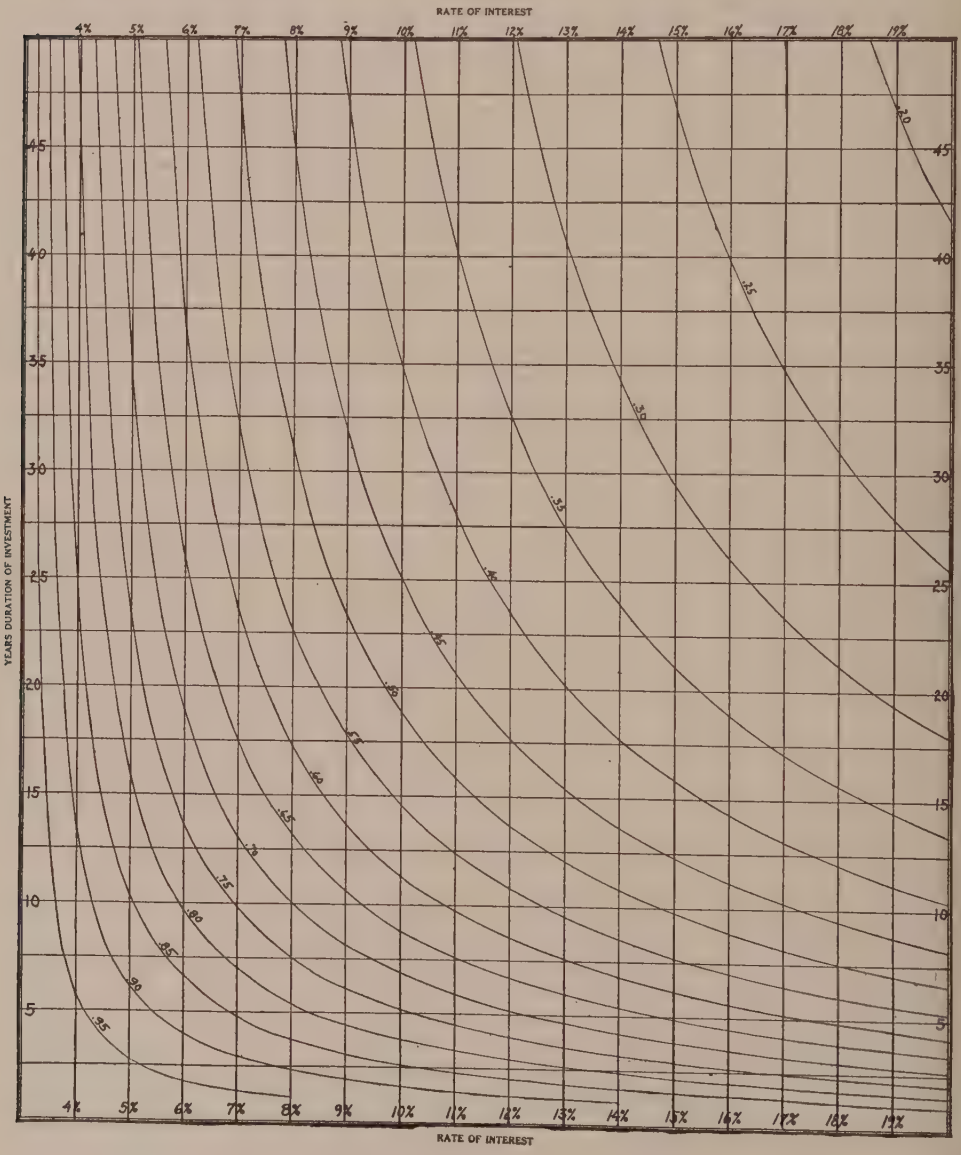


Chart 10.—Hoskold Type Valuation Formula. Factors of safety for speculative rates of interest to 20% and for terms of investment from one to fifty years on the basis of a safe rate of interest of 4%. Each curve represents a factor of safety of the numerical value shown by the figure on the curve.

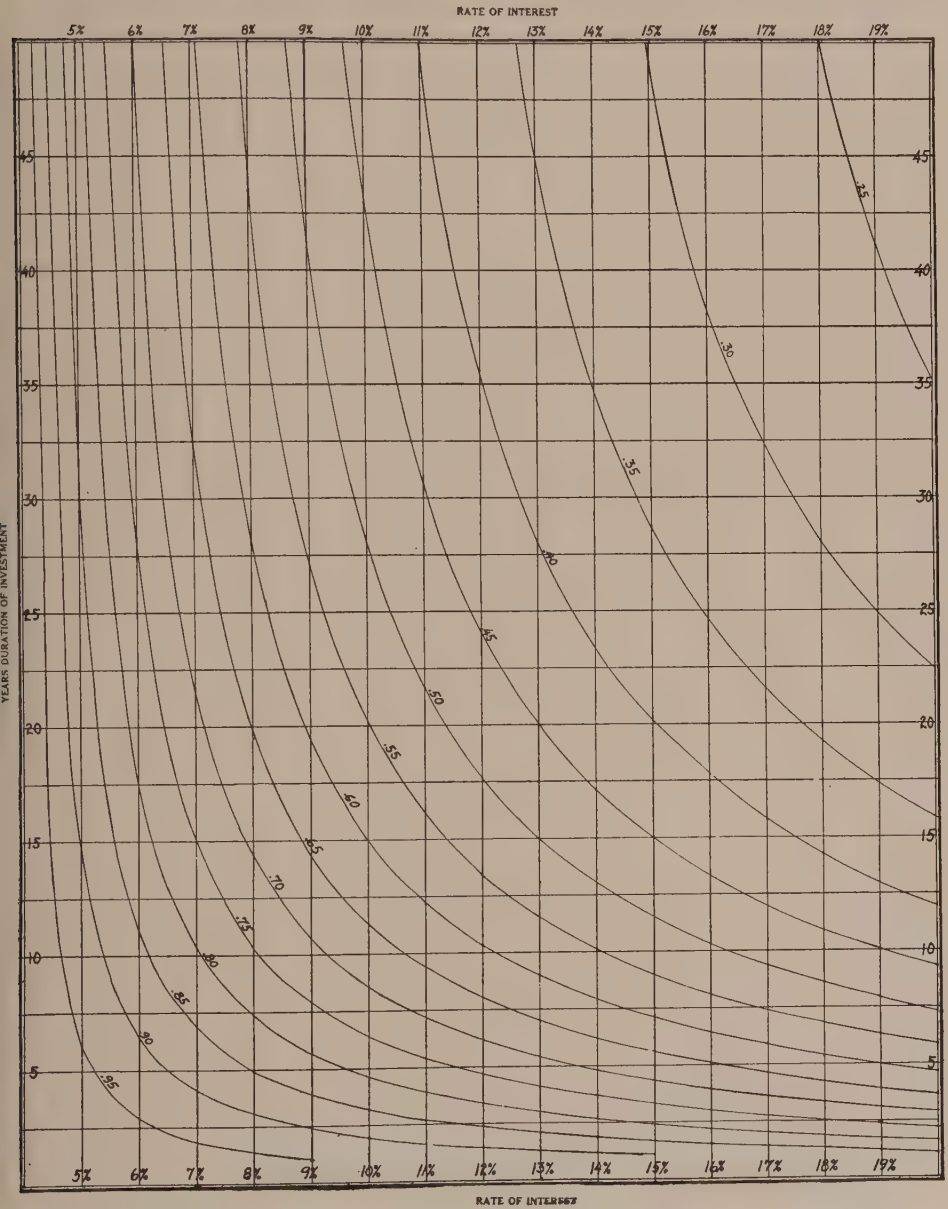

RATE OF INTEREST
5% 6% 7% 8% 9% 10% 11% 12% 13% 14% 15% 16% 17% 18% 19%
YEARS DURATION OF INVESTMENT
45 40 35 30 25 20 15 10 5
.95 .90 .85 .80 .75 .70 .65 .60 .55 .50 .45 .40 .35 .30 .25
RATE OF INTEREST
YEARS DURATION OF INVESTMENT

Chart 11.—Hoskold Type Valuation Formula. Factors of safety for speculative rates of interest to 20% and for terms of investment from one to fifty years on the basis of a safe rate of interest of 5%. Each curve represents a factor of safety of the numerical value shown by the figure on the curve.

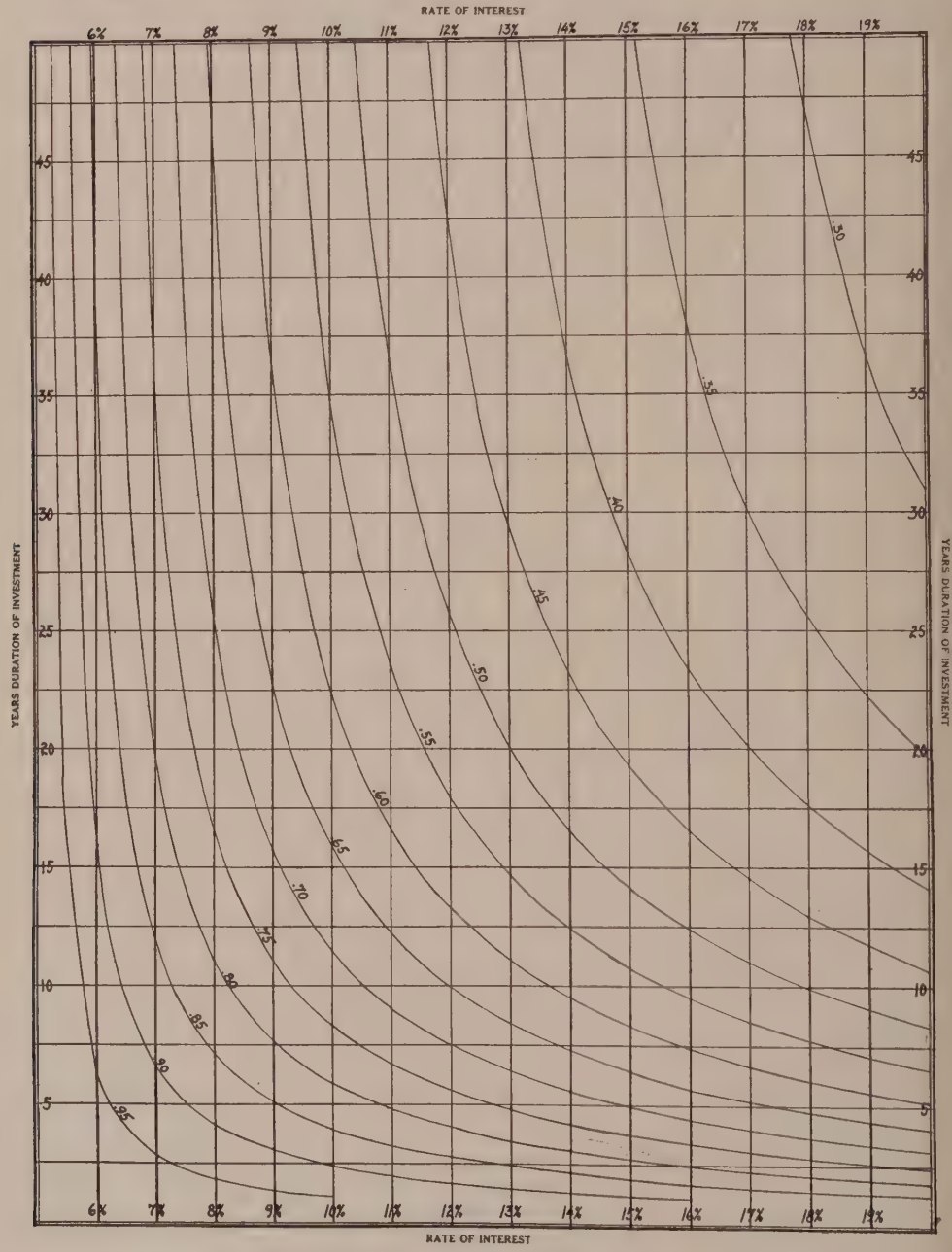
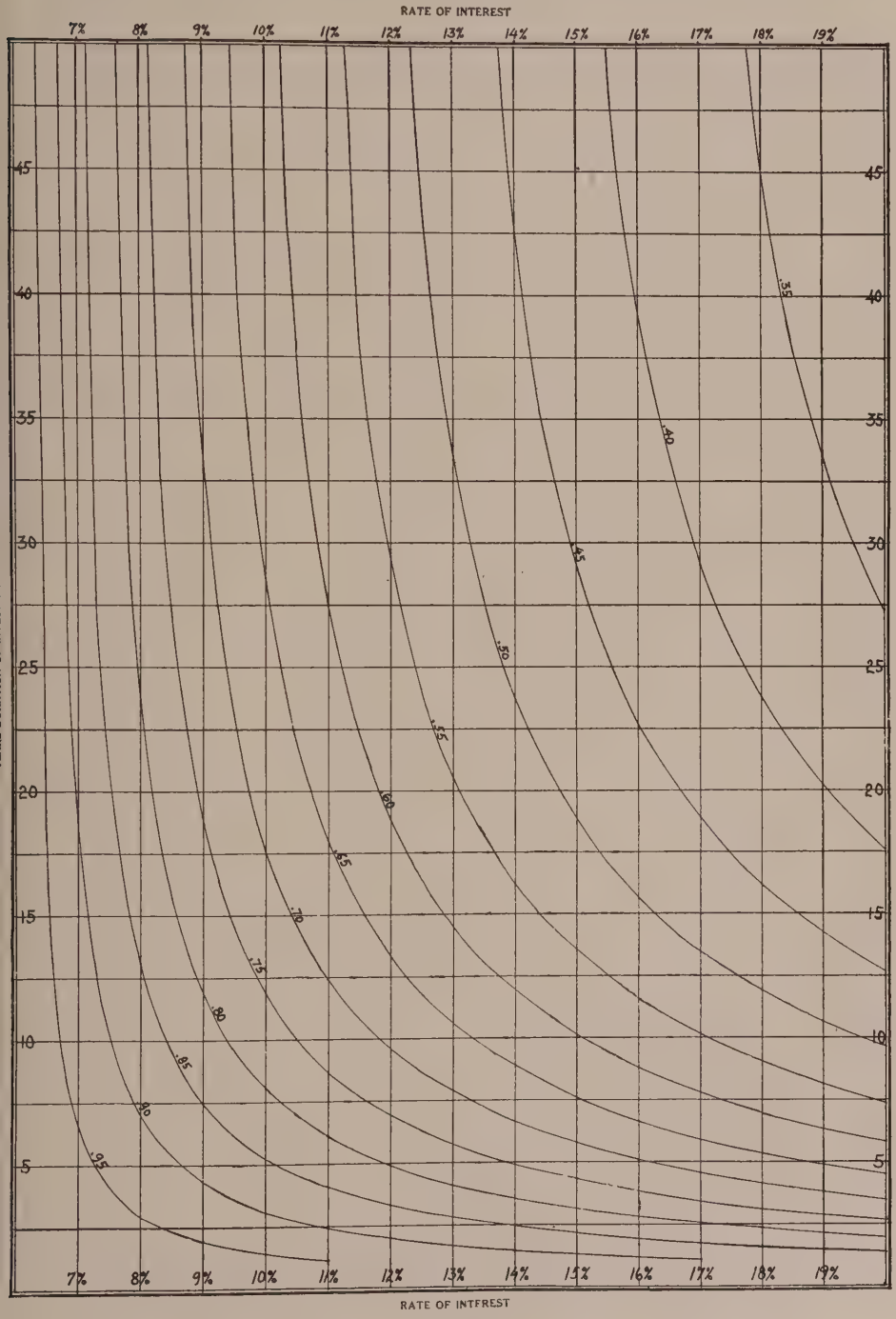

RATE OF INTEREST
6% 7% 8% 9% 10% 11% 12% 13% 14% 15% 16% 17% 18% 19%
YEARS DURATION OF INVESTMENT
45 40 35 30 25 20 15 10 5
.30 .35 .40 .45 .50 .55 .60 .65 .70 .75 .80 .85 .90 .95
RATE OF INTEREST
YEARS DURATION OF INVESTMENT

Chart 12.—Hoskold Type Valuation Formula. Factors of safety for speculative rates of interest to 20% and for terms of investment from one to fifty years on the basis of a safe rate of interest of 6%. Each curve represents a factor of safety of the numerical value shown by the figure on the curve.



Factors of safety applied to gross income, net income, and operating cost.—Usually these factors are applied in addition to factors of safety reflected in the speculative rate of interest used in discounting to present worth. A not infrequent method of valuation is providing such heavy discounts for hazard at various places in the estimates upon which the computation is based that a conservative value would be found even if no further factor of safety should be introduced by the use of a speculative rate of interest in determining present worth. Passing mention has been made previously with respect to such discounts for hazard, which usually consist in arbitrary changes of the best estimates of the evaluator, for the sake of safety and conservatism. These changes most frequently appear in purposeful reduction of the estimated amount of gross income expected from the investment and in overestimates of charges against gross income, both of which are indirect methods of reducing net income, or in arbitrary direct reductions of the estimated net income. It is obvious that it is only necessary to make the sum of such direct and indirect arbitrary deductions equivalent to the actual estimate of expected net income in order to reduce the computed value of any amount of income to zero. It is equally obvious that direct comparisons between speculative rates of interest, used in discounting anticipated income of the same duration to present value, are worthless if arbitrary deductions from estimated net income have been made in varying degree in the several valuations prior to the application of the speculative interest discount. But separate factors of safety may be determined for the speculative rate of interest and for the aggregate reduction of the net income, the product of these giving the net factor of safety used in determining each value. These net factors of safety may be compared directly, one with another.

The following formulas show the simple means by which factors of safety other than those reflected in the speculative rate of interest may be determined for almost any combination of direct and indirect discounts for hazard, which tend to reduce the estimated net income.

Notation.

Let:

G = gross income computed as accurately as possible from available data and future expectancy, without the introduction of any factor of safety. For convenience assume G is made equal to unity.

C = operating cost of producing each unit of gross income G before any factor of safety is applied. If the operating cost is 80% of the gross income, then C is equal to 0.80.

A = net income for unit of gross income before any factor of safety is applied. If 20% of gross income is net income, then A is equal to 0.20.

X = factor of safety as applied to gross income. Thus with G based on an 80% economic recovery or an 80% productive efficiency, a 76% recovery or efficiency represents a 95% safety factor (0.95 times 0.80 = 0.76), and X is equal to 0.95.

X_1 = a second factor of safety as applied to gross income. Thus, if in addition to the prior factor of safety X, there is deducted 10% of the expected sales price of the product (not already taken into account in G), then $X_1 = 0.90$. Other factors of safety as applied to gross income may be represented by X_2, X_3, etc.

x = equivalent factor of safety as applied to gross income, when any number of other factors of safety are applied to cost or net income, or a combination of both, or to combinations of cost, net income, and gross income.

Y = factor of safety as applied to operating cost. If cost is increased 10%, then $Y = 1.10$. Ordinarily, all reasons for increase or decrease in operating cost are considered as to their combined effect. Y_1, Y_2, etc., are unnecessary.

y = equivalent factor of safety as applied to operating cost, when other factors of safety to any number are applied to gross income, or to net income, or to a combination of both, or to combinations of gross income, net income, and cost.

F = factor of safety as applied to net income. If net income is reduced by 20% then F is equal to 0.80. For example, a discount for hazard of 20% or a factor of safety of 80%, reflected in the speculative rate of interest used in discounting to present worth at date of valuation.

F_1 = a second factor of safety as applied to net income. For example, excessive deductions for initial capital expend-

itures after reduction to present worth. Other factors of safety as applied to net income may be represented by F_1, F_2, etc.

f = equivalent factor of safety as applied to net income, when any number of other factors of safety are applied to gross income, or to cost, or to a combination of both, or to combinations of gross income, cost, and net income. If f is negative instead of positive, a loss to that extent is indicated.

Accordingly, $G - C = A$; $G = 1$; and a study of the notation will disclose the basis of the following relations, written as various f, y, and x formulas.

These formulas will be numbered separately for each group of three containing f, y, and x values.

Formulas X-A.

Assume that five factors of safety are involved: two are applied to gross income; one, to cost; and two, to net income. Then:

$$f = \frac{FF_1A - (1 - XX_1) - (YC - C)}{A}$$

$$y = \frac{A - FF_1A + YC + 1 - XX_1}{C}$$

$$x = \frac{XX_1 - (YC - C) - (A - FF_1A)}{G}$$

Since $G = 1$, the x formula may be written:

$$x = XX_1 - (YC - C) - (A - FF_1A)$$

The above three formulas are all that are required for any combination of the five safety factors. By substituting unity for X, X_1, F, F_1, or Y in the above equations, a formula may be derived for the value of the equivalent factors, f, y, or x for any combination desired. For instance, if the value f is required, where only one factor of safety as applied to gross income is involved, then X_1, Y, F, and F_1 become unity, and the above f equation reduces to:

$$f = \frac{A - (1 - X) - O}{A}$$

or, since $1 - A = C$, then

$$f = \frac{X - C}{A}$$

By substituting unity for other factors of safety, the following formulas are derived when only one factor of safety is involved as applied to gross income:

$$f = \frac{X - C}{A}$$

$$y = \frac{1 - X + C}{C}$$

$$x = X$$

Other adaptations of formulas cited above.—By similar processes of elimination it will be possible to make other adaptations of the formulas to almost any conditions involving factors of safety which are directly or indirectly applied to estimates of net income. The discount for hazard (or factor of safety) method of valuation thus combines statistical and mathematical processes and is considered by the authors to be the most suitable means for consistent and intelligent valuation of future terminable income of a speculative nature. It separates the elements of interest from those of risk and profit, allowing separate judgment to be exercised with respect to each of these elements of a speculative yield. Evaluators using this method are confronted with definite questions requiring specific answers as to the investment rate of interest which should be expected during the term of the investment and as to the profit in excess of interest which the speculative investment should promise in order that it may compete with safe investments for capital. More detailed analysis of the weight attributed to the several components of the speculative yield may also be undertaken if the evaluator so desires.

The chief advantage in using this method of valuation, either directly by means of separate discounts for interest and hazard, or indirectly in the selection of a speculative yield which properly combines the separate discounts, is that due recognition must be accorded to the fundamental conception that the same degree of hazard is expressed only for varying terms of investment, through speculative yields varying obversely with the duration of the period of investment. The common error of assuming that the same specula-

tive rate of interest reflects the same degree of hazard, regardless of the length of the investment, is impossible when the factor of safety process of valuation is employed.

Another great advantage of the discount for hazard method is that it provides a common basis for the comparison of all methods of valuation, even when the differences between methods are numerous, such as the inclusion of varying discounts for hazard in factors other than the rate of interest, the use of different rates of speculative yield, and the necessity for comparison of results obtained by various appraisal methods with those reflected in completed transactions. Each appraisal method and each actual investment may be expressed as a numerical amount by a factor of safety or discount for hazard, and the consensus of opinion with respect to the degree of risk inherent in any type of speculative venture may then be determined within quite reasonable limits of variation. The optimism or conservatism of a specific method or transaction in comparison with others is immediately apparent and those which are not representative may be eliminated from consideration.

A further attribute of importance is the facility afforded for comparison between different countries or between different parts of the same country affected by diverse economic conditions. If the discount for hazard in comparison with any safe type of investment has been determined for a certain type of speculative venture in one locality, it may be easily determined that in another locality the same type of speculative enterprise affords a greater or less opportunity for profit in comparison with the selected safe type of investment with which it is being measured. As an example the rates of interest on well secured first mortgage loans are 5% and 8% in two separated localities and the yields promised by identical types of speculative enterprises in the respective localities are 8% for a 20-year period in the first locality and 15% for a 5-year term of investment in the second locality. Using the compound interest actuarial premises: a safe rate of 5% compared to a speculative yield of 8% for 20 years represents a discount for hazard of 21%, while the 15% yield for 5 years compared with an 8% safe rate gives 16% as the discount for hazard.

The preceding comparison is on the basis of terminable income for both speculative and safe investments, although comparison might be made with equal facility between perpetual safe income and terminable speculative income. But, leaving the details of comparison to the individual evaluator, the discount for hazard method of valuation is a ready means by which an authority on speculative investment in one locality may apply his experience in other localities with which he is less familiar.

Rates of interest paid by governmental securities are usually fair reflections of varying political and economic conditions and of financial stability in different countries, and these conditions merit consideration in determinations of the relative yields which should be obtained from speculative business investments of similar nature under diverse economic and political influences. Other standard bases for comparison are first mortgage loans, rates on commercial paper, bank rates of discount, and other standard types of essentially safe investments.

The safe interest value from which a discount for hazard is determined should be for investments of nearly the same type as that for which a value is sought. The elements of marketability, management, geographical variation, etc., included in the safe and speculative yields, should also correspond to the greatest possible extent, and the safe interest rate should be for a form of investment which is almost universally available for comparison regardless of locality. There are several types of investment which approach these requirements; but, for the valuation of investments other than readily marketable stocks and bonds, the prevalent rate of interest on first mortage real estate loans in any locality and at any time will be hard to excel as a basis of comparison.

Expression of the hazard of speculation in terms of the odds for and against success is an advantage to the evaluator, not only because it necessitates a careful review of all estimates made with a view to contingencies, but because the results of his labors are presented in a concrete form, which enables the speculative investor to decide whether such odds are attractive in view of the hazard involved. The use of a speculative rate of interest in determining value of future

speculative income is less illuminating in this respect, as the discount for hazard method of valuation conforms closely to the actual mental processes of large classes of speculative investors, to whom the venture of capital in new speculative enterprises is rarely attractive, unless there is a reasonable prospect for doubling or otherwise multiplying their capital in the event that success attends such ventures.

Chapter VII

SUMMARY AND CONCLUSIONS

When lack of positive evidence precludes other definite prediction, evaluators of speculative future income commonly assume that both the rate of interest and the quantity of income will be constant in yearly amount throughout a period of investment. While it is certain that each of these forecasts will be more or less in error, the combination of the two assumptions is apt to be more nearly correct than any other prognostication, since resultant errors from the two assumptions tend to be both compensating and proportionate. This condition is the result of several causes, among others are: the tendency noted in Chapter V for interest rates and commodity prices to have the same trend; the customary increase or decrease of profits with corresponding alterations in price levels; the lag of cost of production behind commodity price variation, which reflects rapid price modification in exaggerated profit changes; and the fact that the value of income is directly proportionate to the amount of income and variant inversely with the rate of interest. The combined effect of all of these related phenomena is dependent upon the rate of change, as the slower the rate and the smaller the amount of a change in economic conditions the more perfect is the balance maintained between the several elements while adjustment is in process. Interest rates and income thus have tendencies to increase or decrease simultaneously and to equalize each other in effect upon value when modifications are slow and slight; but the effects of great and rapid changes are unbalanced and result in temporary changes of values similar to the income modifications in direction and extent.

Periods of abnormal prosperity or depression are seldom of long duration and may be considered to balance each other in effect upon income over long periods of time. It is, therefore, permissible in valuation to assume that over considerable periods of time economic conditions will be normal and that

income dependent upon economic conditions will also be normal. The longer the term of the investment the greater is the probability that the departures from normal conditions in either direction will balance, and, consequently, the more certain is the realization to approach the estimate of an average yearly income of normal amount. But during any normal period it is impossible to foretell whether the next departure from normal will be a period of prosperity or depression, and, since the time distribution of the same amount of income is reflected in its value, the present worth of income fluctuating with economic conditions is more speculative than the value of income which is received in constant yearly amount independent of the prosperity or depression of business. This speculative element is a portion of the business risk, which also includes the element of uncertainty with respect to the time during which the status at date of valuation will continue. When the amounts of future income must be estimated for long periods of investment, it is thus most reasonable to assume that, when conditions are normal, they will continue to be normal in the future, and that, when conditions are abnormal at the time of valuation, they will return to normal at some future time, usually not far distant. The valuation under abnormal conditions should take account of an immediate period of the abnormal income and a future period of normal income; while valuations when conditions are normal may properly assume normal annual amounts for all future income.

The time distribution of income is always one of the more important elements in an appraisal, and probably receives less consideration than any other element of equal importance. For example, it is improper to value, on an equal annual basis, income which is expected to increase or decrease constantly or periodically, yet this is very frequently done. Many real estate valuations fail to consider decreases of income attributable to decreasing rents and increasing costs of maintenance as the age of a building increases. It is expected of nearly every important new mine that production and profits will increase for a time, remain fairly constant for a later period, and then decline as the ore bodies approach exhaustion; yet valuations of mines rarely recognize the state of maturity of the mine or probable future variations in its

earning capacity other than those arising from contemplated additions to plant and equipment. Even in the event that one has been cognizant of the importance of the time distribution of income over the period of its receipt, he has had but one premise of valuation which was a mathematically correct means for ascertaining values for irregular or periodically changing income, that is, the compound interest deferment valuation method. This work has developed the mathematics involved in the valuation of unequal annual income of all descriptions according to all known premises of valuation—the compound interest actuarial method and various sinking fund methods in addition to the compound interest deferment and sinking fund methods—and places these new tools at the disposal of the evaluator. Although the assumption of an equal annual amount is a proper one for many appraisals of future income, there are frequent occasions when the past history of properties gives a sound basis for other assumptions, such as the progressive increase or decrease of income over a future period of time. This is especially true in the valuation of industries whose income increases with the growth of population, or of the more stable competitive units of such industries. The valuation of income changing every year has its sphere of usefulness in the comparison of actual results with those anticipated at prior dates of valuation, and in the application of the theory of least squares and statistical records to the valuation of future income. With the growth of statistical records in business it is believed that future evaluators will find extensive need for appraisals based on the laws of probability in connection with past experience, as such appraisals will be less often in error than are the usual present methods.

Six distinct premises have been developed and compared, together with a commercial variation which combines two of the premises in the valuation of deferred income by a sinking fund method. It has been demonstrated that the same value is determined for identical income by all premises of valuation, when a safe rate of interest is employed in discounting to present worth, but that with these identical values there are three principal methods by which yearly income may be allocated to the payment of interest and the return of capital—the compound interest deferment, com-

pound interest actuarial, and sinking fund methods. When a speculative rate is selected for the payment of interest on capital value in connection with a safe rate of interest for the accumulation of a sinking fund to provide for return of capital, the six premises of valuation determine three differing values and allow six distinct methods to be employed in the allocation of yearly income to interest and capital accounts. The unequal values demonstrate the fact that the same speculative rate of interest represents a different degree of risk, according to the premise of valuation with which it is used. It has also been shown that for all premises of valuation the same speculative rate of interest is representative of a different degree of risk for each variation in the duration of an investment period, and that for all save the Hoskold premises it represents a variant risk for different time distributions of income within an identical period. A means for determining speculative rates of interest representative of a fixed degree of risk, irrespective of premises of valuation, duration of investment, and time distribution of income, has been outlined, and this discount for hazard method of valuation may also be used directly as a substitute for any method in which a speculative yield is employed for discounting to present worth.

The selection of a mathematical premise of valuation, at least in so far as the allocation of future income to interest payment and return of capital is concerned, should be governed by the financial policy which will be followed by the prospective investor. Each of the six mathematical premises represents a definite financial policy, while the discount for hazard method of valuation may be adapted to any financial policy. An individual investor may usually make a selection according to personal preference, but corporations must comply with certain legal restrictions. One of the most common of the laws for the protection of stockholders requires a corporation to maintain assets equivalent to the par value of outstanding stock as a prerequisite to the distribution of a dividend. Mining companies are the notable exception to this general type of legislation, but in some instances even mining companies must comply with this rule. Several alternatives may be considered, such as the maintenance of initial capital by means of a sinking fund or similar reserve; the reduction of capital either by the retirement of stock or by

lowering its par value; or the replacement of depreciated assets with undepreciated ones of similar type or of equivalent initial value. An efficiently managed corporation also has the incentive of perpetuating and enlarging its activities to consider in connection with the selection of a financial policy, and in opposition to this incentive it must meet the demand of many of its stockholders for the largest possible dividends which may be distributed without jeopardizing the future stability and normal growth of the corporate enterprise. The desirability, and, upon occasion, the necessity, for the selection of a valuation method suited to future financial policy will be apparent without further elaboration, while the following descriptions of the advantages and disadvantages of the several premises of valuation will assist in that selection.

The compound interest deferment premises of Division I for the valuation of a series of equal annual payments have been in commercial use for more than three centuries, and compound interest discounts for single future payments have much greater antiquity. Not only is the compound interest discount method the oldest process of appraisal, but it is the common commercial method of the present time. It is the simplest of all the premises to demonstrate by algebraic processes. Certain objectionable features may be noted with respect to this method, which consists in the determination of a sum for the separate values of each year's income and a deduction of the value determined for the income of each year from the income for that year in arriving at the amount available for interest payment. The total investment must be regarded as a series of separate purchases of individual yearly increments of income, in order that a uniform rate of interest may obtain. If the total investment is the basis of consideration, this premise of valuation is not in accord with commercial usage, as the amount available for the payment of interest increases from year to year while the unreturned amount of the investment decreases with yearly return of capital. No investor would look with favor upon a proposal that he should purchase long term future income upon the basis of a fraction of 1% as an initial rate of interest upon his total investment and that the early deficiencies of interest at the average expected rate would be overcome by payments of several hundred per cent interest on outstanding capital

during the final years of the investment period. The compound interest deferment method for valuing a series of future annual receipts, therefore, conforms to commercial practice only in the event that the purchase of such income is regarded as a multiple investment rather than as a single transaction. At any time during the period of investment, after date of valuation, the sum of the amounts of capital returned plus the value of future receipts of income will never equal, but will always exceed, the initial value, if computations are made on a comparable basis.

The compound interest actuarial premises of Division II require the annual payment of simple interest upon capital outstanding at the beginning of each year. Whenever the income for any year is insufficient for the payment of interest, the unpaid interest due in that year is added to capital, and interest is thus compounded by adding unpaid interest to capital. These premises always determine the same value for identical income as would be determined with the compound interest deferment premises of Division I, the difference between actuarial and deferment types of compound interest valuation being in the allocation of yearly income to capital and interest accounts. Interest on outstanding capital is the first charge against income with the compound interest actuarial premises, and any difference between interest due and income received is added to, or deducted from, the capital account, as the case may be, an excess of income over the interest requirement serving to reduce outstanding capital, while an excess of interest payable over income received will necessitate an increase of capital. The use of this method for allocating yearly income to capital and interest accounts has all the merits and none of the disadvantages of the compound interest deferment method. The value of the income yet to be received plus the capital already returned, at any time during the investment period, will always equal the value determined at a prior date of appraisal, if appreciation or depreciation in value since date of appraisal be excepted. The investor's accounts will always show the exact status of his investment, with interest paid at the stipulated rate to date and an unretired balance of investment upon which future interest must be paid. If an appreciation over appraised value takes place, it is indicated by a retirement of capital

greater than was anticipated, and if depreciation in value below the appraised value occurs it will be indicated by capital reduction less rapid than was anticipated. If a loss must be written off at the termination of the investment period, the exact amount of that loss is the unretired capital, which is shown by the capital account without further computation. Accounting by this method is the simplest of any of the premises, as compound interest discount and value factors are not required after the initial determination of value. Interest is always payable at the stipulated rate upon total unretired capital, and, if the income is received at the times and in the amounts anticipated at date of valuation, the excess of amounts of capital retirements over the amounts of capital increases during the period of investment will have equaled the initial investment by the end of the period during which income is received. If constant capital during the term of an investment is not a condition imposed by financial policy, the compound interest actuarial premises of interest payment and capital return, as expounded in Division II of the several chapters of this work, will be difficult to excel, although Division IV sinking fund premises will be found to afford greater facility for making certain computations, such as the rate of interest which stated amounts of terminable future income will pay upon a fixed value. In selecting premises of valuation, it is important to recall that differing degrees of risk are expressed by the same speculative rate of interest, according to the premises selected.

The development of sinking fund premises, other than the "hybrid" alternative premises, into algebraic formulas for the valuation of irregular annual income has followed the principle that a surplus of income in excess of the interest requirement is a positive contribution to the sinking fund, while a deficiency of income below the amount needed for the payment of interest on outstanding capital is a negative contribution to the sinking fund. The treatment of positive and negative contributions to the sinking fund in an identical manner is the only principle by which the development of formulas for the valuation of irregular annual income is possible. Another very important principle is that the entire initial capital is outstanding without change for the entire period of an investment, and that a payment of interest on

initial capital is required in each year of that period. Nearly all formulas previously developed for the valuation of varying income by sinking fund premises have fallen into the common error of disregarding this principle with the result that such algebraic formulas never can be proved by the acid test—arithmetic. These two principles are of general application in the development of any of the sinking fund premises into algebraic formulas.

The single interest rate compound interest sinking fund premises of Division III, in common with all sinking fund premises, provide for equal outstanding capital for an entire term of investment. Interest on unaltered initial capital is the first charge against income for any year, and the difference between the income received and the interest payable at the end of any year goes into the sinking fund account, which may either be increased or decreased by such contribution, and which may be positive or negative in total amount, except at the end of the term of investment, when it must just equal the initial investment, if the original estimates of future income were correct. The sinking fund is invested to earn interest, such interest being compounded yearly and accumulating to the end of the investment period, when contributions from income to the sinking fund plus the interest earned by such contributions will just equal the initial investment, if anticipated income has been realized. If results have differed from estimates as at date of valuation, the profit or loss on the investment is shown by the difference between the initial investment and the amount in the sinking fund at the termination of the investment period, but at no other date will this condition obtain. The amount in the sinking fund at the beginning of any year plus the value as at the same time of future income yet to be received will equal the initial investment at any time during the period of investment, if income was correctly predicted at date of valuation; or the extent of either appreciation or depreciation in value will be exactly shown at any date by the difference between the initial value and the sum of the sinking fund accumulation plus the value of income yet to be received—the excess of the initial value over such sum signifying depreciation and the excess of such sum over the initial value denoting appreciation, both to the amount of the difference. The fallacy that a sinking fund

may be safely invested to earn a speculative rate of interest is responsible for this peculiarity of Division III premises applied to valuations by means of speculative rates of interest; for, when the rate of interest used in valuation is a safe rate, the preceding statements are true for any sinking fund valuation premises. These premises are suitable for the valuation of future safe income, which is discounted to present worth at date of appraisal by means of a safe rate of interest. Discounting by means of speculative rates of interest is another matter, since a condition of the premises peculiar to sinking fund valuation with one rate of interest is that the sinking fund earning rate and the yield to the investor are identical, and, since the suggestion that an ultimate return of capital could be assured through the speculative investment of a sinking fund, would not appeal to intelligent investors. The value of any type of income under these premises is identical with the value of the same income determined under the premises of Divisions I and II; but the manner in which yearly income is allocated to the payment of interest and to the return of capital differs from either of the preceding premises of valuation.

The Hoskold type sinking fund formulas of Division IV follow premises identical with those of the single interest rate sinking fund method, with this important exception—in valuing speculative income, the sinking fund is invested to earn a safe, rather than a speculative, rate of interest. These premises of valuation have been widely used for the commercial valuation of speculative income, especially in the valuation of mines in many countries, and more recently in connection with real estate valuation. The formulas of this type have a peculiar and most important feature distinguishing them from all other types of valuation formulas, that is, the speculative rate of interest (r) appears only in the denominators of the formulas, never in exponential form. The absence of different exponential functions of r permits the ready determination of the speculative rate of interest which will be paid by a stated series of yearly amounts of income upon a fixed value, the safe rate of interest and the duration of the investment being known. The determination of the speculative rate of interest under similar conditions with any other premises of valuation is possible only by means of "cut

and try" arithmetical methods. The appearance of the speculative rate of interest only in the denominators of the formulas, while the yearly amounts of income appear only in the numerators, is responsible for the other peculiar and valuable attribute of the Hoskold type formulas—the fact that the same speculative rate of interest is equivalent to an identical discount for hazard, regardless of the time distribution of income within any fixed term of investment. By reason of this it is possible to compute discounts for hazard and their complementary factors of safety from formulas much simpler than those required when other premises of valuation are involved; and discount for hazard factors determined for equal annual income apply to any type of unequal annual income of similar duration with the Hoskold premises of valuation, but with no other premises. These exceedingly important and absolutely distinctive features of the Hoskold valuation method, combined with the absence of the objectionable features of the one interest rate sinking fund premises, will operate to extend the general use of this formula in the commercial valuation of speculative income, especially when it is a desirable financial policy to maintain constant capital. It is essential, however, to note at this time that, when the premises of this method during periods of income receipt are used in combination with those of the compound interest method during periods of deferment or inadequate receipt of income, these distinctive and important attributes of the Hoskold premises vanish. Methods for the use of the Hoskold premises in combination with compound interest actuarial premises for sinking fund deficiencies have been developed as alternative premises, for the reason that such combination is frequently used, and generally erroneously, in commercial valuation. The proper use of the combined premises, except for simple deferment, involves algebraic and arithmetical "cut and try" computations and is not recommended by the authors, both because the method is quite complicated and because there is nothing connected with the method which gives it any superiority over other methods.

Two objections which have been made to sinking fund formulas in general, and to the Hoskold type in particular, are worthy of discussion. One of these objections is that sinking funds are seldom accumulated in commercial practice by

corporations, mining corporations especially, but capital is reduced through the payment of dividends without separation of the elements of interest payment and capital return. While this may be generally true, it is not a valid objection to sinking fund premises of valuation, since it is immaterial whether the stockholder or the corporation makes provision for the return of capital; in fact, it is preferable that stockholders should do so individually, as the amounts of their investments in the same amount of future income will be variable and will differ from the investment of the corporation in the same future income. The other objection was to the Hoskold method of valuation in particular, and was based upon an exaggerated example of the misuse of a rather high speculative rate of interest in the valuation of a very long term income. This illustration involved the valuation of deep coal seams which mining operations were expected to reach in 80 years and from which equal annual production was expected for a subsequent period of 36 years. Compound interest deferment at 10% interest was used for the 80-year period, and the Hoskold formula for valuing equal annual income on the basis of a 10% yield by the investment and a 3% earning rate for the sinking fund was used in valuing income receipts for the 36-year period. The value determined on this basis was approximately 0.0117% of the amount of the expected income, in comparison with a value of 5.7% of the expected income had a 3% rate of interest been used in discounting to present worth. This means, in other words, that the evaluator using the 10% speculative rate of interest was assuming, whether he knew it or not, that his estimates had but one chance out of 485 of being correct. The conclusion was then forcibly stated that the value determined was so ridiculously low that a trustee selling the property at the appraised value would be liable to criminal prosecution. It must be quite apparent that the error should properly be ascribed to poor judgment in the selection of the speculative rate of interest rather than to an unsound mathematical formula.

The Morkill type sinking fund premises of Division V are identical with premises of Division II, except that an extraneous sinking fund is added which earns interest at a safe rate and upon which interest is paid to the investor from receipts of income at the same safe rate. The interest paid

to the investor on that portion of his initial investment which has been retired to the sinking fund might as well be paid from earnings of the sinking fund as from the income valued, and this would permit the diversion to the sinking fund of an identical amount of interest paid to the investor from income. Since the sum of the amount of interest paid to the investor on the sinking fund and the contribution to the sinking fund from income, under these premises, is equal to the amount available for the retirement of capital under premises of Division II, it is apparent that the Morkill premises comprise two investments: the one a speculative investment under Division II premises, and the other an independent safe investment. The Morkill sinking fund method has no merits that are not found in other methods of valuation, unless it might be said that the extension in the use of Division II premises with a financial policy requiring constant capital is an advantage. Yet the premises of the Morkill method form a connecting link between the compound interest actuarial method and the sinking fund method of valuation which is enlightening. It is also worthy of note that the imposition of an income tax upon sinking fund earnings changes formulas of Morkill type to those of the three interest rate type of sinking fund formulas next described.

The three interest rate sinking fund valuation premises of Division VI are new, and it is doubtful if they will ever be accorded an extended use in commercial valuation. It is not probable that they have points of superiority over valuation formulas now in use for commercial purposes. But, mathematically, the formulas of Division VI are the most important of all the premises of valuation, for absolutely any valuation formula under any of the other premises of valuation and for any time distribution of income may be obtained by letting two of the three rates of interest, or all three of them, be equal, and by proper summation of the terms in which the amounts of yearly income are shown. This feature of the three interest rate sinking fund formulas is so thoroughly demonstrated in Chapters II and III that no further discussion is needed.

We thus arrive at the conclusion that the compound interest actuarial premises of Division II are superior for valuation, in connection with a financial policy requiring the diminution

of outstanding capital, while the Hoskold sinking fund premises of Division IV are superior, when the financial policy requires constant outstanding capital. The Hoskold premises are more adaptable than any other premises when applied in connection with the discount for hazard method of selecting a speculative rate of interest representative of a fixed degree of risk, and offer certain advantages in the facility with which other mathematical problems may be solved. On the other hand, the compound interest actuarial premises of valuation have an undoubted superiority over other premises when investment accounting is considered.

The discount for hazard method of valuation has been so exhaustively described that little need be added here except a reiteration of the authors' firm belief that an intelligent selection of a speculative rate of interest for the purposes of valuation can be accomplished through use of this method with greater consistency than is possible with any other method. The discount for hazard method is also the best available means for comparing diverse methods advocated and used in the valuation of the same types of speculative income, and may be used for valuation either independently or in connection with any of the mathematical premises of valuation. Its flexibility gives it superiority for purposes of valuation and its adaptability preserves all the advantages which may be ascribed to any or all of the six algebraic premises of valuation. Its ultimate advantage is simplicity.

Finally, the selection of one of the mathematical premises will be preferable for the valuation of safe income, and the alternative or complementary use of the discount for hazard method will be preferable when the income to be valued is highly speculative. For valuing intermediate types of income either the mathematical method, or the discount for hazard method, or a combination of both methods, may be used according to the judgment of the evaluator.

APPENDICES

APPENDIX A

The interest rate tabulations hereafter appearing are for the primary purpose of furnishing the figures upon which the charts of Chapter V are based. Other tabulations are added in further illustration of differences in the yields of investments by reason of risk, geographical variations in rates of interest, and variations in the length of investment period. It is regrettable that reliable statistics on rates of interest are so fragmentary and so difficult to obtain. Those of this Appendix are presented in the hope that they may be helpful to others interested in the study of rates of interest.

More recent annual rates may be obtained from the same sources as those noted for rates appearing in the following tabulations. *The Harvard Review of Economic Statistics*, the "Proceedings of the Association of Life Insurance Presidents," the "Monthly Reports of the Government Actuary," issued by the Treasury Department, the "Annual Reports of the Comptroller of the Currency," issued by the Treasury Department, and the reports of the Federal Reserve Board will be valuable sources of reference.

The "Rate of Interest," a book by Professor Irving Fisher, and studies made by Professor Frederick McCauley for the Federal Reserve Bank of New York are worthy of particular mention.

An excellent study by Herbert B. Dorau of the yields of new securities at prices of issue for the period 1919 to 1924 appeared in the July and the October, 1925, issues of the *Journal of Land and Public Utility Economics*. Continuation of the same statistics for later years was indicated. Yields were classified by type of security, type of industry, etc., and both simple and weighted average yields were given.

Table I.—Interest Rates and Wholesale Commodity Index Figures in the United States.

Year	Col. I Approximate yield of U. S. A. government securities at current market prices	Col. II Approximate rate of interest earned by the investments of New York mutual savings banks	Col. III Approximate rate of interest earned by life insurance companies on total invested assets	Col. IV Approximate rate of interest earned by life insurance companies on first mortgage loans	Col. V Market rate of interest on 60 to 90 day commercial paper at New York (Federal Reserve Bank of New York)	Col. VI Wholesale commodity price indices (Federal Reserve Bank of New York)
1800	.....	.....	....		.....	....
1801	.....	.....	....	....	.....	181
1802	5.78	.....	....	....	.....	149
1803	5.95	.....	....	....	.....	153
1804	6.14	.....	....	....	.....	164
1805	6.35	.....	....	....	.....	169
1806	5.86	.....	....	....	.....	165
1807	5.83	.....	....	....	.....	157
1808	5.80	.....	....	....	.....	153
1809	5.38	.....	....	....	.....	(164)*
1810	5.39	.....	....	....	.....	175
1811	5.49	.....	....	....	.....	170
1812	5.69	.....	....	....	.....	173
1813	6.43	.....	....	....	.....	201
1814	6.89	.....	....	....	.....	251
1815	6.74	.....	....	....	.....	197
1816	6.91	.....	....	....	.....	167
1817	5.71	.....	....	....	.....	170
1818	5.44	.....	....	....	.....	165
1819	5.71	.....	....	....	.....	147
1820	5.05	.....	....	....	.....	129
1821	4.20	.....	....	....	.....	124
1822	4.57	.....	....	....	.....	128
1823	4.69	.....	....	....	.....	124
1824	4.54	.....	....	....	.....	123
1825	4.55	.....	....	....	.....	125
1826	4.65	.....	....	....	.....	121
1827	4.18	.....	....	....	.....	120
1828	4.16	.....	....	....	.....	117
1829	4.18	.....	....	....	.....	117
1830	2.88	.....	....	....	.....	114
1831	2.92	.....	....	....	6.13	118
1832	4.15	.....	....	....	6.25	118
1833	4.87	.....	....	....	7.83	115
1834	4.65	.....	....	....	14.50	105
1835	(4.65)(*)	.....	....	....	7.00	117
1836	(4.65)(*)	.....	....	....	18.41	129
1837	5.0	.....	....	....	14.18	126
1838	6.0	.....	....	....	9.03	121
1839	6.0	.....	....	....	13.22	124
1840	6.0	.....	....	....	7.69	110
1841	5.7	.....	....	....	6.77	104
1842	6.1	.....	....	....	8.15	95
1843	4.87	.....	....	....	4.48	93
1844	4.63	.....	....	....	4.83	93
1845	4.72	.....	....	....	6.03	93
1846	5.52	.....	....	....	8.35	100
1847	5.85	.....	....	....	9.54	107
1848	5.90	.....	....	....	15.13	92
1849	4.96	.....	....	....	10.08	94
1850	4.64	.....	5.55	....	8.02	100
1851	4.63	.....	5.08	....	9.69	100
1852	4.26	.....	6.08	....	6.42	101
1853	3.96	.....	5.12	....	10.22	110
1854	4.00	.....	5.88	....	10.38	118
1855	4.23	.....	5.56	....	8.96	121
1856	4.21	.....	6.06	....	8.92	120
1857	4.49	.....	6.19	....	12.77	126
1858	4.42	5.48	5.40	.....	4.99	104
1859	4.77	5.57	5.96	....	6.59	106
1860	5.03	5.69	5.02	7.88	6.80	104
1861	7.47	5.36	4.43	7.14	6.38	102
1862	6.48	6.11	7.20	7.15	5.48	121
1863	6.12	6.54	7.44	6.84	5.00	166

Table I (Continued).—Interest Rates and Wholesale Commodity Index Figures in the United States.

Year	Col. I Approximate yield of U. S. A. government securities at current market prices	Col. II Approximate rate of interest earned by the investments of New York mutual savings banks	Col. III Approximate rate of interest earned by life insurance companies on total invested assets	Col. IV Approximate rate of interest earned by life insurance companies on first mortgage loans	Col. V Market rate of interest on 60 to 90 day commercial paper at New York (Federal Reserve Bank of New York)	Col. VI Wholesale commodity price indices (Federal Reserve Bank of New York)
1864	5.98	7.64	9.28	6.67	6.06	235
1865	6.38	6.70	5.55	7.19	7.50	216
1866	5.44	5.87	7.78	6.20	6.28	193
1867	5.43	6.68	6.89	7.40	7.33	172
1868	5.01	6.54	6.50	7.67	7.24	166
1869	4.44	7.31	7.15	7.31	9.71	164
1870	4.79	6.36	5.71	8.44	7.15	146
1871	4.59	6.39	7.00	7.90	6.97	136
1872	4.36	8.01	6.90	8.59	8.63	141
1873	4.07	7.35	6.93	8.30	10.24	140
1874	3.88	10.48	6.89	7.93	5.98	133
1875	3.24	5.00	6.79	7.30	5.43	125
1876	3.05	5.80	6.55	7.73	5.14	116
1877	3.75	4.55	6.37	7.28	5.17	113
1878	4.11	5.55	5.94	6.77	4.81	102
1879	3.86	4.88	5.83	7.01	5.07	98
1880	3.28	8.29	5.48	6.74	5.22	109
1881	3.06	6.23	5.51	6.66	5.20	106
1882	2.91	5.32	5.53	6.49	5.67	108
1883	2.77	5.62	5.52	6.36	5.56	102
1884	2.64	3.95	5.45	6.37	5.21	93
1885	2.65	5.70	5.41	6.08	4.07	86
1886	2.30	5.53	5.37	6.38	4.76	84
1887	2.30	3.64	5.44	6.55	5.75	85
1888	2.28	5.14	5.41	6.56	4.90	87
1889	2.07	4.23	5.23	6.43	4.88	84
1890	2.32	2.87	5.15	6.38	5.68	81
1891	2.66	3.36	5.28	6.20	5.37	82
1892	2.75	4.07	5.08	6.06	4.12	76
1893	3.03	3.66	4.95	5.72	6.92	77
1894	2.78	4.68	4.93	5.70	3.04	69
1895	2.88	3.88	5.00	5.60	3.68	70.0
1896	3.20	4.34	4.91	5.66	5.81	66.7
1897	3.36	5.13	4.86	5.45	3.51	66.8
1898	2.56	4.43	4.87	5.57	3.84	69.6
1899	2.27	4.40	4.81	5.37	4.14	74.9
1900	1.79	4.44	4.67	5.25	4.38	80.5
1901	1.73	3.61	4.61	5.19	4.29	79.3
1902	1.72	3.32	4.42	5.18	4.93	84.4
1903	1.61	3.08	4.61	5.03	5.47	85.5
1904	1.77	4.32	4.63	5.14	4.21	85.6
1905	1.78	3.84	4.68	4.73	4.40	86.2
1906	1.71	3.07	4.68	4.89	5.68	88.6
1907	1.74	2.60	4.80	4.93	6.38	93.5
1908	1.94	5.63	4.77	4.97	4.38	90.1
1909	2.11	4.14	4.79	4.97	3.98	96.9
1910	2.11	3.97	4.78	5.06	5.00	100.9
1911	2.14	4.53	4.79	5.14	4.04	93.0
1912	2.11	4.02	4.79	5.15	4.74	99.1
1913	2.22	3.79	4.84	5.23	5.58	100.0
1914	2.29	5.29	4.83	5.34	4.79	98.1
1915	2.32	4.82	4.88	6.13	3.45	100.8
1916	2.20	5.75	4.91	6.13	3.42	126.8
1917	3.14	3.37	4.94	6.10	4.73	177.2
1918	4.10	4.29	4.89	6.12	5.88	194.3
1919	4.42	4.81	4.85	5.90	5.42	206.4
1920	5.28	4.46	5.02	6.08	7.35	226.2
1921	5.15	5.28	5.29	6.27	6.54	146.9
1922	4.16	7.51	5.23	5.93	4.42	148.8
1923	4.08	4.87	5.38	5.94	4.97	153.7
1924	3.85	6.75	5.38	5.83	3.76	149.7
1925	3.78	5.55	5.34	5.72	.....	158.7
1926	3.75	.....	....	5.57	.....	151.0
1927	3.53	.....	....	....	.....	146.4

*Interpolated figures are indicated by a star (*).

Comparable figures for farm mortgage loans from Proceedings of the Association of Life Insurance Presidents, the Bureau of the Census, and the Department of Agriculture:

Year	*Total Loans, %*	*New Loans, %*
1914	5.55	
1915		
1916	5.63	
1917	5.63	
1918	5.64	
1919		
1920	6.10	
1921	5.86	6.46
1922	5.93	6.03
1923	5.96	5.36

Comparable figures for percentages of gross earnings to total investments of National Banks, from reports of the Comptroller of the Currency:

	Per Cent
1914	6.17
1915	6.05
1916	5.92

Table I A.—Rates of Interest Paid to Depositors by Mutual Savings Banks of the State of New York from 1819 to 1874.

Year	Rate %	Year	Rate %	Year	Rate %	Year	Rate %
1819	1.92	1833	3.86	1847	4.91	1861	4.70
1820	3.89	1834	3.79	1848	4.65	1862	4.38
1821	4.29	1835	3.84	1849	4.81	1863	4.42
1822	4.36	1836	3.79	1850	5.00	1864	4.47
1823	4.18	1837	3.73	1851	4.82	1865	4.97
1824	4.22	1838	3.91	1852	5.39	1866	4.59
1825	3.70	1839	4.04	1853	4.89	1867	5.23
1826	4.20	1840	4.46	1854	4.56	1868	5.40
1827	4.18	1841	4.44	1855	5.48	1869	5.67
1828	4.22	1842	4.10	1856	4.94	1870	5.74
1829	4.24	1843	4.83	1857	4.98	1871	5.68
1830	4.34	1844	5.03	1858	4.90	1872	5.63
1831	4.15	1845	4.89	1859	4.91	1873	6.26
1832	3.92	1846	4.73	1860	4.51	1874	5.48

able II.—Yearly Average Rates of Interest on Collateral Time Loans and Commercial Paper at New York.

Sources of Information:
(1) The Financial Review - (2) "The Rate of Interest" by Irving Fisher.
(3) The Commercial and Financial Chronicle, Bank and Quotation Section.
(4) The Harvard Review of Economic Statistics.
(5) Senate Document No. 262 - 66th Congress - 2nd Session.
Figures are averages of the weekly high and low quotations for each year

Year	Call Money	Collateral Time Loans							Over Year	Commercial Paper (choice)	
		30 days#	60 days	90 days	4 months	5 months	6 months	7 months#		60-90 day	4-6 months
1849	(2)4.5									*7.8	
1850	4.8									7.2	
1851	5.9									8.3	
1852	5.1									7.3	
1853	6.9									10.1	
1854	7.7									12.5	
1855	6.6									9.3	
1856	7.0									9.9	
1857	6.9									10.4	
1858	3.8									6.7	
1859	5.0									7.2	
1860	6.1									**7.7	
1861	5.4									6.6	
1862	5.6									5.4	
1863	5.0									5.8	
1864	7.2									8.0	
1865	6.1									8.2	
1866	(4)5.23									(4)6.75	
1867	6.39									8.21	
1868	10.75									7.69	
1869	9.79									9.60	
1870	6.35									7.00	
1871	6.62									6.84	
1872	18.06									8.85	
1873	23.67									10.13	
1874	3.44									6.01	
1875	3.05									5.51	
1876	3.35									5.18	
1877	4.30									5.21	
1878	4.03									4.80	
1879	12.00									5.05	
1880	7.74									5.23	
1881	4.16									5.19	
1882	5.96									5.67	
1883	3.84									5.57	
1884	15.51									5.21	
1885	1.72									4.06	
1886	5.81									4.77	
1887	7.18									5.75	
1888	2.50									4.89	
1889	4.73									4.86	
1890	5.85	(1)5.15	(1)5.15#	(1)5.29#	(1)5.46	(1)5.58	(1)5.67	(1)5.67		5.75	(4)6.91
1891	3.41	4.55	4.69	5.00	5.31	5.49	5.53	5.58		5.39	6.46
1892	3.08	3.17	3.49	3.75	4.06	4.40	4.53	4.58		4.11	5.40
1893	4.56	4.83#	5.07#	5.10#	5.18#	5.41#	5.43#	5.43#		6.66	7.64
1894	1.07	1.53	1.96	2.30	2.51	3.03	3.13	3.13		3.04	5.22
1895	1.55	2.21	2.63	2.61	3.24	3.57	3.71	3.71		3.66	5.80
1896	4.29#	4.35#	4.43#	4.56#	4.89#	5.03#	5.09#	5.44#		5.82	7.02
1897	1.76	2.14	2.42	2.73	3.00	3.32	3.36	3.53		3.53	4.72
1898	2.17	2.99	3.15	3.32	3.60	3.73	3.79	3.87		3.83	5.34
1899	5.08	4.08	4.13	4.16	4.42	4.49	4.50	4.50		4.12	5.50
1900	2.94	3.91	3.79	4.01	4.25	4.45	4.45	4.45		4.39	5.71
1901	3.99	4.20	4.19	4.23	4.32	4.38	4.39	4.39		4.28	5.40
1902	5.15	4.98	5.03	5.05	5.06	5.07	5.06	5.06		3.92	5.81
1903	3.71	5.01	4.98	5.05	5.19	5.28	5.36	5.75		5.47	6.16
1904	1.78	2.60	2.88	3.11	3.44	3.58	3.71	3.92		4.21	5.14
1905	4.43	3.83	3.83	3.84	3.94	3.96	4.02	4.31		4.40	5.18
1906	6.55	6.10	5.57	5.70	5.69	5.58	5.63	5.76	(5)5.89	5.68	6.25
1907	7.02	6.18	6.37	6.51	5.50(a)	5.62(a)	5.79(a)	6.08	7.24	6.36	6.66
1908	1.92	3.13	3.29	3.12	3.53	3.80	3.93	4.49	4.12	4.38	5.00
1909	2.71	2.95	3.08	3.29	3.45	3.45	3.67	3.85	4.04	3.98	4.67
1910	2.98	3.68	3.74	4.08	4.19	4.32	4.37	4.62	4.62	5.00	5.72
1911	2.57	3.08	3.02	3.22	3.42	3.52	3.59	3.85	3.90	4.03	4.75
1912	3.50	3.88	3.98	4.14	4.25	4.29	4.35	4.36	4.73	4.74	5.41
1913	3.22		4.31	4.74	4.74	4.90	4.98		5.23	5.60	6.20
1914	3.44		4.13	4.25	4.38	4.48	4.53		5.01	4.78	5.47
1915	1.91		2.55	2.81	3.00	3.13	3.19		3.43	3.45	4.01
1916	2.62		3.07	3.23	3.38	3.45	3.47		3.79	3.43	3.84
1917	3.43		4.48	4.58	4.65	4.73	4.76		4.94	4.74	5.07
1918	5.28	5.81	5.87	5.88	5.94	5.95	5.95		5.97	5.87	6.02
1919	6.49		5.96	5.97	5.99	5.99	5.99			5.42	5.56
1920	7.72		8.02	8.02	7.97	7.96	7.96			7.37	7.54
1921	5.97	(3)	(3)6.13	(3)6.16	(3)6.16	(3)6.25	(3)6.27	(3)		6.53	6.75
1922	4.47		4.50	4.52	4.56	4.58	4.61			4.42	4.68
1923	4.84		5.07	5.08	5.09	5.10	5.11			4.97	5.17
1924	2.93		3.49	3.66	3.81	3.88	3.93			3.76	4.14
1925	4.21		4.10	4.19	4.26	4.29	4.31			(b)	4.13
1926	4.51		4.57	4.58	4.60	4.61	4.62				4.50
1927	4.06		4.25	4.32	4.38	4.40	4.40				4.29

* Sixty Day Commercial Paper, Irving Fisher's "The Rate of Interest".
** Prime 2 Name 60 Day Paper, Irving Fisher's "The Rate of Interest".
# Complete quotations for 30 day and for 7 months collateral time loans not available in the publications cited. The yearly average quotations given in this tabulation are determined by computing the average differences between 30 and 60 day loans and between 6 months and 7 months loans for the number of weeks for which comparative quotations are available, and then assuming that these differences would obtain throughout the year. Other quotations designated by # for individual years were determined in the same manner by comparison with the next nearest type of time loan for which complete quotations were available, and for the same reason. When only one or two weeks quotations were missing such quotations have been assumed by interpolation with due regard to changes in the rate of interest indicated by time loans of other types which were available for the weeks of missing quotations. Indicates that commissions were added to the quotations given, for some weeks of the years in which quotations are followed by this symbol.

(a) 42 weeks, quotations incomplete during 10 panic weeks.
(b) Not quoted by the Commercial and Financial Chronicle.

Table III.—Annual Average Minimum Rates of Discount of European Banks.

"The Bankers Almanac and Year Book" 1924-1925 Edition, Edited by Thomas Skinner and Published by Thomas Skinner and Company, London.

The average of the following rates to and including 1923 is as follows:

England	France	Germany	Holland	Belgium
3.7875	3.70833	4.85833	3.53750	3.66667

	Bank of England	Bank of France	Imperial Bank of Germany	Nether-lands Bank	Nat'l Bank of Belgium
1845	2.683	4.000	4.335	3.113	-
1846	3.325	4.000	4.679	4.313-	-
1847	5.175	5.008	4.842	4.163-	-
1848	3.721	4.000	4.663-	3.738-	-
1849	2.929	4.000	4.042	2.567	-
1850	2.504	4.000	4.000	2.092	-
1851	3.000	4.000	4.000	2.000	4.000
1852	2.150	3.179	4.000	2.000	3.413-
1853	3.692	3.238-	4.250	2.150	3.000
1854	5.113-	4.325	4.371	3.000	3.000
1855	4.892	4.446	4.071	3.188-	3.000
1856	6.058	5.508	4.942	4.271	3.267
1857	6.663-	6.163-	5.779	4.938-	4.000
1858	3.229	3.700	4.129	3.750	3.192
1859	2.729	3.467	4.513-	3.000	3.179
1860	4.179	3.629	4.000	3.000	3.225
1861	5.267	5.554	4.000	3.071	3.800
1862	2.429	3.754	4.000	3.729	3.175
1863	4.408	4.654	4.075	3.621	3.563-
1864	7.400	6.529	5.321	5.342	5.563-
1865	4.767	3.696	4.946	3.971	4.075
1866	6.950	3.575	6.221	5.913-	4.338-
1867	2.538-	2.700	4.000	3.054	2.958
1868	2.096	2.500	4.000	2.658	2.500
1869	3.208	2.500	4.246	3.479	2.500
1870	3.100	3.983	4.879	4.296	3.375
1871	2.883	3.713-	4.138-	3.279	4.050
1872	4.100	5.138-	4.283	3.213-	3.779
1873	4.792	5.142	4.967	4.829	5.050
1874	3.692	4.308	4.383	3.633	4.367
1875	3.233	4.000	4.579	3.325	3.796
1876	2.604	3.408	4.163-	3.000	2.708
1877	2.900	2.263-	4.429	3.000	2.650
1878	3.763	2.208	4.338-	3.446	3.163-
1879	2.517	2.592	3.700	3.250	3.004
1880	2.767	2.842	4.242	3.000	3.304
1881	3.500	3.875	4.425	3.263-	4.067
1882	4.133	3.767	4.521	4.521	4.358
1883	3.567	3.071	4.133	4.133	3.554
1884	2.954	3.000	4.000	3.183	3.279
1885	2.879	3.000	4.121	2.704	3.229
1886	3.050	3.000	3.263	2.500	2.754
1887	3.350	3.000	3.417	2.500	3.054
1888	3.296	3.096	3.325	2.500	3.275
1889	3.546	3.083	3.679	2.500	3.525
1890	4.521	3.000	4.521	2.779	3.175
1891	3.258	3.000	3.788-	3.121	3.000
1892	2.525	2.700	3.213	2.704	2.700
1893	3.054	2.500	4.075	3.408	2.829
1894	2.113-	2.500	3.125	2.588-	3.000
1895	2.000	2.100	3.138-	2.500	2.604
1896	2.483	2.000	3.667	3.038-	2.850
1897	2.633	2.000	3.808	3.133	3.000
1898	3.246	2.200	4.271	2.833	3.038-
1899	3.754	3.063-	5.042	3.583	3.921
1900	3.963-	3.238-	5.325	3.600	4.088-
1901	3.717	3.000	4.096	3.225	3.279
1902	3.329	3.000	3.325	3.000	3.000
1903	3.750	3.000	3.838-	3.396	3.171
1904	3.304	3.000	4.229	3.242	3.000
1905	3.004	3.000	3.813-	2.675	3.171
1906	4.263-	3.000	5.146	4.108	3.838-
1907	4.921	3.463-	6.029	5.100	4.942
1908	3.017	3.050	4.775	3.392	3.575
1909	3.013-	3.000	3.921	2.879	3.108
1910	3.721	3.000	4.346	4.229	4.113-
1911	3.467	3.138-	4.396	3.442	4.150
1912	3.775	3.371	4.942	4.000	4.413-
1913	4.771	4.000	5.883	5.000	4.517
1914	4.038-	4.221	4.892	4.433	4.563
1915	5.000	5.000	5.000	4.750	5.000
1916	5.479	5.000	5.000	4.500	5.000
1917	5.000	5.000	5.000	4.500	5.000
1918	5.000	5.000	5.000	4.500	5.000
1919	5.150	5.000	5.000	4.500	4.442
1920	6.713-	5.729	5.000	4.500	5.046
1921	6.121	5.792	5.000	4.500	5.192
1922	3.692	5.096	6.292	4.054	4.721
1923	3.488-	5.000	38.375	4.500	5.442
1924	4.000	6.000	9.521	4.942	5.500
1925	4.575	6.525	8.842	3.879	5.692

Table IV.—Yields of Mortgage Loan Investments of Life Insurance Companies.

SUB-COLUMN (A) INDICATES RATE ON TOTAL OUTSTANDING LOANS.
SUB-COLUMN (B) INDICATES RATE ON NEW LOANS DURING YEAR.

SOURCES OF INFORMATION:
Columns I, II, III, IV, V, VIII, IX, A, from Proceedings of the Association of Life Insurance Presidents, Vol. 16, P. 159; Vol. 9, p. 143; Vol. 17, p. 144; Vol. 18, p. 170.

Column VII (a) from 1920 Census Reports.

Column VII (b) from U. S. Department of Agriculture Bulletin.

(1) A later publication gives rate of 5.20
(2) A later publication gives rate of 6.20
(3) A later publication gives rate of 7.06

Column	I	II	III	IV	V	VI	VII		VIII		IX			
	1914	1915	1916	1917	1918	1919	1920		1921		1922		1923	
Sub-Column							(a)	(b)	(A)	(B)	(A)	(B)	(A)	(B)
New England States	5.00										5.29	6.00	5.53	6.00
Maine	5.80	6.10	6.00	6.00	6.00		6.1	6.32	6.00	-	6.00	-	6.00	-
New Hampshire	-	5.30	-	-	-		5.1	5.39	-	-	-	-	-	-
Vermont	5.26	5.60	5.22	5.00	5.00		5.4	6.05	5.00	-	5.00	-	5.00	-
Massachusetts	5.19	5.60	5.00	5.00	5.08		5.7	5.98	5.00	-	5.50	6.00	5.60	6.00
Rhode Island	-	5.70	-	-	-		5.8	6.00	-	-	-	-	-	-
Connecticut	5.26	5.70	5.00	-	-		5.7	5.99	5.48	6.50	5.14	-	5.48	-
Middle Atlantic States	5.79	-	-	-	-		-	-	-	-	5.99	-	6.61	7.44
New York	5.86	5.50	5.59	5.50	5.50		5.4	5.94	5.85	6.00	5.94	-	5.94	6.00
New Jersey	6.00	5.50	5.52	5.70	5.67		5.4	6.00	6.00	6.00	6.00	-	6.76	7.50
Pennsylvania	5.75	5.50	5.64	5.77	5.80		5.3	5.97	5.88	-	6.00	-	6.00	-
Delaware	5.97	5.60	5.91	5.88	5.66		5.7	6.00	-	-	-	-	-	-
Maryland	5.84	5.70	5.80	5.80	5.81		5.6	6.00	5.64	-	6.00	-	5.61	-
Central Northern States	5.20	-	-	-	-		-	-	-	-	5.79	5.89	5.62	5.17
Ohio	5.30	5.30	5.34	5.55	5.43		5.9	6.40	5.67	6.37	5.78	5.83	5.70	5.30
Michigan	5.41	6.30	5.72	6.13	6.24		6.0	6.62	5.43	6.50	5.78	5.85	5.58	5.17
Indiana	5.31	5.80	5.42	5.33	5.38		5.8	6.56	5.78	6.27	5.80	5.92	5.66	5.24
Illinois	5.16	5.70	5.23	5.16	5.21		5.5	6.32	5.68	6.36	5.78	5.89	5.56	5.09
Wisconsin	5.53	5.70	5.50	5.76	5.76		5.3	6.31	5.72	6.46	5.85	5.87	5.58	5.13
South Atlantic States	6.00	-	-	-	-		-	-	-	-	6.34	6.54	6.39	6.03
Virginia	6.00	6.10	5.99	5.99	6.00		5.9	6.17	6.09	6.18	6.08	6.22	5.93	5.36
West Virginia	5.70	6.20	5.95	5.87	6.00		5.9	6.06	6.00	6.00	-	-	6.00	-
North Carolina	5.79	6.30	5.99	5.65	5.61		6.0	6.12	5.96	6.17	5.97	6.01	5.91	5.49
South Carolina	6.47	7.80	6.74	6.51	6.42		7.3	7.98	6.94	7.12	6.52	6.99	6.73	5.95
Georgia	6.28	7.60	6.22	6.18	6.06		7.3	8.28	6.36	6.85	6.39	6.71	6.44	6.27
Florida	6.00	9.00	6.21	7.27	6.94		7.3	8.59	6.33	7.00	6.58	7.08	7.24	7.19
Gulf and Mississippi Valley States	5.89	-	-	-	-		-	-	-	-	6.27	6.39	6.20	5.68
Alabama	7.34	8.70	7.05	6.78	6.68		7.5	8.18	6.61	6.77	6.49	6.69	6.58	6.18
Mississippi	6.99	8.00	6.90	6.38	6.31		6.5	7.99	6.56	6.52	6.59	6.95	6.65	6.21
Tennessee	5.56	7.30	5.69	5.76	5.67		6.2	7.51	5.92	6.38	6.06	6.27	6.00	5.50
Kentucky	5.41	6.70	5.41	5.45	5.48		6.0	6.45	5.66	5.97	5.82	5.86	5.65	5.34
Louisiana	7.64	8.20	7.18	7.14	7.08		7.2	8.24	6.68	6.94	6.91	6.86	6.63	6.12
Southwestern States	5.74	-	-	-	-		-	-	-	-	6.01	6.11	6.03	5.51
Missouri	5.35	6.20	5.31	5.39	5.39		6.1	7.13	5.71	6.30	5.82	5.98	5.76	5.28
Arkansas	6.99	9.00	6.81	7.04	6.84		7.8	9.34	6.61	6.93	6.49	6.90	6.99	6.56
Texas	6.99	8.40	7.07	7.01	6.92		7.3	9.05	6.71	7.31	6.51	6.53	6.83	6.35
Kansas	5.46	6.10	5.49	5.38	5.41		6.0	7.30	5.67	6.28	5.75	5.90	5.69	5.25
Colorado	6.92	8.30	6.99	7.07	7.03		6.8	8.58	6.82	7.22	6.88	7.00	7.25	7.07
New Mexico	7.55	9.70	7.52	8.19	8.04		7.3	9.52	6.97	7.27	6.93	6.81	8.46	8.48
Oklahoma	5.91	6.60	5.95	5.83	5.82		6.6	8.98	5.87	6.60	5.93	6.04	5.96	5.55
Northwestern States	5.40	-	-	-	-		-	-	-	-	5.83	5.95	5.83	5.23
Iowa	5.32	5.60	5.28	5.30	5.26		5.5	6.44	5.71	6.32	5.79	5.91	5.81	5.15
Minnesota	5.36	6.30	5.33	5.35	5.35		5.8	6.86	5.77	6.25	5.85	5.94	5.75	5.24
Nebraska	5.34	6.30	5.41	5.30	5.42		5.7	7.19	5.77	6.34	5.85	5.92	5.80	5.24
North Dakota	5.88	6.90	5.70	5.82	5.79		6.7	8.35	6.16	6.96	6.12	6.60	6.20	6.11
South Dakota	5.44	7.00	5.47	5.26	5.39		5.9	7.56	5.79	6.23	5.84	5.98	5.76	5.29
Wyoming	7.71	9.20	7.97	7.17	7.44		7.7	9.21	-	7.00	6.91	-	-	-
Montana	7.29	8.40	7.58	6.58	6.66		7.6	9.50	6.14	7.08	6.78	6.72	7.20	6.82
Pacific States	7.00	-	-	-	-		-	-	-	-	6.52	6.43	6.70	6.29
Washington	6.97	7.90	6.59	5.89	6.10		6.6	7.95	6.41	6.76	6.20	6.18	5.94	5.82
Oregon	6.66	7.70	6.86	7.16	6.34		6.5	7.96	6.32	6.67	6.37	6.17	6.44	5.82
California	6.42	7.40	6.30	6.48	6.41		6.6	7.20	6.50	7.00	6.56	6.56	6.87	7.02
Nevada	8.00	-	-	6.07	6.33		6.7	8.62	6.00	-	6.00	-	6.00	-
Idaho	8.53	8.20	8.12	8.47	8.15		7.3	8.83	7.11	7.25	6.99	6.58	7.45	6.43
Arizona	7.20	9.10	7.04	7.00	7.06		7.5	9.00	7.62	7.48	7.58	7.00	7.16	7.28
Utah	8.74	8.60	8.42	8.59	7.22		7.1	8.71	-	-	-	6.98	8.45	6.74
Miscellaneous														
Porto Rico	-	-	-	-	-		-	-	-	-	-	-	-	-
District of Columbia	-	-	6.00	-	-		5.8	6.0	-	-	-	-	-	-
Average Farm Mortgage Rate of Interest for the U. S.	5.55	-	5.63	5.63	5.64		6.1	-	5.86	6.46	5.93	6.03	5.82	5.36
No. of Companies	126								50		52			
Percentage of Total Loans	97%								93.84		93.8			
Outstanding Loans for the year	647		829	923	954		859	1,086	1,306		1,454			

Table V.—Life Insurance Company Investments and First Mortgage Loans on City Real Estate (Proceedings of the Association of Life Insurance Presidents).

	1914	1921
New England States	4.68	
Maine		4.72
New Hampshire		
Vermont		5.11
Massachusetts		5.61
Rhode Island		5.92
Connecticut		5.78
Middle Atlantic States	5.03	
New York		5.49
New Jersey		5.86
Pennsylvania		5.99
Delaware		
Maryland		5.69
Central Northern States	5.00	
Ohio		6.03
Michigan		5.86
Indiana		5.63
Illinois		5.55
Wisconsin		5.47
South Atlantic States	5.58	
Virginia		5.93
W. Virginia		6.03
North Carolina		6.04
South Carolina		6.05
Georgia		6.09
Florida		6.16
Gulf and Miss. Valley States	5.49	
Alabama		5.92
Mississippi		6.71
Tennessee		5.97
Kentucky		5.60
Louisiana		5.97
Southwestern States	5.23	
Missouri		5.70
Arkansas		6.06
Texas		6.41
Kansas		6.03
Colorado		5.92
New Mexico		6.64
Oklahoma		7.59
Northwestern States	5.29	
Iowa		5.98
Minnesota		5.81
Nebraska		5.88
North Dakota		6.09
South Dakota		6.21
Wyoming		
Montana		7.04
Pacific States	5.24	
Washington		5.92
Oregon		5.97
California		5.88
Nevada		
Idaho		6.00
Arizona		6.24
Utah		5.96
Miscellaneous		
District of Columbia		5.63
Porto Rico		
Average of above	5.13	5.69
Number of Companies		
% of total assets of all Co's.		
Total loans in Millions of $		

Table VI.—Approximate Rates of Interest Realized on Investments by Mutual Savings Banks of the State of New York from 1875 to 1925.

Year	Rate of interest paid to depositors	Percentage of expenses to average deposits for year	Percentage of increase or decrease of surplus to average deposits for year	Approximate percentage yield on investments (sum of three preceding columns)
1875	4.70	0.50	−0.20	5.00
1876	5.19	0.46	0.15	5.80
1877	4.74	0.48	−0.67	4.55
1878	4.29	0.46	0.80	5.55
1879	4.35	0.45	0.08	4.88
1880	4.06	0.37	3.86	8.29
1881	3.64	0.34	2.25	6.23
1882	3.54	0.34	1.44	5.32
1883	3.49	0.34	1.79	5.62
1884	3.46	0.34	0.15	3.95
1885	3.39	0.34	1.97	5.70
1886	3.36	0.34	1.83	5.53
1887	3.39	0.33	−0.08	3.64
1888	3.38	0.42	1.34	5.14
1889	3.41	0.33	0.49	4.23
1890	3.42	0.33	−0.88	2.87
1891	3.45	0.33	−0.42	3.36
1892	3.49	0.34	0.24	4.07
1893	3.57	0.35	−0.26	3.66
1894	3.60	0.36	0.72	4.68
1895	3.63	0.35	−0.10	3.88
1896	3.61	0.34	0.39	4.34
1897	3.58	0.33	1.22	5.13
1898	3.53	0.32	0.58	4.43
1899	3.47	0.31	0.62	4.40
1900	3.42	0.30	0.72	4.44
1901	3.43	0.34	−0.16	3.61
1902	3.34	0.32	−0.34	3.32
1903	3.34	0.32	−0.58	3.08
1904	3.43	0.38	0.51	4.32
1905	3.47	0.37	−0.00	3.84
1906	3.54	0.35	−0.82	3.07
1907	3.64	0.35	−1.39	2.60
1908	3.67	0.35	1.61	5.63
1909	3.69	0.35	0.10	4.14
1910	3.61	0.34	0.02	3.97
1911	3.59	0.35	0.59	4.53
1912	3.54	0.35	0.13	4.02
1913	3.53	0.36	−0.10	3.79
1914	3.60	0.37	1.32	5.29
1915	3.63	0.38	0.81	4.82
1916	3.65	0.41	1.69	5.75
1917	3.67	0.43	−0.73	3.37
1918	3.66	0.45	0.18	4.29
1919	3.70	0.48	0.63	4.81
1920	3.67	0.43	0.36	4.46
1921	3.74	0.39	1.15	5.28
1922	3.78	0.51	3.22	7.51
1923	3.88	0.54	0.45	4.87
1924	3.87	0.58	2.30	6.75
1925	3.91	0.60	1.04	5.55

Table VII.—Average Rates Charged by Member Banks on Customers Paper Rediscounted with Federal Reserve Bank. Rates for 1921 and 1922 are for December only. Rates for later years are the arithmetical averages of rates published in reports of the Federal Reserve Board for March, June, September and December.

Year	Size of Community	All Districts	Boston	New York	Philadelphia	Cleveland	Richmond	Atlanta	Chicago	St. Louis	Minneapolis	Kansas City	Dallas	San Francisco
1921	Under 15,000	8.0	6.2	6.1	6.0	6.3	6.9	7.9	7.5	8.0	8.7	8.9	9.7	8.2
	15,000 to 100,000	7.2	6.1	6.0	6.0	6.4	6.8	7.5	7.1	7.6	8.9	7.3	8.3	7.3
	Over 100,000	6.1	6.1	6.0	6.0	6.3	6.1	7.1	6.4	6.4	6.8	7.2	7.6	6.1
1922	Under 15,000	7.7	5.8	5.7	5.9	6.2	6.7	7.9	7.5	7.8	9.0	8.8	9.8	8.0
	15,000 to 100,000	6.4	5.5	6.0	6.0	6.1	6.2	7.0	6.3	6.6	7.4	6.4	8.6	7.2
	Over 100,000	5.1	4.8	4.9	5.0	5.6	5.9	5.8	5.0	5.3	5.7	6.3	6.3	5.2
1923	Under 15,000	7.7	6.0	6.0	6.0	6.1	6.4	7.9	7.4	7.8	8.9	8.7	9.6	8.1
	15,000 to 100,000	6.4	5.5	5.9	6.0	6.2	6.1	7.1	6.1	6.0	7.0	6.8	8.2	7.1
	Over 100,000	5.3	5.1	5.3	5.4	5.6	5.9	5.8	5.4	5.4	5.7	6.4	6.5	5.6
1924	Under 15,000	7.4	6.1	5.9	5.9	6.2	6.4	7.7	7.3	7.8	8.7	8.8	9.5	8.0
	15,000 to 100,000	6.3	5.6	5.8	6.0	6.1	6.2	6.9	6.3	6.4	7.2	7.0	7.5	6.9
	Over 100,000	5.4	5.1	5.4	5.5	5.7	6.0	5.8	5.3	5.2	5.8	6.6	6.2	6.0
1925	Under 15,000	7.1	5.9	5.9	5.9	6.3	6.3	7.6	7.3	7.8	8.5	8.7	9.1	7.7
	15,000 to 100,000	6.1	5.2	5.7	6.0	6.0	6.2	6.6	6.0	6.2	7.1	7.8	8.0	7.4
	Over 100,000	5.1	4.9	5.1	5.6	5.4	6.0	6.0	4.7	4.8	5.8	5.8	5.5	6.0
1926	Under 15,000	7.1	5.9	5.9	6.0	6.1	6.3	7.8	7.2	7.9	8.3	8.5	9.2	7.8
	15,000 to 100,000	6.1	5.5	5.8	6.0	6.1	6.1	6.7	6.0	6.5	6.9	6.3	6.6	5.7
	Over 100,000	5.2	5.4	5.6	5.4	5.5	6.0	5.9	4.9	4.9	5.5	6.7	5.9	5.7

Table VIII.—Average Interest Rates Charged by Banks on Loans and Discounts.

Locality	Average Interest Rates Charged by Banks on Loans and Discounts as at July 12, 1889. From Reports of the U. S. Comptroller of the Currency				Average Interest Rates Charged by Banks on Loans and Discounts as at July 18, 1894 From Reports of the U. S. Comptroller of the Currency				Average Interest Rates Charged by Banks on Loans and Discounts as at June 30, 1899 From Reports of the U. S. Comptroller of the Currency			
	National Banks	State and Private Banks	Loan and Trust Co's.	Total all Banks	National Banks	State and Private Banks	Loan and Trust Co's.	Total all Banks	National Banks	State and Private Banks	Loan and Trust Co's.	Total all Banks
Maine	6.3	6.0	0.0	6.0	6.00	6.0	6.2	6.2	5.8	6.0	6.8	5.5
New Hampshire	6.0			6.1	5.9			6.8	5.8			5.0
Vermont	6.0		6.0	6.1	5.9		6.0	5.6	5.9		5.7	5.8
Massachusetts	5.3	7.3	5.8	5.1	5.3	6.7	5.1	4.9	5.0	6.8	4.7	4.7
Boston	5.1		4.1	4.4	4.4		3.4	4.3	4.2		3.7	4.0
Rhode Island	5.4	-	6.0	5.7	5.2	-	5.7	5.5	4.5	8.0	5.0	5.1
Connecticut	6.0	6.0	6.0	5.4	5.9	6.0	6.0	5.1	5.0	5.7	5.8	4.9
Total New England States	5.5	7.1	4.6	5.2	5.6	6.6	4.4	5.0	4.7	6.3	4.3	4.7
New York	6.0	6.4	5.4	5.0	5.9	6.2	5.5	4.9	5.7	6.0	5.2	4.8
New York City	4.8	5.3	4.2	4.6	4.1	4.9	4.5	4.6	4.2	5.0	4.3	4.6
Albany	5.5	6.0		4.8	5.0	6.0		4.6	4.7	6.0		4.6
Brooklyn	5.4	5.1	4.2	5.2	4.2	5.5	4.5	4.9	4.0	4.8	4.1	4.8
New Jersey	5.9	6.0	6.0	5.4	5.3	5.8	5.4	5.2	5.1	5.9	5.0	5.0
Pennsylvania	6.0	6.1	5.5	6.6	5.8	6.1	6.0	6.2	5.4	5.9	5.6	5.6
Philadelphia	5.1	4.7	4.8	4.5	4.7	4.2	4.8	4.0	4.2	3.7	4.3	3.9
Pittsburg	6.0	6.0	6.0	6.0	6.0	6.0	6.0	6.0	5.5	5.4	5.2	5.8
Delaware	6.0		5.7	6.2	6.0		5.5	5.8	5.3		5.1	5.2
Maryland	6.0	6.0		6.7	5.9	5.9		6.5	5.4	5.6		5.9
Baltimore	5.1	5.5		5.3	5.2	5.5		4.8	4.9	5.1		4.8
Washington,D.C.	5.7		-	-	5.8		5.6	-	5.3		4.6	6.0
Total Eastern States	5.3	5.6	4.6	4.8	4.8	5.4	4.7	4.8	4.7	5.2	4.5	4.6
Virginia	6.4	7.5	8.0	7.0	6.5	7.5	6.6	7.0	6.1	8.4	6.3	5.8
W. "	6.7	6.0			6.7	6.2			6.5	6.4		
N. Carolina	7.6	8.0			7.6	8.1			6.5	6.5		
S. "	8.0	-			7.7	-			7.3	8.0		
Georgia	9.3	10.3			8.4	8.9			7.2	7.7		
Savannah	8.0				7.0				6.0			
Florida	9.7	10.8			8.8	8.1			8.6	8.1		
Alabama	8.5	12.0			8.3	10.0			7.9	10.3		
Mississippi	9.3	10.0			9.1	9.7		8.0	8.4	9.2		8.5
Louisiana	-	-			8.0	8.5			8.0	8.8		
Texas	11.6	11.6			10.6	11.2			9.8	10.8		
Houston	8.5				8.5				7.8			
Arkansas	10.8	9.8			10.4	9.6			10.3	9.0		
Kentucky	7.4	7.9	-		6.6	7.8	8.4		6.4	7.3	7.1	
Louisville	6.8	7.2			6.5	6.7			5.6	6.1		
Tennessee	8.1	8.1	-	7.5	7.5	8.9	8.6	7.9	6.2	8.3	7.7	7.4
Total Southern States	8.3	8.2	8.0	7.1	8.0	7.9	7.6	7.2	7.4	7.4	6.9	6.4
Ohio	7.3	7.4	-	6.8	7.6	7.4		6.6	6.3	6.7	-	5.2
Cincinnati	6.0	6.0		5.3	5.7	6.2		5.4	4.2	5.9		5.0
Cleveland	6.7	-	6.6		6.4	7.0	6.5		5.4	6.4	5.7	
Indiana	7.7	7.9			7.4	7.7			7.1	7.4		
Indianapolis	7.0				6.9				6.1			
Illinois	7.5	7.8	-	7.0	7.0	7.0	-	7.0	6.3	6.3	-	6.0
Chicago	5.6	6.7	5.5		5.1	6.3	5.0		4.4	5.0	4.5	
Michigan	7.4	7.5		7.3	7.2	7.2		7.2	6.2	5.5		6.4
Detroit	7.1			6.9	6.3			6.4	4.9			5.5
Wisconsin	7.3	7.9		7.0	6.9	6.0		6.4	5.9	6.3		5.7
Milwaukee	6.8				5.9				4.8			
Minnesota	8.5	9.4	-		8.2	9.3		-	6.8	8.2		-
1st. Paul	7.1	8.0	-		6.4	7.0		7.0	5.2	5.5		5.8
Minneapolis	7.6	9.0			6.5	8.0			4.9	6.5		
Iowa	8.5	9.6	7.4	7.4	8.0	8.2	7.3	7.0	7.1	7.7	6.6	6.2
Des Moines	9.0				7.5				6.7			
Missouri	8.9	8.2	-	8.4	8.2	8.2	-	8.0	7.4	7.9	-	8.0
St. Louis	6.5	7.0	-		6.3	6.6	6.0		5.2	5.8	5.6	
Kansas City	7.7	0.0			7.4	8.0			6.1	6.9		
St. Joseph	9.0	7.3			7.0	6.6			6.2	6.2		
Total Middle States	7.5	7.9	6.5	7.0	6.9	7.5	6.2	6.8	5.8	6.8	5.6	6.1
North Dakota	12.0	12.7			11.2	11.8			9.7	10.9		
South "	12.4	13.8			10.9	12.2			9.0	10.6		
Nebraska	10.5	12.1		9.5	10.1	10.9		8.7	8.7	9.6		7.8
[illegible]	8.4				8.0				6.9			
Kansas	10.4	12.5		9.8	9.6	10.9		8.7	8.5	9.7		7.3
Montana	12.0	10.5			11.2	11.4			10.3	10.0		
Wyoming	13.3	12.0			12.9	12.1			11.2	11.4		
Colorado	10.7	11.0			10.4	10.2			9.1	10.5		
New Mexico	12.0	-			11.8	11.6			10.9	10.7		
Oklahoma		-		-	12.0	19.3		11.6	14.4	16.3		10.7
Indian Territory	-	10.0			12.2	10.0			10.3	10.0		
Total Western States	10.5	12.3		9.7	10.0	11.2		9.5	8.9	10.1		8.5
Washington	11.5	11.9			10.7	10.6			10.0	9.7		
Oregon	10.3	10.0			10.2	10.0			9.2	9.4		
Portland	7.3	-			8.2	-			7.5	0.0		
California	8.8	8.5	5.5	8.1	8.1	8.3	6.5	8.1	7.2	7.1	5.8	7.2
San Francisco	5.7	7.9		5.9	6.0	6.9		6.5	5.2	6.0		5.9
Idaho	13.4	-			12.5	-			9.6	12.0		
Utah	10.5	-		8.0	10.5	11.0		8.4	9.0	9.6		7.9
Nevada	12.0	-			10.0	-			8.0	12.0		
Arizona	15.0				12.4				10.4			
Alaska	-								10.0			
Total Pacific States	9.2	8.4	5.5	6.1	9.0	7.9	6.5	6.7	7.8	7.3	5.8	6.2
Total United States	6.1	7.6	4.9	5.2	5.8	7.2	5.0	5.2	5.3	6.7	4.8	4.9
Number of Banks	1026	824	44	443	1377	1002	115	574	1478	1506	165	584
Report of Comptroller for	1899	1899	1899	1899	1899	1899	1899	1899	1899	1899	1899	1899

Table IX A.—Average Interest Rates Charged by Banks on Time Loans in 1910 (from Reports of the U. S. Comptroller of the Currency).

Locality	National Banks	State Banks	Mutual Savings Banks	Stock Savings Banks	Loan and Trust Companies	Private Banks
Maine	5.85	-	5.72	-	5.98	-
New Hampshire	5.48	5.75	5.39	5.44	-	-
Vermont	5.88	-	-	-	-	-
Massachusetts	5.31	-	5.32	-	5.23	-
Rhode Island	5.30	5.58	5.35	-	5.40	-
Connecticut	5.56	5.17	5.01	-	5.57	-
Av.New England States	5.53	5.50	5.36	5.44	5.55	-
New York	5.85	5.83	4.95	-	5.40	6.00
New Jersey	5.80	5.85	5.19	5.25	5.75	5.33
Pennsylvania	5.51	5.88	6.06	-	5.75	6.03
Delaware	5.80	-	-	-	-	6.00
Maryland	5.83	-	5.48	-	-	-
District of Columbia	5.70	-	-	5.76	5.31	-
Av. Eastern States	5.66	5.85	5.42	5.51	5.55	5.84
Virginia	6.06	6.24	-	6.14	6.38	7.00
W. "	4.82	6.42	6.00	6.00	6.00	-
N. Carolina	6.50	6.15	-	6.05	6.20	-
S. "	7.32	7.95	-	7.50	7.80	-
Georgia	8.49	9.08	-	7.93	-	9.00
Florida	8.35	8.91	-	8.20	-	8.93
Alabama	8.74	9.05	-	8.80	-	10.88
Mississippi	8.60	9.13	-	8.43	-	-
Louisiana	7.72	8.32	-	7.13	-	-
Texas	9.57	9.68	-	-	8.82	9.88
Arkansas	9.10	9.38	-	-	8.14	-
Kentucky	6.58	6.69	-	6.60	6.33	-
Tennessee	7.29	7.99	-	6.68	-	8.50
Av. Southern States	7.99	8.08	6.00	7.22	7.09	9.03
Ohio	6.09	5.92	5.37	6.07	5.79	6.42
Indiana	6.32	6.51	5.75	-	6.21	6.71
Illinois	6.51	6.08	-	-	5.47	6.41
Michigan	6.16	6.52	-	6.31	6.27	7.33
Wisconsin	6.01	6.32	6.00	5.97	5.53	-
Minnesota	6.67	7.92	6.23	-	5.75	-
Iowa	7.36	7.05	-	6.74	6.40	7.34
Missouri	7.60	7.22	-	-	6.31	7.49
Av.Mid.West States	6.55	6.69	5.84	6.27	5.97	6.95
North Dakota	10.43	10.73	-	-	7.00	-
S. "	8.69	-	-	-	-	-
Nebraska	7.87	8.11	-	6.37	-	-
Kansas	8.03	8.26	-	7.53	7.00	8.00
Montana	9.78	10.45	-	9.00	8.50	11.00
Wyoming	9.15	9.93	-	9.00	11.00	10.67
Colorado	8.94	9.71	-	8.28	8.02	10.28
N. Mexico	10.12	10.97	-	11.33	-	-
Oklahoma	12.60	12.02	-	11.00	8.50	-
Av.Western States	9.27	10.02	-	8.93	8.34	9.99
Washington	8.57	9.04	-	8.08	7.93	-
Oregon	8.14	8.30	-	7.77	7.00	8.45
California	6.61	7.26	5.50	6.90	6.13	8.06
Idaho	9.63	9.99	-	9.50	8.67	9.33
Utah	8.34	8.71	-	7.80	-	9.00
Nevada	9.40	9.53	-	8.00	-	-
Arizona	9.76	10.14	-	8.00	-	-
Alaska	11.50	12.00	-	-	-	-
Av.Pacific States	7.83	9.37	5.50	8.01	7.43	8.71
Hawaii	8.33	7.06	-	-	-	-
Porto Rico	9.14	10.11	-	-	-	-
Av. Island Possessions	8.53	8.59	-	-	-	-
Number of Banks	60,69	10,293	3.98	1,010	954	801
Av. United States	7.33	7.73	5.62	6.89	6.66	8.10

Table IX B.—Average Interest Rates Charged by Banks on Demand Loans in 1910 (from Reports of the U. S. Comptroller of the Currency).

Locality	National Banks	State Banks	Mutual Savings Banks	Stock Savings Banks	Loan and Trust Companies	Private Banks
Maine	5.67	-	5.67	-	5.58	-
New Hampshire	5.55	5.71	5.22	5.21	-	-
Vermont	5.78	-	-	-	-	-
Massachusetts	5.10	-	4.70	-	4.97	-
Rhode Island	5.21	4.31	4.40	-	5.04	-
Connecticut	5.11	4.92	5.08	-	5.20	-
Av.New England States	5.31	4.98	5.01	5.21	5.20	-
New York	5.63	5.51	4.50	-	4.89	6.19
New Jersey	5.52	5.57	5.26	5.00	5.33	4.00
Pennsylvania	5.67	5.82	5.00	-	5.57	5.75
Delaware	5.51	-	-	-	-	6.00
Maryland	5.53	-	5.30	-	-	-
District of Columbia	5.07	-	-	5.47	5.12	-
Av. Eastern States	5.63	5.63	5.01	5.23	5.23	5.48
Virginia	5.93	6.13	-	6.06	6.31	7.00
W. "	4.54	6.04	6.00	6.00	6.00	-
N. Carolina	6.28	6.14	-	6.00	6.40	-
S. "	7.01	7.76	-	7.06	7.67	-
Georgia	8.10	8.53	-	7.23	-	10.00
Florida	8.07	8.58	-	8.00	-	8.33
Alabama	8.08	8.20	-	7.50	-	10.67
Mississippi	8.26	8.78	-	8.42	-	-
Louisiana	7.22	7.78	-	6.59	-	-
Texas	9.19	8.94	-	-	8.92	9.43
Arkansas	8.10	9.22	-	-	7.71	-
Kentucky	6.14	6.36	-	6.33	6.01	-
Tennessee	6.87	7.53	-	5.88	-	10.00
Av. Southern States	7.53	7.69	6.00	6.82	7.00	9.24
Ohio	5.80	6.09	5.69	5.94	5.74	5.37
Indiana	5.50	6.34	5.50	-	6.17	6.49
Illinois	6.08	6.28	-	-	5.36	6.41
Michigan	5.99	6.34	-	5.82	5.72	6.80
Wisconsin	5.84	6.32	-	6.20	5.37	-
Minnesota	6.93	8.18	6.00	-	5.50	-
Iowa	7.19	7.34	-	7.20	6.00	7.67
Missouri	6.68	6.85	-		6.06	7.83
Av.Mid.West States	6.25	6.72	5.73	6.29	5.74	6.76
North Dakota	9.72	10.96	-	-	6.25	-
S. "	8.51	-	-	-	-	-
Nebraska	7.55	8.38	-	6.65	-	-
Kansas	7.19	8.28	-	7.67	6.25	6.67
Montana	9.58	10.50	-	10.00	8.80	11.33
Wyoming	9.30	10.22	-	8.00	11.00	10.00
Colorado	8.69	10.13	-	8.44	8.00	10.45
N. Mexico	10.12	10.69	-	11.00	-	-
Oklahoma	9.85	10.59	-	10.00	8.50	-
Av.Western States	8.70	9.97	-	8.82	8.13	9.61
Washington	8.55	9.21	-	8.39	7.72	-
Oregon	7.99	8.64	-	7.95	8.00	8.50
California	7.14	7.33	5.50	6.34	5.83	8.33
Idaho	9.50	10.07	-	10.00	8.83	10.17
Utah	8.16	8.28	-	7.85	-	9.00
Nevada	9.82	10.10	-	8.00	-	-
Arizona	9.70	9.54	-	8.00	-	-
Alaska	11.50	12.00	-	-	-	-
Av.Pacific States	7.76	9.39	5.50	8.08	7.59	9.00
Hawaii	7.45	7.19	-	-	-	-
Porto Rico	9.00	9.00	-	-	-	-
Av. Island Possessions	7.76	8.09	-	-	-	-
Number of Banks	5,680	8,190	334	823	830	556
Av. United States	7.00	7.49	5.45	6.75	6.49	8.02

Table X.—Rate of Interest Paid to Depositors by Savings Departments of National Banks in Cities (Reports of the U. S. Comptroller of the Currency, Years Ending, June 30th).

Year	1921	1922	1923	1924	1925	1926	1927
City							
New York	3.35	3.50	3.50	3.46	3.45	3.29	3.37
St. Louis	3.00	3.00	-	-	-	-	-
Chicago	3.00	3.00	2.25	3.00	3.00	3.00	3.00
Total Central Reserve Cities	3.18	3.25	2.88	3.23	3.23	3.15	3.19
Boston	3.83	4.50	3.75	3.79	3.98	3.92	3.72
Albany	2.17	3.50	3.25	3.36	3.42	3.50	3.75
Brooklyn and Bronx	-	-	3.50	3.50	3.50	3.67	3.83
Buffalo	4.00	4.00	4.00	4.00	4.00	4.00	3.83
Philadelphia	3.18	3.25	2.50	3.75	3.75	3.75	3.80
Pittsburg	3.82	3.75	3.75	3.90	4.00	3.86	4.00
Baltimore	3.81	4.00	4.00	4.00	3.90	3.90	3.90
Washington	3.14	3.25	3.00	3.12	3.15	3.22	3.13
Richmond	3.00	3.00	3.00	2.91	3.00	3.00	3.00
Charleston	5.00	4.00	-	-	-	-	-
Atlanta	3.56	3.50	3.75	3.50	3.50	3.33	3.25
Savannah	-	4.00	-	-	-	-	4.17
Jacksonville	4.00	4.00	3.75	4.00	3.25	3.50	3.75
Birmingham	4.00	4.00	4.00	4.00	4.00	4.00	4.00
Dallas	4.00	4.00	4.00	4.00	3.90	4.00	4.00
El Paso	4.00	4.00	4.00	4.00	3.00	3.00	3.00
Fort Worth	4.00	4.00	4.00	4.00	4.00	4.00	4.00
Galveston	4.00	4.00	4.00	4.00	4.00	4.00	4.00
Houston	3.92	4.00	4.00	4.00	4.00	4.00	4.00
San Antonio	4.00	4.00	4.00	4.00	3.63	4.00	4.00
Waco	4.00	4.00	4.00	4.00	4.00	4.00	4.00
Little Rock	4.00	4.00	4.00	4.00	4.00	4.00	4.00
Louisville	3.25	3.25	3.75	3.25	3.25	3.22	3.63
Chattanooga	4.00	4.00	-	-	-	-	-
Memphis	3.00	3.00	3.00	3.00	3.00	3.00	3.00
Nashville	3.75	4.00	3.50	4.00	3.50	4.00	3.75
Cincinnati	3.07	3.00	4.00	4.00	4.00	4.00	4.00
Cleveland	4.00	4.00	4.50	4.00	4.00	4.00	4.00
Columbus	3.00	3.00	3.00	3.00	3.00	3.00	3.00
Toledo	3.00	3.00	3.00	3.00	3.00	3.50	3.50
Indianapolis	3.40	3.00	3.00	3.00	3.00	3.75	3.63
Chicago	3.00	3.00	3.00	3.00	3.00	3.00	3.00
Peoria	3.00	3.00	3.00	3.00	3.00	3.00	3.00
Detroit	3.50	3.00	3.00	3.00	3.00	3.00	3.00
Grand Rapids	3.08	3.25	3.00	3.25	3.25	3.00	3.00
Milwaukee	4.00	3.00	3.00	2.90	2.93	2.97	2.85
Minneapolis	3.81	4.00	3.75	3.44	3.50	3.50	3.50
St. Paul	3.83	3.25	3.75	3.75	3.78	4.33	3.25
Cedar Rapids	4.00	4.00	4.00	4.00	4.00	3.50	3.50
Des Moines	5.00	4.00	4.00	4.00	4.00	3.50	3.50
Dubuque	4.00	4.00	4.00	4.00	3.50	3.50	3.50
Sioux City	4.00	3.00	4.00	3.80	3.70	3.50	3.50
Kansas City, Mo.	3.00	3.00	3.00	3.00	3.00	3.00	3.00
St. Joseph	4.00	4.00	4.00	4.00	4.00	3.00	3.00
St. Louis	-	-	3.00	3.03	4.03	3.05	3.09
Lincoln	4.00	4.00	4.00	4.00	4.00	4.00	4.00
Omaha	3.33	3.25	3.25	3.25	3.14	3.17	3.18
Kansas City, Kans.	3.00	3.00	3.00	3.00	3.00	3.00	3.00
Topeka	4.00	4.00	4.00	4.00	3.00	3.00	3.00
Helena	4.00	4.00	4.00	4.00	4.00	4.00	4.00
Denver	4.00	4.00	4.00	3.83	3.61	3.50	3.54
Pueblo	3.50	3.00	4.00	4.00	4.00	4.00	4.00
Muskogee	4.00	4.00	4.00	4.00	4.00	4.00	4.00
Oklahoma City	4.00	4.00	4.00	4.00	4.00	4.00	4.00
Tulsa	4.00	4.00	4.00	4.00	4.00	4.00	3.92
Seattle	3.00	3.00	2.75	3.05	3.00	3.00	3.06
Spokane	3.00	3.50	3.00	4.00	4.00	4.00	3.70
Tacoma	3.00	3.00	3.00	-	-	-	-
Portland	3.00	3.00	3.00	3.00	3.50	3.33	3.60
Los Angeles	3.69	3.75	3.50	3.67	3.50	3.71	3.50
Oakland	3.70	3.50	4.00	4.00	4.13	4.00	4.00
San Francisco	4.00	3.75	3.75	3.75	4.00	3.88	3.94
Ogden	4.00	4.00	4.00	4.00	4.00	4.00	4.00
Salt Lake City	4.00	4.00	3.00	4.00	4.00	4.00	4.00
Total other Reserve Cities	3.57	3.50	3.58	3.64	3.59	3.60	3.58
Total all Reserve Cities	3.55	3.50	3.56	3.44	3.41	3.38	3.39

Table XI.—Rate of Interest Paid to Depositors by Savings Departments of National Banks in Country Districts (Reports of the U. S. Comptroller of the Currency, Years Ending June 30th).

Year	1921	1922	1923	1924	1925	1926	1927
State							
Maine	3.95	4.00	4.00	3.97	3.94	3.98	3.96
New Hampshire	3.21	3.50	3.25	3.37	3.00	3.50	3.68
Vermont	3.97	4.00	4.00	4.01	4.00	4.00	4.08
Massachusetts	4.34	4.50	4.00	4.10	4.01	4.06	3.86
Rhode Island	3.95	4.00	4.00	4.13	4.17	4.40	4.25
Connecticut	4.00	4.00	4.25	4.00	4.00	4.00	4.32
Total New England States	4.07	4.00	3.92	3.93	3.85	3.99	4.03
New York	3.76	3.75	3.75	3.84	3.87	3.94	3.92
New Jersey	3.58	3.50	3.50	3.63	3.75	4.00	3.77
Pennsylvania	3.34	3.50	3.50	3.50	3.50	3.49	3.85
Delaware	3.87	4.00	4.00	4.00	3.98	3.83	4.00
Maryland	3.63	3.75	3.75	3.66	3.72	3.04	3.82
Total Eastern States	3.51	3.75	3.70	3.73	3.76	3.66	3.87
Virginia	3.61	3.75	3.75	3.76	3.75	4.00	3.73
West Virginia	3.55	3.50	3.75	3.75	3.75	3.70	3.79
North Carolina	4.05	4.00	4.00	4.00	4.00	4.00	4.00
South Carolina	4.49	4.50	4.50	4.38	4.25	4.33	4.30
Georgia	4.21	4.50	4.25	4.19	4.00	4.16	4.14
Florida	4.14	4.00	4.00	4.11	4.00	4.00	4.08
Alabama	4.01	4.00	4.00	4.00	4.00	4.00	3.99
Mississippi	-	4.00	4.00	3.94	4.00	4.00	3.94
Louisiana	3.85	4.00	4.00	3.87	4.00	3.85	3.89
Texas	4.03	4.00	4.00	4.00	3.93	4.00	3.95
Arkansas	4.12	4.00	4.00	4.01	4.00	4.00	3.95
Kentucky	3.36	3.50	3.50	3.65	3.69	4.10	3.66
Tennessee	3.95	3.50	3.25	3.78	3.75	3.75	3.82
Total Southern States	3.89	4.00	3.92	3.96	3.93	3.99	3.94
Ohio	3.66	4.00	3.75	3.80	4.00	4.00	3.83
Indiana	3.64	3.75	3.75	3.54	3.58	3.63	3.70
Illinois	3.40	3.50	3.25	3.45	3.46	3.42	3.46
Michigan	3.40	3.25	3.25	3.08	3.25	3.50	3.34
Wisconsin	3.29	3.50	3.50	3.41	3.26	3.34	3.28
Minnesota	4.45	4.50	4.50	4.33	3.90	3.83	3.84
Iowa	4.31	4.25	4.25	4.30	4.22	4.14	4.01
Missouri	3.42	3.50	3.50	3.45	3.38	3.45	6.92
Total Middle Western States	3.77	3.75	3.72	3.67	3.63	3.66	4.05
North Dakota	4.91	5.00	5.00	4.99	4.50	4.40	4.11
South Dakota	4.90	4.95	4.75	4.70	4.42	4.20	4.09
Nebraska	4.31	4.25	4.50	4.39	4.25	4.04	4.11
Kansas	3.70	3.25	3.75	3.84	3.78	3.70	3.71
Montana	4.60	4.25	4.75	4.31	4.40	4.30	4.38
Wyoming	4.25	4.25	4.25	4.18	4.24	4.02	3.98
Colorado	4.02	4.00	4.00	4.05	4.00	4.03	3.96
New Mexico	4.10	4.00	4.25	4.00	4.10	4.09	4.19
Oklahoma	4.09	4.00	4.00	4.08	4.00	4.00	3.99
Total Western States	4.34	4.22	4.36	4.28	4.19	4.09	4.06
Washington	3.82	4.00	4.00	3.89	4.00	3.89	3.89
Oregon	3.80	3.75	3.75	3.78	3.50	3.75	3.64
California	3.97	4.00	4.00	4.02	4.00	3.75	3.97
Idaho	4.07	4.50	4.00	4.13	4.00	4.00	3.93
Utah	4.08	4.00	4.00	4.07	4.00	4.00	4.10
Nevada	4.00	4.00	4.00	4.00	4.00	4.00	4.00
Arizona	4.05	4.00	4.00	4.09	4.00	3.90	4.08
Total Pacific States	3.95	4.00	3.96	4.00	3.93	3.90	3.94
Non Member Banks							
Alaska	-	3.50	3.25	3.75	3.63	3.63	3.67
Hawaii	-	4.00	4.00	4.00	4.00	4.00	4.00
Total non member Banks	-	3.75	3.63	3.88	3.82	3.82	3.84
Country Banks Average of U. S. Member Banks also Alaska and Hawaii	3.99	3.93	3.89	3.92	3.87	3.87	3.96
City and Country Banks Total Member and non-member banks, U.S., Alaska and Hawaii	3.81	3.71	3.73	3.68	3.64	3.63	3.68

Table XII.—Federal Reserve Bank Rates, being Average Rates Earned on Total Bills and Securities (from Annual Reports of the Federal Reserve Board).

	All Banks	Boston	New York	Philadelphia	Cleveland	Richmond	Atlanta	Chicago	St. Louis	Minneapolis	Kansas City	Dallas	San Francisco
1916	2.67												
1917	3.29												
1918	4.12	4.02	3.97	4.19	4.14	4.29	4.17	4.19	4.17	4.45	4.35	4.30	4.41
1919	4.04	4.01	4.00	3.92	4.02	4.16	4.03	4.02	3.92	4.11	4.26	4.28	4.26
1920	5.50	5.48	5.59	5.02	5.45	5.29	5.33	5.75	5.29	5.76	5.38	5.14	5.49
1921	5.61	5.42	5.69	4.95	5.39	5.61	5.63	5.91	5.43	6.09	5.39	5.79	5.56
1922	4.11	3.87	3.82	4.11	4.10	4.57	4.40	4.23	4.14	4.64	4.25	4.56	4.08
1923	4.33	4.28	4.30	4.37	4.29	4.44	4.41	4.26	4.40	4.42	4.37	4.26	4.32
1924	3.83	3.69	3.53	4.02	3.79	4.24	4.24	3.92	4.01	4.10	4.02	3.84	3.78
1925	3.51	3.35	3.39	3.58	3.47	3.83	3.63	3.69	3.59	3.65	3.56	3.50	3.39
1926	3.76	3.73	3.70	3.82	3.79	3.84	3.80	3.82	3.78	3.79	3.73	3.70	3.72

Table XIII.—Federal Reserve Bank Rates, being Average Rates Earned on Bills Bought in the Open Market (from Annual Reports of the Federal Reserve Board).

	All Banks	Boston	New York	Philadelphia	Cleveland	Richmond	Atlanta	Chicago	St. Louis	Minneapolis	Kansas City	Dallas	San Francisco
1916	2.36	2.22	2.38	2.31	2.29	3.09	2.71	2.28	2.34	2.32	2.39	2.36	2.39
1917	3.26	3.25	3.33	3.18	3.24	3.22	3.34	3.11	3.16	3.16	3.20	3.40	3.25
1918	4.14	4.13	4.07	4.12	4.20	4.27	4.20	4.24	4.18	4.36	4.26	4.10	4.20
1919	4.30	4.25	4.25	4.24	4.27	4.57	4.57	4.33	4.36	4.27	4.36	4.79	4.29
1920	5.66	5.81	5.69	5.85	5.67	5.74	5.55	5.64	5.30	5.26	5.45	5.49	5.61
1921	5.70	5.61	5.59	5.61	5.90	6.03	6.28	5.67	5.63	-	6.40	6.10	5.70
1922	3.54	3.52	3.54	3.50	3.48	4.83	4.15	3.50	3.36	-	4.96	3.67	3.47
1923	4.14	4.13	4.11	4.13	4.15	4.27	4.19	4.15	4.07	4.13	4.26	4.13	4.14
1924	3.31	3.18	3.01	3.58	3.52	4.18	3.77	3.48	2.91	3.73	3.06	3.66	3.26
1925	3.17	3.19	3.08	3.17	3.12	3.23	3.33	3.16	3.16	3.24	3.20	3.18	3.12
1926	3.55	3.54	3.53	3.56	3.56	3.55	3.55	3.54	3.52	3.55	3.55	3.55	3.55

Table XIV.—Federal Reserve Bank Rates, being Average Rates Earned on Discounted Bills (from Annual Reports of the Federal Reserve Boar

	All Banks	Boston	New York	Philadelphia	Cleve-land	Rich-mond	Atlanta	Chicago	St. Louis	Minne-apolis	Kansas City	Dallas	San Francis
1916	4.20	3.52	3.69	3.75	4.25	4.00	3.77	4.60	4.04	4.80	4.77	4.49	5.01
1917	3.61	3.66	3.12	3.73	3.94	3.91	3.99	3.85	3.85	4.22	4.14	4.48	4.53
1918	4.24	4.01	4.04	4.29	4.28	4.40	4.27	4.30	4.29	4.63	4.74	4.67	4.65
1919	4.23	4.21	4.12	4.13	4.22	4.34	4.25	4.26	4.25	4.33	4.68	4.64	4.51
1920	5.88	5.91	5.88	5.48	5.88	5.69	5.77	6.17	5.83	6.22	6.05	5.67	5.79
1921	6.07	6.03	6.13	5.49	5.85	5.92	6.12	6.40	5.97	6.48	6.06	6.25	5.82
1922	4.63	4.24	4.25	4.50	4.64	4.78	4.76	4.76	4.71	5.12	4.95	5.08	4.42
1923	4.46	4.41	4.41	4.50	4.50	4.50	4.50	4.50	4.50	4.52	4.50	4.50	4.42
1924	4.25	4.18	3.97	4.12	4.17	4.32	4.36	4.37	4.39	4.49	4.46	4.40	4.33
1925	3.67	3.58	3.43	3.56	3.58	4.00	4.00	4.00	4.00	4.00	4.00	4.00	3.57
1926	3.95	4.00	3.82	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00

Table XV.—Federal Reserve Bank Rates, being Average Rates Earned on U. S. Securities (from Annual Reports of the Federal Reserve Board).

	All Banks	Boston	New York	Philadelphia	Cleveland	Rich-mond	Atlanta	Chicago	St. Louis	Minne-apolis	Kansas City	Dallas	San Francisco
1916	2.35												
1917	2.68												
1918	2.99												
1919	2.26	2.03	2.55	2.13	2.21	2.01	2.03	2.10	2.10	2.46	2.26	2.26	2.27
1920	2.21	2.20	2.41	2.19	2.20	2.01	2.01	2.12	2.11	2.02	2.22	2.11	2.12
1921	2.37	2.16	2.70	2.22	2.21	2.04	2.79	2.23	2.17	2.06	2.16	2.17	2.16
1922	3.67	3.67	3.63	3.83	3.84	2.06	2.54	3.68	3.70	3.43	3.68	2.83	4.00
1923	4.01	3.93	3.94	4.25	3.96	2.36	3.71	3.68	4.21	4.25	4.16	3.72	4.19
1924	3.67	3.71	3.50	4.09	3.64	3.47	3.69	3.72	3.54	3.91	3.87	3.67	3.60
1925	3.56	3.31	3.48	3.95	3.52	3.18	3.47	3.70	3.47	3.84	3.63	3.58	3.41
1926	3.60	3.43	3.51	3.58	3.60	3.34	3.62	3.73	3.58	3.85	3.67	3.61	3.53

Table XVI.—Geographic Variation of Interest Rates* (Shown by Percentage of Gross Earnings to Total Investments of National Banks from Reports of the U. S. A. Comptroller of the Currency).

Locality	1914	1915	1916
New England States	5.65	5.52	5.47
Eastern States	5.49	5.35	5.35
Southern States	7.17	7.13	7.06
Mid-western States	6.16	6.06	5.78
Western States	8.50	8.37	8.08
Pacific States	6.79	6.71	6.67
Hawaii	6.36	6.88	6.33
Total U. S. A.	6.17	6.05	5.92

* Average of losses for these three years was 0.139 % of the total assets, and cost of bank operation would be less than 0.4 %, say 0.5 % for total deduction from gross income to determine net income.

Table XVII.—Rate of Capitalization on the Basis of Total Earnings for the Year, Divided by the Total Quotational Values for the Month Succeeding the Income Period of Securities Outstanding at the End of the Year, Including Outstanding Bonds at Par Value.

COMPANY	ADDRESS	CAPITALIZATION RATES FOR THE FOLLOWING YEARS & PERIODS									
Machinery Manufacture		1908	1909	1910	1911	1912	1908 to 1912	1913	1914	1915	1916	1913 to 1916
American Car and Foundry Co.	New York, N.Y.	17.17	8.72	7.86	8.21	8.33	8.44	6.84	7.48	4.58	5.64	6.16
American Locomotive Co.	New York, N.Y.	10.91	∮	5.79	9.01	4.77	7.58	15.29	6.31	∮	24.08	15.73
Baldwin Locomotive Works	Philadelphia, Pa.	●	●	●	●	10.76	10.76	11.54	∮	6.19	13.63	10.05
General Electric Company	Schenectady, N.Y.	8.50	6.71*	10.20	8.76	8.30	7.93	9.46	7.66	4.56	10.48	9.28
International Harvester Co. Corp.	Chicago, Ill.	9.00	10.34	10.27	10.34	10.86	10.28	9.68	6.39	5.90	7.07	7.08
International Steam Pump Co.	New York, N.Y.	7.72	8.14	6.14	6.34*	∮	6.24	11.00	●	●	●	11.00
The New York Airbrake Co.	New York, N.Y.	3.07	3.88	6.18	5.66	6.91	4.57	8.78	9.33	8.64	47.03	21.52
Pressed Steel Car Co.	Pittsburgh, Pa.	∮	9.80	9.71	5.30	5.78	7.64	12.84	5.30	6.53	12.08	9.31
Underwood Typewriter Co.	Jersey City, N.J.	●	●	11.19*	9.00	12.44	10.84	13.23	7.31	10.90	17.86	12.71
The Westinghouse Electric & Mfg. Co.	Pittsburgh, Pa.	●	∮	6.28	7.10	4.09	5.45	5.47	5.13	5.28	12.24	7.41
Westinghouse Airbrake Co.	Pittsburgh, Pa.	7.93	8.76	11.26	6.73	9.12	8.20	5.05	4.37	2.85	15.94	6.69
Average rate		8.68	7.83	8.68	8.46	8.01	8.33	8.54	6.94	6.70	13.13	8.92
Metal Working												
American Brake Shoe & Foundry Co.	Mahwah, N. J.	●	15.08	11.89	13.60	9.94	11.76	11.58	9.14	7.76	10.86	9.76
The American Brass Co.	Waterbury, Conn.	8.06	9.01	9.81	7.89	11.23	8.64	9.66	7.14	15.68	21.46	15.71
American Can Co.	New York, N.Y.	∮	∮	∮	6.28	10.80	9.16	7.21	7.86	6.68	11.08	8.22
American Radiator Co.	Chicago, Ill.	6.47	6.02	5.94	5.48	4.67	5.50	5.78	6.45	6.46	∮	6.23
American Smelting & Refining Co.	New York, N.Y.	8.14	7.31	7.76	6.91	8.47	7.78	7.56	7.45	8.97	14.12	9.74
American Steel Foundries	St. Louis, Mo.	●	●	11.40	∮	11.59	11.48	15.40	4.04	3.44	29.47	13.75
National Enameling and Stamping Co.	Milwaukee, Wisc.	∮	7.49	7.74	7.16	8.41*	6.93	9.04	7.31	8.04	17.44	10.83
National Lead Co.	New York, N.Y.	6.88	6.80	6.85	6.63	6.79	6.79	6.86	5.95	6.42	7.46	6.91
Average rate		7.33	7.48	7.99	6.96	8.81	7.75	7.79	7.30	8.38	14.37	9.59
Iron and Steel Making												
Bethlehem Steel Corp.	New York, N. Y.	∮	∮	8.08	9.29	7.65	8.29	13.82	12.75	16.29	29.99	20.99
Crucible Steel Co. of America	Pittsburgh, Pa.	∮	7.32	16.62	10.82	12.26	11.49	19.03	∮	7.51	27.05	17.78
Lackawanna Steel Co.	West Seneca, N.Y.	∮	6.03	9.10	4.70	6.42	6.69	9.41	∮	6.66	22.80	12.94
Republic Iron and Steel Co.	Youngstown, Ohio	8.43	8.16	8.06	6.81	∮	6.93	9.01	∮	7.35	24.28	14.27
United States Steel Corp.	New York, N.Y.	6.58	7.91	6.96	6.99	6.95	7.61	10.90	6.69	9.63	21.38	12.69
Average rate		6.52	7.77	9.01	7.01	7.06	7.53	11.03	6.85	9.84	22.36	13.24
Chemicals												
The American Agricultural Chemical Co.	New York, N.Y.	12.14	8.04	9.91	8.08	6.35	8.42	7.06	7.11	8.63	10.98	8.57
General Chemical Co.	Philipstown, N.Y.	6.43	9.60	10.17	8.95	7.64	8.56	7.04	7.04	9.63	22.38	12.41
Virginia-Carolina Chemical Co.	Richmond, Va.	9.88	8.24	9.96	8.76	8.88	7.66	6.24	8.89	9.00	9.86	8.60
Average rate		9.65	8.42	9.99	7.24	6.33	8.11	6.77	7.68	9.08	14.22	9.74
Foods, etc.												
American Beet Sugar Co.	New York, N.Y.	12.02	11.46	13.41	11.84	10.63	11.70	9.75	5.31	12.52	16.39	12.16
The American Cotton Oil Co.	New York, N.Y.	6.98	9.66	7.87	∮	7.66	8.13	6.24	5.46	8.26	7.96	7.04
The American Sugar Refining Co.	Jersey City, N.J.	5.56	4.49	4.74	10.74	6.78	6.44	3.62	5.33	5.84	11.27	6.56
The American Tobacco Co.	Jersey City, N.J.	8.93	8.65	12.92	12.14	9.10	10.45	9.30	7.92	8.11	8.46	8.47
National Biscuit Co.	New York, N.Y.	5.52	6.60	6.80	6.30	7.12	6.64	7.46	5.81	6.12	6.97	6.84
Swift and Co.	Chicago, Ill.	14.04	12.87	11.41	8.29	11.82	11.51	12.97	11.51	13.34	14.98	13.47
Average rate		8.47	8.30	10.52	10.86	12.20	9.33	8.27	7.69	8.71	10.94	8.97
Miscellaneous												
American Ice Securities Co.	New York, N.Y.	8.31	11.16	6.73	7.31	5.37	7.74	13.05	4.04	5.07	6.20	6.68
American Snuff Co.	New York, N.Y.	9.40	9.92	10.02	15.39	8.14	10.44	8.56	8.38	8.22	9.92	8.75
American Woolen Co.	Boston, Mass.	∮	8.23	7.43	7.86	9.02	8.09	∮	∮	8.56	12.29	10.43
Central Leather Co.	Jersey City, N.J.	6.65	7.35	∮	∮	9.48	7.52	7.91	7.97	9.07	16.12	10.64
Harbison Walker Refractories Co.	Pittsburgh, Pa.	8.93	8.58	11.62	7.66	6.85	8.60	8.75	6.15	7.43	11.14	8.77
International Salt Co.	New York, N.Y.	●	●	●	10.33	10.15	10.23	7.51	8.89	8.83	21.16	13.15
United States Industrial Alcohol Co.	New York, N.Y.	9.26	8.08	9.17	8.73	7.88	8.59	9.25	6.87	7.50	6.81	8.14
United States Realty and Improvement Co.	New York, N.Y.	7.92	8.01	8.94	6.68	7.50	7.78	9.43	7.37	7.37	8.97	6.91
United States Rubber Co.	New York, N.Y.							7.29	6.91	5.81	20.91	10.49
x Pittsburgh Plate Glass Co.	Pittsburgh, Pa.							7.31	4.03	8.76	13.76	8.59
x United Fruit Co.	Boston, Mass.							8.56	6.72	8.16	12.69	8.67
Average rate		7.56	8.27	8.93	8.56	8.28	8.32	8.56	6.72	8.16	12.69	8.67
Merchandising												
The May Department Stores Co.	St. Louis, Mo.	●	●	6.31	10.10	11.51	10.19	10.25	9.18	10.94	17.94	12.16
Sears Roebuck and Co.	Chicago, Ill.	11.01	10.65	10.13	10.43	8.64	9.98	10.52	10.60	9.24	11.04	10.38
Average rate		11.01	10.65	9.44	10.14	9.29	10.01	10.57	10.38	9.44	11.75	10.62
Grand Average rate		7.52	7.98	9.21	8.06	8.22	8.13	9.48	7.14	8.62	16.99	10.85

x—These two companies were not included for 1908 to 1912, i. e.
*—Adjusted for periods other than 12 months.
∮ Deficit
○ Accurate information not available.

Table XVIII.—Rates of Capitalization on the Basis of Common Stock Earnings for the Year, Divided by the Total Quotational Value for the Month Succeeding the Income Period of the Common Stock Outstanding at the End of the Year.

COMPANY	ADDRESS	CAPITALIZATION RATES FOR THE FOLLOWING YEARS AND PERIODS										
		1908	1909	1910	1911	1912	1908 to 1912	1913	1914	1915	1916	1913 to 1916
Machinery Manufacture												
x International Harvester Corporation	Chicago, Ill.							12.12	7.40	5.70	3.85	6.66
American Car and Foundry Co.	New York, N. Y.	43.06	4.98	11.24	13.06	6.18	13.15	9.41	11.08	1.47	4.24	6.18
American Locomotive Co.	New York, N. Y.	21.46	ϕ	3.85	17.71	1.10	11.85	58.77	4.99	ϕ	57.09	4.61
Baldwin Locomotive Works	Philadelphia, Pa.	●	●	●	●	26.84	26.84	31.16	ϕ	6.42	32.75	18.80
General Electric Company	Schenectady, N. Y.	5.60	6.99*	11.01	8.65	8.64	8.22	8.94	7.80	4.67	10.83	8.56
International Harvester Co.	Chicago, Ill.	13.56	15.39	14.00	13.99	14.87	14.36	14.80	15.03	17.86	20.63	17.18
International Steam Pump Co.	New York, N. Y.	8.13	2.47	7.32	7.32*	ϕ	5.92	30.94	-	-	-	30.94
The New York Airbrake Co.	New York, N. Y.	2.76	3.14	6.26	6.06	7.26	4.14	19.07	11.06	9.18	58.32	25.97
Pressed Steel Car Co.	Pittsburgh, Pa.	ϕ	16.36	17.17	0.48	2.84	9.65	29.43	0.41	6.08	19.38	14.21
Underwood Typewriter Co.	Jersey City, N. J.	●	●	16.20*	10.75	16.71	14.28	18.02	7.64	13.95	28.50	18.95
The Westinghouse Electric & Mfg. Co.	Pittsburgh, Pa.	●	ϕ	8.25	8.73	8.36	8.66	6.18	7.34	8.44	14.89	9.21
Westinghouse Airbrake Co.	Pittsburgh, Pa.	7.93	5.76	11.26	8.73	9.12	9.20	8.08	4.37	2.66	16.94	6.69
Average Rates		10.84	9.12	10.82	10.08	9.42	9.94	10.10	7.75	7.32	17.55	10.73
Metal Working												
American Brake Shoe & Foundry Co.	Mahwah, N. J.	●	35.38	25.44	22.26	20.87	24.74	21.96	14.90	15.68	24.87	19.35
The American Brass Co.	Waterbury, Conn.	5.06	9.01	9.61	7.89	11.23	8.54	9.68	7.14	15.88	21.44	15.71
American Can Company	New York, N. Y.	ϕ	ϕ	ϕ	0.60	27.71	20.88	8.90	12.89	8.25	25.67	13.90
American Radiator Co.	Chicago, Ill.	6.84	6.29	6.08	5.81	4.60	5.84	5.84	6.88	6.42	ϕ	6.35
American Smelting & Refining Co.	New York, N. Y.	9.50	8.30	9.17	7.85	15.00	9.72	11.17	10.06	14.94	29.55	18.25
American Steel Foundries	St. Louis, Mo.	●	●	14.96	ϕ	14.90	14.93	25.73	2.92	2.69	37.84	18.67
National Enameling and Stamping Co.	Milwaukee, Wisc.	ϕ	7.10	11.09	9.54	1.15*	7.21	15.71	4.93	11.08	43.11	22.85
National Lead Co.	New York, N. Y.	7.37	7.35	7.71	6.77	7.89	7.31	7.59	6.11	4.97	10.24	6.82
Average Rates		7.79	8.47	9.86	7.28	12.44	9.32	10.01	9.13	11.32	25.80	14.53
Iron and Steel Making												
Bethlehem Steel Corp.	New York, N. Y.	ϕ	ϕ	16.25	25.92	14.53	19.38	87.11	64.79	24.78	62.92	44.40
Crucible Steel Co. of America	Pittsburgh, Pa.	ϕ	5.56	60.62	25.77	41.30	31.09	84.01	ϕ	6.48	56.63	36.40
Lackawanna Steel Co.	West Seneca, N. Y.	ϕ	7.67	21.90	3.26	10.03	10.99	23.82	ϕ	8.51	40.90	24.55
Republic Iron and Steel Co.	Youngstown, Ohio	10.16	2.15	15.44	8.21	ϕ	8.59	21.27	ϕ	12.55	60.27	38.41
United States Steel Corp.	New York, N. Y.	10.19	13.99	18.26	11.49	11.29	13.56	27.96	10.25	12.69	50.42	31.28
Average Rates		10.18	13.47	19.61	11.49	11.66	13.40	29.20	11.92	19.11	51.70	32.21
Chemicals												
The American Agricultural Chemical Co.	New York, N. Y.	33.79	16.82	27.15	15.43	12.05	16.65	11.14	14.59	21.04	30.28	20.25
General Chemical Co.	Phillipstown, N. Y.	7.22	16.55	17.81	13.27	9.38	12.48	8.24	8.81	11.98	31.56	17.04
Virginia-Carolina Chemical Co.	Richmond, Va.	17.11	14.07	18.14	5.42	5.55	11.89	3.37	17.62	18.34	21.51	14.13
Average Rates		16.73	15.13	19.63	10.61	8.79	13.65	7.96	12.45	14.80	29.23	17.18
Foods, &c.												
American Beet Sugar Co.	New York, N. Y.	23.56	19.75	19.09	15.49	12.86	16.60	11.79	1.61	15.85	20.97	15.31
The American Cotton Oil Co.	New York, N. Y.	9.29	13.97	10.94	ϕ	11.86	11.83	7.77	8.41	13.54	12.70	10.37
The American Sugar Refining Co.	Jersey City, N. J.	5.71	3.23	3.34	15.53	7.52	7.03	0.84	4.45	8.74	17.11	7.12
The American Tobacco Co.	Jersey City, N. J.	13.36	11.84	20.84	23.98	10.66	15.96	11.42	9.25	9.82	10.42	10.26
National Biscuit Co.	New York, N. Y.	7.24	7.25	7.99	6.89	8.27	7.52	8.94	7.76	6.72	8.19	7.90
Swift and Co.	Chicago, Ill.	14.93	13.47	11.93	8.58	12.24	12.00	13.48	12.59	15.86	17.65	15.22
Average Rates		11.44	10.58	14.70	13.61	10.38	12.61	9.71	8.96	10.92	14.40	11.12
Miscellaneous												
American Ice Securities Co.	New York, N. Y.	10.87	21.04	8.94	12.06	8.15	11.80	29.42	1.58	3.76	7.04	8.92
American Snuff Co.	New York, N. Y.	10.88	11.37	11.58	23.65	8.87	12.82	9.15	9.06	8.93	11.03	9.51
American Woolen Co.	Boston, Mass.	ϕ	15.14	7.07	7.67	11.64	10.65	ϕ	ϕ	13.19	33.48	23.02
Central Leather Co.	Jersey City, N. J.	4.27	14.27	ϕ	ϕ	30.64	12.82	18.83	17.78	20.11	36.70	26.74
Harbison Walker Refractories Co.	Pittsburgh, Pa.	13.37	14.57	21.21	9.76	8.01	12.52	11.98	6.24	8.62	13.73	10.98
United States Industrial Alcohol Co.	New York, N. Y.	●	●	●	17.19	14.26	15.46	5.69	8.29	9.89	30.27	17.69
United States Realty and Improvement Co.	New York, N. Y.	14.68	11.15	12.96	12.83	10.61	12.39	14.01	13.61	12.85	9.29	12.65
United States Rubber Co.	New York, N. Y.	1.13	12.92	24.83	6.27	10.69	12.00	21.11	14.39	17.69	24.94	19.70
x Pittsburgh Plate Glass Co.	Pittsburgh, Pa.							7.25	6.80	5.73	20.95	10.48
x United Fruit Co.	Boston, Mass.							8.65	3.88	10.67	15.14	10.01
Average Rates		9.47	13.12	13.74	13.51	12.59	12.61	11.86	8.11	11.37	20.90	13.15
Merchandising												
The May Department Stores Co.	St. Louis, Mo.	●	●	8.08	12.86*	16.45	13.08	13.41	11.01	15.08	27.85	17.02
Sears Roebuck and Co.	Chicago, Ill.	13.42	11.68	10.23	11.29	9.19	10.74	11.24	11.22	9.57	11.44	10.86
Average Rates		13.49	11.68	10.52	11.34	10.01	11.00	11.81	11.20	9.94	12.44	11.34
Grand Average Rates		10.54	11.49	14.48	11.72	10.69	11.96	14.92	9.82	12.90	29.83	17.79

x These two companies were not included for 1908 to 1912, inclusive.

* Adjusted on a 12 months basis.

ϕ Deficit.

● Accurate information not available

Table XIX.—Rates of Capitalization on the Basis of Common Stock Dividends for the Year, Divided by the Total Quotational Value for the Month Succeeding the Income Period of the Common Stock Outstanding at the End of the Year.

COMPANY	ADDRESS	CAPITALIZATION RATES FOR THE FOLLOWING YEARS AND PERIODS										
		1908	1909	1910	1911	1912	1908 to 1912	1913	1914	1915	1916	1913 to 1916
Machinery Manufacture												
x International Harvester Corp.	Chicago, Ill.							4.70	3.42	0.00	0.00	2.20
American Car and Foundry Co.	New York, N. Y.	9.64	3.73	3.39	3.67	3.39	4.38	4.11	4.01	3.62	3.57	3.87
American Locomotive Co.	New York, N. Y.	5.66	ϕ	0.00	0.00	0.00	2.94	0.00	0.00	0.00	0.00	0.00
Baldwin Locomotive Works	Philadelphia, Pa.	●	●	●	●	3.52	3.52	4.76	ϕ	0.00	0.00	0.94
General Electric Company	Schenectady, N. Y.	5.19	5.61 *	5.27	4.75	4.27	4.95	5.58	5.63	4.62	4.73	5.10
International Harvester Co.	Chicago, Ill.	0.00	0.00	3.55	4.64	4.50	2.98	4.66	5.22	4.08	4.12	4.60
International Steam Pump Co.	New York, N. Y.	0.00	0.00	0.00	0.00*	ϕ	0.00	0.00	0.00	0.00	0.00	0.00
The New York Airbrake Co.	New York, N. Y.	0.00	0.00	4.19	8.07	1.90	2.36	9.22	10.34	4.10	7.74	7.05
Pressed Steel Car Co.	Pittsburgh, Pa.	ϕ	0.00	0.00	0.00	0.00	0.00	5.36	0.00	0.00	5.87	3.51
Underwood Typewriter Co.	Jersey City, N. J.	●	●	0.00*	2.96	4.11	2.67	4.62	7.26	4.65	4.33	4.98
The Westinghouse Electric & Mfg. Co.	Pittsburgh, Pa.	●	ϕ	0.00	0.00	0.00	0.00	2.26	2.73	4.89	4.01	3.32
Westinghouse Airbrake Co.	Pittsburgh, Pa.	6.05	4.12	7.33	6.44	6.82	6.22	2.87	3.94	5.68	5.66	4.21
Average Rates		4.54	3.12	3.71	3.96	3.71	3.78	4.23	4.58	4.34	4.43	4.28
Metal Working												
American Brake Shoe & Foundry Co.	Mahwah, N. J.	●	6.58	6.91	7.91	7.16	7.21	7.40	7.69	6.73	6.45	7.08
The American Brass Co.	Waterbury, Conn.	4.61	5.02	4.82	4.70	5.19	4.86	5.30	4.43	4.99	7.32	5.87
American Can Co.	New York, N. Y.	ϕ	ϕ	ϕ	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
American Radiator Co.	Chicago, Ill.	2.97	3.30	3.49	3.08	2.02	2.79	2.24	2.89	4.03	ϕ	3.05
American Smelting & Refining Co.	New York, N. Y.	9.47	4.32	5.17	5.15	5.59	5.85	4.98	6.67	3.81	5.41	5.25
American Steel Foundries	St. Louis, Mo.	●	●	5.65	ϕ	0.00	3.16	6.32	7.08	0.00	2.05	2.95
National Enameling and Stamping Co.	Milwaukee, Wisc.	ϕ	0.00	0.00	0.00	0.00*	0.00	0.00	0.00	0.00	0.00	0.00
National Lead Co.	New York, N. Y.	6.36	5.90	7.14	5.66	5.74	6.16	6.25	6.58	4.30	6.67	5.81
Average Rates		6.99	4.53	5.02	4.38	3.62	4.76	4.20	4.48	3.39	5.11	4.25
Iron and Steel Making												
Bethlehem Steel Corp.	New York, N. Y.	ϕ	ϕ	0.00	0.00	0.00	0.00	0.00	0.00	0.00	6.59	3.02
Crucible Steel Co. of America	Pittsburgh, Pa.	ϕ	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Lackawanna Steel Co.	West Seneca, N. Y.	ϕ	0.00	0.00	0.00	0.00	0.00	2.70	0.00	0.00	10.55	4.93
Republic Iron and Steel Co.	Youngstown, Ohio	0.00	0.00	0.00	0.00	ϕ	0.00	0.00	0.00	0.00	1.89	0.97
United States Steel Corp.	New York, N. Y.	3.76	4.65	6.81	7.72	7.68	6.09	8.02	6.43	1.49	7.82	5.90
Average Rates		3.69	4.36	6.14	7.15	7.13	5.72	7.46	6.23	1.14	7.40	5.34
Chemicals												
The American Agricultural Chemical Co.	New York, N. Y.	0.00	0.00	0.00	0.00	3.17	0.89	8.51	7.65	7.67	5.89	7.29
General Chemical Co.	Philipstown, N. Y.	6.45	3.92	4.88	4.62	3.42	4.34	3.37	6.04	3.68	2.40	3.70
Virginia-Carolina Chemical Co.	Richmond, Va.	0.00	0.00	5.22	8.72	6.04	4.55	13.76	0.00	0.00	0.00	2.33
Average Rates		1.91	0.99	4.02	5.16	4.26	3.55	6.42	5.27	3.79	2.68	4.24
Foods, etc.												
American Beet Sugar Co.	New York, N. Y.	0.00	0.00	0.00	0.00	7.60	2.50	3.80	0.00	0.00	8.80	4.26
The American Cotton Oil Co.	New York, N. Y.	8.73	4.73	8.06	ϕ	0.00	5.73	0.00	0.00	0.00	7.26	2.14
The American Sugar Refining Co.	Jersey City, N. J.	5.36	5.78	6.10	5.95	6.05	5.84	6.58	6.50	6.17	6.30	6.38
The American Tobacco Co.	Jersey City, N. J.	9.42	8.21	9.66	7.45	10.68	9.08	14.21	10.71	9.79	9.18	11.09
National Biscuit Co.	New York, N. Y.	5.88	5.67	4.86	6.17	6.04	5.73	5.27	5.70	5.75	5.89	5.64
Swift and Co.	Chicago, Ill.	6.87	6.35	6.81	6.71	6.95	6.81	6.67	6.76	6.10	5.07	6.03
Average Rates		7.67	7.01	7.84	6.62	7.76	7.39	5.29	7.67	6.78	6.71	7.49
Miscellaneous												
American Ice Securities Co.	New York, N. Y.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
American Snuff Co.	New York, N. Y.	6.09	7.12	9.47	20.37	7.62	9.21	8.73	6.23	8.06	8.76	7.96
American Woolen Co.	Boston, Mass.	ϕ	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	10.93	5.30
Central Leather Co.	Jersey City, N. J.	0.00	0.00	ϕ	ϕ	0.00	0.00	7.87	8.31	7.43	9.14	8.30
Harbison Walker Refractories Co.	Pittsburgh, Pa.	0.00	0.00	4.76	4.44	4.21	3.37	4.54	4.09	3.36	5.83	4.77
United States Industrial Alcohol Co.	New York, N. Y.	●	●	●	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
United States Realty and Improvement Co.	New York, N. Y.	12.02	4.85	6.35	6.58	6.42	6.80	7.64	8.38	5.46	0.00	5.94
United States Rubber Co.	New York, N. Y.	0.00	0.0	0.00	0.00	5.18	1.56	9.77	10.79	2.79	0.00	4.26
x Pittsburgh Plate Glass Co.	Pittsburgh, Pa.							2.84	6.28	5.53	5.53	6.99
x United Fruit Co.								4.77	6.27	5.89	4.33	5.09
Average Rates		4.31	3.20	5.54	7.28	4.37	4.78	6.38	6.71	4.55	4.99	5.49
Merchandising												
The May Department Stores Co.	St. Louis, Mo.	●	●	0.00	0.00*	6.71	2.78	7.73	9.59	5.18	4.56	6.73
Sears Roebuck and Co.	Chicago, Ill.	0.00	2.85	3.74	4.64	3.38	3.41	3.72	3.69	3.90	3.02	3.41
Average Rates		0.00	2.85	3.20	3.88	3.76	3.34	4.18	4.24	3.61	3.11	3.67
Grand Average Rate		5.32	4.62	5.67	5.69	5.52	5.37	5.52	5.77	3.56	5.79	5.21

* – Adjusted on a 12 months basis.
ϕ Deficit
● Accurate information not available.
x These companies were not included for 1908 to 1912, inclusive.

Table XX.—Average Annual Yields of Bonds.

Year	I 10 Railway Bonds	II State and Municipal Bonds	III Moody's Municipal Bonds	IV Corporate Bonds	V Bonds of Mining and Metallurgical Companies	VI Weighted Average Yield of New Issues of Public Utility Bonds	U. S. Government Bonds without Special privileges tending to lower the rate of interest
1890	4.72						
1891	4.84						
1892	4.64						
1893	4.76						
1894	4.59						
1895	4.47						
1896	4.53						
1897	4.37						
1898	4.21						
1899	3.96						
1900	3.95	3.00					
1901	3.79	3.15					
1902	3.78	3.25					
1903	3.96	3.35					
1904	3.92	3.40		4.15			
1905	3.83	3.50		4.05			
1906	3.94	3.80		4.20			
1907	4.21	4.10		4.50			
1908	4.17	4.10		4.50			
1909	4.00	4.00		4.30			
1910	4.13	4.10		4.40			
1911	4.16	4.15		4.40			*2.91
1912	4.23	4.20		4.45			2.93
1913	4.44	4.30	4.29	4.65	5.62		2.96
1914	4.44	4.25	4.15	4.65	6.24		2.59
1915	4.56	4.25	4.19	4.80	5.75		2.96
1916	4.43	4.15	3.97	4.65	4.98		2.93
1917	4.78	4.30#	4.22#	5.05	5.43		3.57
1918	5.31	4.55	4.54	5.55	6.56		**4.50
1919	5.36	4.50	4.50	5.55	5.99	6.36	4.59
1920	5.90	4.70	5.04	6.30	6.80	7.49	5.54
1921	5.22	4.80	4.16	6.15	-	7.07	5.29
1922	4.61	4.15	4.31	5.15	-	6.01	4.28
1923	4.75	4.35	4.32	5.25	-	6.02	4.18
1924	4.69	4.25	-	5.15	-	5.98	4.10
1925	4.60	4.09	-	5.00	-	5.58	3.99
1926	4.47					5.53	3.87

\# Hereafter exemption from Federal Income and other taxes reduces yields of these bonds.

I Harvard Review of Economic Statistics.

II E. DeWolf of L. F. Rothschild and Co., 120 Broadway, New York City - (Chart)

III Moody's "Governments and Municipals".

IV Brookmire Economic Service. (Chart)

V Computed from quarterly quotations from Bradstreet, etc.

VI Herbert B. Dorau in "Journal of Land and Public Utility Economics", Oct. 1925.

* 3% Panama Canal Loan of 1961.

** Liberty and Later Treasury Bonds, except First Liberty 3½% Bonds.

APPENDIX B

The following descriptions apply to the computation and presentation of the valuation factors of all tables of this Appendix.

Interest is compounded annually, at the speculative rate of interest for compound interest premises and at the sinking fund rate for sinking fund premises of valuation. The speculative rate per cent of interest is indicated at the head of each column of valuation factors. The sinking fund rate of interest is stated in the heading of each table of sinking fund valuation factors.

The period of years to which a valuation factor applies will be found in a marginal column opposite the factor. The factor applies at the end of the year. Valuation factors applicable to equal annual amounts are computed on the basis of receipt of such amounts at the end of each year for the period.

Tables of factors for the compound interest premises apply to the three compound interest premises of Divisions I, II, and III, and to the Morkill Sinking Fund Premises of Division V.

Tables of factors for the Hoskold Sinking Fund Valuation Premises of Division IV also apply to valuation with the triple interest rate sinking fund premises of Division VI, when the composite rate of interest, j, of Division VI, equals the sinking fund rate, e, of Division IV.

A Years Purchase Valuation Factor is the factor by which the income for one year is multiplied to find the total value of equal yearly income for a period of years.

A Factor of Safety is the value determined for an amount of income by discounting at a speculative rate of interest, divided by the value for the same amount of income determined by discounting at a safe rate of interest.

Repetition of the preceding statements will be avoided in the titles for valuation tables.

Valuation tables serve the sole purpose of saving time for the evaluator. All mathematical formulas for commercial valuation include an exponential term or exponential terms representing interest compounded for a series of time increments, customarily years. Any valuation formula in this work, except those for valuing income decreasing by equal annual percentages, can be solved from the one table showing the amount of 1 in n years, n being the number of years for which interest is compounded. These factors which

appear in Table I are all represented by R^n, the symbol R equaling 1 plus r, and r being the decimal rate of interest. R^n means that R is multiplied by itself n times; for example, if the amount of 1 in 86 years is required, such amount may be found by multiplying the amount of 1 in 50 years by the amount of 1 in 36 years, the product being the amount of 1 in 86 years. The next step in the preparation of valuation tables is the determination of the value of 1 due n years hence. If 1 increases to 10 in n years, with interest compounded annually, it is apparent that the amount which will increase to 1 in the same time is $\frac{1}{10}$. Thus the value of 1 due n years hence is always the reciprocal of R^n, that is, one divided by R^n. If the value of 1 due 86 years hence is required, such amount may be determined by multiplying the value of 1 due 50 years hence by the value of 1 due 36 years hence. Table II gives values for 1 due n years hence.

The amount of 1 per year for n years at compound interest is represented by the mathematical formula, $\frac{R^n - 1}{r}$. This formula is the geometrical progression formula of which the numerator is the last term (R^n) minus the first term (1), and the denominator is R, the common difference between the terms, less 1, the amount $(R-1)$ equaling r, since R equals $1+r$. No table of valuation factors has been included for this formula, but such a table may be found in almost any book of valuation tables based on compound interest. As no portion of this amount would be received until the end of the nth year, it is apparent that each 1 of the amount at the beginning of the income period is worth $\frac{1}{R^n}$. Therefore, the value of 1 per year for n years is $\frac{R^n - 1}{rR^n}$, which is the compound interest annuity valuation formula. The amount per year required to equal 1 in n years is 1 divided by $\frac{R^n - 1}{r}$, or $\frac{r}{R^n - 1}$. This formula is the one from which the factors of Table III have been computed.

Since it is easier to solve a valuation formula once for a single computation of value than it is to solve it hundreds of times in the preparation of a table of valuation factors, such tables are prepared for use only with types of income frequently valued. The tables of this Appendix apply only to the commonest type—equal annual income—as it is believed that a demand for tables for the valuation of income increasing or decreasing by equal annual amounts or percentages will be relatively infrequent, and as a separate table would be required for each rate by which the income either increases or decreases. The only table of valuation factors prepared for the

valuation of unequal annual income is one for W^n, in which W is equal to $1 - w$ and w is the decimal rate of yearly decrease of income. When income is increasing by a ratio of W, the table for R^n is equivalent to a table for W^n, when $W = R$, and needed factors may be obtained from Table I. The infinite variety of unequal annual income makes it impossible to prepare tables for its valuation, other than the tables of factors for R^n and W^n previously described.

The formulas for the valuation of equal annual income all include the term A, which is the algebraic notation representing the yearly amount of income. The formulas, therefore, show the value of income over a period of time in terms of a multiple of A, the remainder of the term in which A appears being that multiple. When A is made equal to unity, the formula is obtained from which "years purchase factors" are computed, a years purchase factor being the amount by which a unit of yearly income is multiplied to determine the value for n units of income received at the rate of a unit at the end of each year of an n year period of investment. Tables IX to XIII, inclusive, are "years purchase factors" computed from valuation formulas for equal annual income in which A is made equal to one. The amount of income for one year is multiplied by a years purchase factors for n years to determine the present worth of n years income.

The "years purchase factors" give the value of n units of income. To determine the value of each unit of income the "years purchase factors" formulas are divided by n. Formulas so obtained are called total income purchase factors, since the value of the total income for n years may be determined by multiplying total income by these factors. Tables numbered from IV to VIII, inclusive, give "total income purchase factors" for equal annual income.

Tables XIV to XXV, inclusive, for factors of safety, are computed, as fully explained in Chapter VI, by dividing the value of equal annual income determined by means of a speculative rate of interest by the value of the same income determined by means of a safe rate of interest.

Table XXVI is for factors computed in a manner identical with that employed in computing factors of Table I, but W is a decimal less than unity, being unity minus w, whereas R is greater than unity, being unity plus r, both w and r being positive decimals. When W is greater than unity, that is, unity plus w, amounts of W^n are equal to amounts of R^n when w equals r, and Table I may be used in such cases for obtaining the amounts of W^n.

The preceding statements explain the methods by which all of the tables of this Appendix have been computed, and by which other similar tables may be computed, while the following arithmetical illustrations are for the purpose of suggesting uses for the tables in

the solution of problems frequently encountered in the practice of valuation. The compound interest deferment and compound interest sinking fund premises of valuation are so well known that their application need not be illustrated; therefore the type problems will be based upon the use of the compound interest actuarial method and its premises or the Hoskold sinking fund premises, which are the two methods recommended by the authors as being completely in accord with the commercial principles of investment. It is particularly desired that the use of compound interest factors in valuation by the actuarial premises should be emphasized in the following illustrations, and that the time saving possible through use of valuation tables may be indicated.

The valuation tables of this Appendix are computed to more decimal places than, ordinarily, should be required in commercial valuation, which at best can be but an approximation. If other elements of the computation, such as the yearly income, cannot be estimated to within 10%, it is an unnecessary refinement to use valuation factors which are accurate to the nearest 0.001 or 0.0001% which is the accuracy of factors in Tables I to VIII, inclusive. An absolute checking of return of capital to the nearest cent in illustrations demands at least as many places of decimals in the valuation factor as there are digits in the number of cents income per year, and even more decimal places are desirable. The lack of these decimal places is the cause of slight discrepancies in the proof of any valuation process by arithmetical means. While it is believed that all tables are accurate to one digit in the last decimal place, and numerous checks have been made which should reveal any inaccuracies, it will be appreciated if the reader will call attention to any errors, as the great number of computations involved suggests the possibility that some few errors have not been found.

Problem 1.

To find the amount of $1 in 135 years with interest at 4%, compounded annually. The factors of Table I extend only to 50 years; but, as noted in previous text, the amount of 1 in 135 years would be the product of the factors for 50 years, 50 years, and 35 years, the sum of these periods being 135 years. Therefore, the amount of $1 in 135 years at 4% interest, compounded annually, may be determined from Table I by multiplying the proper factors under the 4% column as follows:

	Years	Factor	Years	Factor
	50	7.106683	45	5.841176
	50	7.106683	45	5.841176
	35	3.946089	45	5.841176
Sum of years and product of above factors.....	135	199.297000	135	199.297052

Problem 2.

To find the present value of $1 due 60 years hence, if interest is to be paid at 4%, compounded annually. As in Problem 1 the table of factors extends only to 50 years, but the product of any factors under the 4% column of Table II for aggregate periods of 60 years will provide the factor desired, as illustrated below:

Years	Factor	Years	Factor	Years	Factor
43	.185168	50	.140713	30	.308319
17	.513373	10	.675564	30	.308319
60	.095060	60	.095061—	60	.095061—

Problem 3.

To find the value of $1 per year for 62 years on the basis of the compound interest actuarial premises of valuation with interest of 3% paid annually on diminishing capital.

Process:

The amount of $1 in 62 years with 3% interest compounded annually is the product of the R^n factors under the 3% interest column of Table I for the periods of 50 and 12 years. Hence $R^{62} = 4.383906 \times 1.425761 = 6.250402$.

The formula for the valuation is

$$V = \frac{R^n - 1}{rR^n}$$

in which,

$$n = 62$$
$$r = .03$$
$$R = 1.03$$
$$R^n = 6.250402$$

Solving the formula the value is determined to be $28.000343.

Problem 4.

To find the amount of the yearly payment to a sinking fund earning 3% interest compounded annually, in order to amortize an investment of $100,000 in 62 years.

The formula for determining the amount of the yearly payment to amortize $1 is $\frac{r}{R^n - 1}$, and the quantities for r and R^n in this formula are the same as in Problem 3, that is:

$$R^n = 6.250402$$
$$r = 0.03$$

Solving the formula when these amounts are substituted gives $0.00571385 per year as the amount required to amortize $1 in 62 years by means of a sinking fund earning 3% interest compounded annually. Multiplying this amount by 100,000 gives $571.385 as the amount of the yearly payment to a sinking fund required to amortize $100,000 under the same conditions.

A check on this result is readily obtained from the figures of Problem 3, since the value of an income of $1 a year for 62 years under the conditions stated was found to be $28.0003, and 3% interest on this amount is $0.84001029 per annum. Of the yearly income of $1 there thus remains $0.15998971 available for the amortization of the value of $28.000343, or $0.00571385 as the amount available for the amortization of each dollar of value.

The identity of the valuation formulas for the compound interest actuarial and the compound interest sinking fund premises permits this basis of comparison.

Problem 5.

Given:

Yearly income	$1,000
Period for which income is expected	10 years
Speculative rate of interest required	10%

Required:

The value under the above conditions with the compound interest actuarial premises; also the amounts of capital returned at the end of each year of the investment period.

Process:

Total income	$10,000
Total income valuation factor (from Table IV under 10% and opposite 10 years)	.61446
Value (product of above amounts)	$ 6,144.60

The capital return by years may be taken directly from Table II, the value of \$1 due n years hence, as demonstrated on page 57 on the comparison of valuation formulas. For an n year period of investment with the compound interest actuarial premises the value \$1 due n years hence, with interest compounded annually, is the amount of capital return at the end of the first year, the value of \$1 due $(n - 1)$ years hence is the capital return for the second year, and so forth, to the final amount of the capital return at the end of the nth year, which is the value of \$1 due one year hence. It is only necessary to reverse the order of the factors given under the 10% interest column of Table II opposite the years 1 to 10, inclusive, and to multiply each factor by the yearly income in order to ascertain return of capital under the compound interest actuarial premises of valuation. The capital return by years is then:

Year after date of investment	Capital return
1	\$ 385.543
2	424.098
3	466.507
4	513.158
5	564.474
6	620.921
7	683.013
8	751.315
9	826.446
10	909.091
Total	\$6,144.566

Problem 6.

Given:

Yearly income	\$1,000
Period for which income is expected	10 years
Speculative rate of interest required	10%
Safe rate of interest for 10 year investments	4%

Required:

The value under the above conditions with the Hoskold sinking fund valuation premises.

Process:

Total income	\$100,000
Total income valuation factor (from Table VI under 10% and opposite 10 years)	.545581
Value (product of above amounts)	\$ 5,455.81

In Problems 5 and 6 total income valuation factors were employed. The values might have been determined by the use of years purchase factors from Table IX for Problem 5 and from Table XI for Problem 6, but the years' purchase factors have not been carried to as many places of decimals as the total income purchase factors and were not used for that reason.

Problem 7.

Given:

Yearly income	$1,000
Speculative rate of interest required	10%
Price asked for property	$8,000

Required:

The number of years income required to return the price of the property, together with 10% interest under the compound interest actuarial valuation premises.

Process:

The price of $8,000 is divided by the yearly income (8,000 ÷ 1,000) and the quotient of 8 is the years purchase factor, that is, the value is 8 times one years income. The evaluator then turns to Table IX giving years purchase factors for all the compound interest premises of valuation and follows down the factors under the column for 10%, the required rate of interest, until he finds a factor of 8, such factor in this case representing an investment period slightly less than 17 years in duration. As a check, multiply the total income of $17,000 for 17 years by the valuation factor for a like investment period under the 10% interest column of Table IV, this factor being 0.47186 and the product of 0.47186 times $17,000 being $8,021.62, or slightly in excess of the price asked, as an income for 17 years is slightly in excess of the amount required to justify the investment under the required terms.

This type of problem is frequently encountered in the valuation of terminable income from patents, natural resources, and other similar types of investment. If 10 years income from a natural resource, such as a mine, can be secured from known ore reserves, the investor would be exhibiting his confidence that an additional 7 years supply of ore could be found by additional development, if he should invest on these terms.

Problem 8.

Given:

Yearly income	$1,000
Price asked	5,000
Period for which income is expected	10 years

Required:

The rate of interest which such an investment will pay under the compound interest actuarial premises of valuation.

Process:

An investment on these terms is a payment of five times one year's income. From Table IX, opposite 10 years, the years purchase factors of 5.019 and 4.833 are found under the columns of 15% and 16% interest, respectively. The difference between these factors, or 0.186, is the difference for 1% interest, hence 0.019 divided by 0.186 gives 0.102% as the approximate rate of interest in excess of 15% which the investment would pay in addition to providing for a return of capital. This result may be checked as follows:

	Investment outstanding at first of each year	Income per year	Interest on investment outstanding at first of year	Capital returned at end of year
	Col. I	Col. II	Col. III	Col. IV
Year	Initial value minus sum of amounts under Column IV for preceding years	From premises	15.102% of opposite amount under Column I	Col. II minus Col. III
1	$ 5,000.00	$ 1,000.00	$ 755.10	$ 244.90
2	4,755.10	1,000.00	718.12	281.88
3	4,473.22	1,000.00	675.55	324.45
4	4,148.77	1,000.00	626.55	373.45
5	3,775.32	1,000.00	570.15	429.85
6	3,345.47	1,000.00	505.23	494.77
7	2,850.70	1,000.00	430.51	569.49
8	2,281.21	1,000.00	344.51	655.49
9	1,625.72	1,000.00	245.52	754.48
10	871.24	1,000.00	131.57	868.43
Total.....	$33,126.75	$10,000.00	$5,002.81	$4,997.19

The above computation may be checked as follows:

$$\frac{5,002.81}{33,126.75} = 15.102\%, \text{ the rate of interest paid.}$$

$5,002.81 + 4,997.19 = 10,000$, the total income

This income of $1,000 a year for 10 years will then yield 15.102% interest on an initial investment of $5,000 and return all but $2.81

of the initial investment within the same period. The exact rate of interest which could be paid if the investment was exactly returned could be found by "cut and try" methods, but an approximation such as the preceding is accurate enough for any practical purpose.

Problem 9.

Given:

Yearly income	$1,000
Price asked	$5,000
Period for which income is expected	10 years
Safe sinking fund rate of interest	4%

Required:

The rate of speculative interest which will be paid by such an investment under the Hoskold sinking fund premises of valuation.

Process:

The amount per year which must be contributed from income to a sinking fund accumulating at 4% interest, compounded annually, in order to amortize $1 in ten years is found under the 4% interest column and opposite ten years on Table III. Since this amount is $0.083291, it will require a yearly contribution of $416.455 to a sinking fund in order to amortize $5,000 in ten years. Subtracting $416.455 from $1,000 gives $583.545, the amount of the yearly income available for the payment of interest. The amount of $583.545 divided by $5,000, the amount of the investment gives 11.6709% as the rate of interest required. Under the Hoskold sinking fund premises it is thus evident that no "cut and try" methods are necessary to the determination of the exact speculative rate of interest which an investment will pay, other factors of the valuation being known. Such a computation is impossible under any other premises of valuation.

Problem 10.

Given:

Income per year	$1,000
Period for which income will be received	10 years
Period elapsing before income receipt begins	2 years
Rate of interest required	10%

Required:

The present value of the income in accordance with the compound interest actuarial premises of valuation.

Process:

Obtain the deferment valuation factor for two years under the 10% column of Table II and the total income valuation factor for 10 years under the 10% column of Table IV. These factors are 0.826446 and 0.61446, respectively, and their product of 0.50782 is the valuation factor by which the total income of $10,000 is multiplied to obtain the required value, or $5,078.20 under the conditions stated.

Illustration of the process for proving this result arithmetically will be found in Chapter III, under solution and income allocation with respect to Premises II-E, where the annual income is $1,250 instead of $1,000. It must be noted that unpaid interest is compounded by adding such interest to outstanding capital at the end of each year during the period of deferment and that thereafter simple interest is paid annually on reducing balances of the investment.

Problem 11.

Given:

Price of property yielding income	$9,000
Income per year	1,000
Rate of interest required	10%
Sinking fund rate of interest	4%

Required:

The salable value which the property must possess at the end of 10 years in order that the investment may be justified, valuation being by means of the Hoskold sinking fund premises.

Process:

Annual income expected	$1,000	
Annual interest payment required (10% of $9,000)	$ 900	
Annual amount available for contribution to sinking fund	$ 100	
Amount of annual contribution to a sinking fund accumulating at 4% interest, which is required to amortize $1 in 10 years (amount from Table III opposite 10 years and under 4% column)	$ .083291	
Amount of investment	$9,000.00	
Portion of investment amortized in 10 years by contribution of $100 a year to the sinking fund ($100 ÷ .083291)	$1,200.61	
Unamortized balance of the investment, which is the required salable value of the property at the end of 10 years	$7,799.39	

The amount of $1,200.61 for ten years gives the amount of $120.06 per year and this sum divided by the amount of the investment, or $9,000, gives 1.334%, which represents the maximum possible yearly depreciation, if the investment is to meet with success.

Problem 12.

Given:

Yearly income	$1,000
Term of speculative investment	10 years
Yield of successful investments of the same type	6%
Estimated probability of success	75%

The compound interest actuarials premises of valuation are to be used.

Required:

The value of the income and the speculative rate of interest which the speculation must yield to fulfill the above conditions of paying 6% interest, together with a profit of one third of the amount invested, in the event that success attends the venture.

Process:

The valuation factor is found under the 6% interest column of Table IV, opposite 10 years, the factor of 0.73601 being obtained. The total income of $10,000 multiplied by this factor gives $7,360.10, as the value of the income which would be returned in 10 years together with 6% interest.

The factor of safety to be used is 0.75 (75%) and 0.75 × $7,360.10 = $5,520.08, which is the value sought. Turning to Table XXI of factors of safety for speculative rates of interest in comparison with a safe interest rate of 6% with the simple interest premises of valuation, and opposite 10 years there will be found the factors of safety of 0.768 under the 12% interest column and of 0.737 under the 13% interest column. As the factor of safety of 0.75 lies about midway between 0.768 and 0.737, the speculative investment should pay approximately 12.5% interest in order to meet the requirements. This rate may be determined more accurately by reference to the proper factor of safety chart appearing in Chapter VI.

Problem 13.

Given:

The same conditions as in Problem 12, except that a 2-year period of deferment elapses before the initial year of income receipt.

Required:

The value and the rate of interest, as in Problem 12.

Process:

The 6% safe interest value for the 10-year annuity period is further reduced by a deferment valuation factor of 0.889996, which will be found on Table II, opposite 2 years, under the 6% column.

Multiplying $7,360.10 by 0.889996 gives $6,550.46 as the present worth of the income at 6% interest, and multiplying this amount by the factor of safety of 0.75 gives $4,912.85 as the value sought.

The speculative rate of interest by which the income must be reduced to present worth to obtain that value may be approximated by finding the speculative rates of interest between which a factor of safety of 0.75 would obtain. It is necessary to multiply the factor of safety for deferment opposite 2 years on Table XVII by the factor of safety for the annuity period which will be found opposite 10 years on Table XXI to find the factor of safety which a speculative rate of interest represents for the combined deferment and annuity periods. The combined period factors of safety are 0.730 for an 11% rate of interest and 0.776 for a 10% rate of interest, hence the speculative rate of interest corresponding to a factor of safety of 0.75 is about halfway between these rates, or, approximately, the required rate of interest is 10.5%. A more accurate determination of this rate may be made if the factor of safety charts of Chapter VI are used, and as accurate a determination as desired may be made by means of "cut and try" methods.

Problem 14.

Given:

Income as follows:

First year	none
Second year	$ 1,000
Third year	$ 2,000
Fourth year	$ 3,000
Fifth and all later years	$ 4,000
Negotiated purchase price of investment	$57,000
Rate of interest required	6%

Required:

The salable value at the end of 50 years, if the investment is to be successful.

Process with the compound interest actuarial premises of valuation:

Deferment factors are obtained from Table II under the 6% interest column and opposite the respective years of deferment. The annuity valuation factor for 46 years is obtained from Table IV under the 6% interest column. The values of the respective amounts of income are determined separately by multiplying the total income for the period by the product of the valuation factors for the annuity and deferment periods, and the total value is the sum of the separate values so determined. The computation of value follows:

Years	Yearly income	Total income for period	6% deferment valuation factor	6% annuity valuation factor	Product of valuation factors	Value of income for period opposite
1	none	none	.....	.....	.....	none
2	$1,000.00	$ 1,000.00	.889996	1.00000	.889996	$ 890.00
3	2,000.00	2,000.00	.839619	1.00000	.839619	1,679.24
4	3,000.00	3,000.00	.792095	1.00000	.792095	2,376.28
5 to 50	4,000.00	184,000.00	.792095	0.33749	.267324	49,187.62
Total.....	.....	$190,000.00	.....	.....	.....	$54,133.14

Initial investment	$57,000.00
Present value of income	54,133.14
Cost unreturned and upon which no interest will be paid for 50 years	$ 2,866.86

In order that 6% interest may be paid on the amount of $2,866.86 for 50 years, it is necessary that the salable value of the property at the end of that time shall be great enough to pay both the unreturned portion of that cost at date of valuation plus the interest, which will not be paid by the anticipated income. From Table I the amount of $1 in 50 years with 6% interest, compounded annually for 50 years, is determined to be 18.420154, and $2,866.86 multiplied by this amount gives $52,808 as the required salable value of the property at the end of 50 years.

Problem 15.

Given:

Period of investment	50 years
Rate of interest required	6%
Value of income, as in Problem No. 14	$54,133.14

Required:

The price which can be paid if the salable value at the end of 50 years will be $20,000.

Process:

The amount of 1 due n years hence with 6% interest, compounded annually, is 18.420154 (as in Problem 14). Dividing $20,000 by 18.420154 gives $1,085.77 as the present value at date of investment of the $20,000 salvage value. The value of the income, $54,133.14 plus $1,085.77, gives $55,218.91 for the maximum price which can be paid under the conditions stated.

Problem 16.

Given:

Income as follows:

First year	none
Second year	$ 1,000
Third year	2,000
Fourth year	3,000
Fifth and all later years	4,000
Price asked for investment	50,000

Required:

The average rate of interest payable in perpetuity if interest is compounded on deficiencies of interest at the same rate during earlier years.

Process:

Discount the amounts of income to value at date of appraisal by means of any rate of interest, say 6%, as follows:

Year	Income	Valuation factor	First trial value
1	none		
2	1,000.00	.889996	$ 890.00
3	2,000.00	.839619	1,679.24
4	3,000.00	.792095	2,376.28
5	4,000.00	13.201584*	52,806.34
Total value	.....	.....	$57,751.86

* The years purchase factor for a perpetuity paying 6 % is $\frac{1.00}{0.06}$, or 16.666667, multiplying this amount by the deferment factor of 0.792095 gives 13.201584 as the 6% years purchase factor for perpetual income deferred four years. It should be noted that total income purchase factors cannot be determined for perpetuities.

It is evident that the income will pay more than 6% interest on the price asked for the investment and by a computation similar to the preceding we next determine that the value of the income at 7% would be $48,388.72. The rate of interest sought then lies between 6% and 7% and the approximate rate may be determined as follows:

Value at 6% interest	$57,751.86
Value at 7% interest	48,388.72
Difference	$ 9,363.14
Value at 6% interest	$57,751.86
Price asked	$50,000.00
Difference	$ 7,751.86

$\frac{7751.86}{9363.14}$ = 0.83 of 1% interest to be added to the 6% rate for further trial.

Year	Investment at first of year	Interest at 6.83%	Income for the year	Interest deficiency added to capital
1	$50,000.00	$3,415.00	none	$3,415.00
2	53.415.00	3,648.24	$1,000.00	2,648.24
3	56,063.24	3,829.12	2,000.00	1,829.12
4	57,892.36	3,954.05	3,000.00	954.05
5	58,846.41	4,019.21*	4,000.00*	19.21

* It is necessary that the amounts marked with stars shall be equal in order that the rate of interest may be paid in perpetuity, and it is obvious that the rate of 6.83% is slightly high. By further "cut and try" computations similar to the preceding, it is determined that the rate of interest sought is very slightly less than 6.8045%, as proved by the following computation.

Year	Investment at first of year	Interest at 6.8045%	Income for the year	Interest deficiency added to capital
1	$50,000.00	$3,402.25	none	$3,402.25
2	53,402.25	3,633.76	$1,000.00	2,633.76
3	56,036.01	3,812.97	2,000.00	1,812.97
4	57,848.98	3,936.33	3,000.00	936.33
5	58,785.31	4,000.05	4,000.00	0.05

At times it may be desirable to determine the rate of interest which a stated amount of income received in variable amounts over a period of years will pay on a fixed value, leaving that value unchanged at the termination of the investment period. Such a computation would be identical in procedure with that just preceding except as to object, which would be to have the investments equal at the beginning and at the end of the period.

Table I.—Amount of 1.00 with Interest for a Given Time.

Year	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%	Year
1	1.02	1.03	1.04	1.05	1.06	1.07	1.08	1.09	1.10	1.11	1.12	1.13	1.14	1.15	1.16	1.17	1.18	1.19	1.20	1
2	1.0404	1.0609	1.0816	1.1025	1.1236	1.1449	1.1664	1.1881	1.21	1.2321	1.2544	1.2769	1.2996	1.3225	1.3456	1.3689	1.3924	1.4161	1.440	2
3	1.061208	1.092727	1.124864	1.157625	1.191016	1.225043	1.259712	1.295029	1.331	1.367631	1.404928	1.442897	1.481544	1.520875	1.56090	1.60161	1.64303	1.68516	1.7280	3
4	1.082432	1.125509	1.169859	1.215506	1.262477	1.310796	1.360489	1.411582	1.4641	1.518070	1.573519	1.630474	1.688960	1.749006	1.81064	1.87388	1.93878	2.00534	2.07360	4
5	1.104081	1.159274	1.216653	1.276282	1.338226	1.402552	1.469328	1.538624	1.61051	1.685058	1.762342	1.842435	1.925415	2.011357	2.10034	2.19244	2.28776	2.38635	2.48832	5
6	1.126162	1.194052	1.265319	1.340096	1.418519	1.500730	1.586874	1.677100	1.771561	1.870415	1.973823	2.081952	2.194973	2.313061	2.43640	2.56516	2.69955	2.83976	2.98598	6
7	1.148686	1.229874	1.315932	1.407100	1.503631	1.605781	1.713824	1.828039	1.948717	2.076160	2.210681	2.352605	2.502269	2.660020	2.82621	3.00124	3.18547	3.37932	3.58319	7
8	1.171659	1.266770	1.368569	1.477455	1.593848	1.718186	1.850930	1.992563	2.143589	2.304538	2.475963	2.658444	2.852586	3.059023	3.27841	3.51145	3.75886	4.02139	4.29982	8
9	1.195093	1.304773	1.423312	1.551328	1.689479	1.838459	1.999005	2.171893	2.357948	2.558037	2.773079	3.004042	3.251949	3.517876	3.80296	4.10840	4.43545	4.78545	5.15978	9
10	1.218994	1.343916	1.480244	1.628895	1.790848	1.967151	2.158925	2.367364	2.593742	2.839421	3.105848	3.394567	3.707221	4.045558	4.41144	4.80682	5.23384	5.69468	6.19174	10
11	1.243374	1.384234	1.539454	1.710339	1.898299	2.104852	2.331639	2.580426	2.853117	3.151757	3.478550	3.835861	4.226232	4.652391	5.11726	5.62399	6.17593	6.77667	7.43008	11
12	1.268242	1.425761	1.601032	1.795856	2.012196	2.252192	2.518170	2.812665	3.138428	3.498450	3.895976	4.334523	4.817905	5.350250	5.93603	6.58007	7.28759	8.06424	8.91610	12
13	1.293607	1.468534	1.665074	1.885649	2.132928	2.409845	2.719624	3.065805	3.452271	3.883280	4.363493	4.898011	5.492411	6.152788	6.88579	7.69868	8.59936	9.59645	10.69932	13
14	1.319479	1.512590	1.731676	1.979932	2.260904	2.578534	2.937194	3.341727	3.797498	4.310441	4.887112	5.534753	6.261349	7.075706	7.98752	9.00745	10.14724	11.41977	12.83918	14
15	1.345868	1.557967	1.800944	2.078928	2.396558	2.759032	3.172169	3.642482	4.177248	4.784589	5.473566	6.254270	7.137938	8.137062	9.26552	10.53872	11.97375	13.58953	15.40702	15
16	1.372786	1.604706	1.872981	2.182875	2.540352	2.952164	3.425943	3.970306	4.594973	5.310894	6.130394	7.067326	8.137249	9.357621	10.74800	12.33030	14.12902	16.17154	18.48843	16
17	1.400241	1.652848	1.947900	2.292018	2.692773	3.158815	3.700018	4.327633	5.054470	5.895093	6.866041	7.986078	9.276464	10.761264	12.46768	14.42646	16.67225	19.24613	22.18611	17
18	1.428246	1.702433	2.025817	2.406619	2.854339	3.379932	3.996019	4.717120	5.559917	6.543553	7.689966	9.024268	10.575169	12.375454	14.46251	16.87895	19.67326	22.90052	26.62333	18
19	1.456811	1.753506	2.106849	2.526950	3.025600	3.616528	4.315701	5.141661	6.115909	7.263344	8.612762	10.197422	12.055693	14.231772	16.77652	19.74838	23.21444	27.25162	31.94800	19
20	1.485947	1.806111	2.191123	2.653298	3.207135	3.869684	4.660957	5.604411	6.727500	8.062312	9.646293	11.523088	13.743490	16.366537	19.46076	23.10560	27.39303	32.42942	38.33760	20
21	1.515666	1.860295	2.278768	2.785963	3.399564	4.140562	5.033834	6.108808	7.400250	8.949166	10.803848	13.021089	15.667578	18.821518	22.57448	27.03355	32.32378	38.59101	46.00512	21
22	1.545980	1.916103	2.369919	2.925260	3.603537	4.430402	5.436540	6.658600	8.140275	9.933574	12.100310	14.713831	17.861039	21.644746	26.18640	31.62925	38.14206	45.92331	55.20614	22
23	1.576899	1.973587	2.464716	3.071524	3.819750	4.740530	5.871464	7.257874	8.954302	11.026267	13.552347	16.626629	20.361585	24.891458	30.37622	37.00623	45.00763	54.64873	66.24737	23
24	1.608437	2.032794	2.563304	3.225100	4.048935	5.072367	6.341181	7.911083	9.849733	12.239157	15.178629	18.788091	23.212207	28.625176	35.23642	43.29729	53.10901	65.03199	79.49685	24
25	1.640606	2.093778	2.665836	3.386355	4.291871	5.427433	6.848475	8.623081	10.834706	13.585464	17.000064	21.230542	25.461916	32.918953	40.87424	50.65783	62.66863	77.38807	95.39622	25
26	1.673418	2.156591	2.772470	3.555673	4.549383	5.807353	7.396353	9.399158	11.918177	15.079865	19.040072	23.990513	30.166584	37.856796	47.41412	59.26966	73.94898	92.09181	114.47546	26
27	1.706866	2.221289	2.883369	3.733456	4.822346	6.213868	7.988061	10.245082	13.109994	16.736650	21.324881	27.109279	34.389906	43.535315	55.00038	69.34550	87.25980	109.58925	137.37055	27
28	1.741024	2.287928	2.998703	3.920129	5.111687	6.648838	8.627106	11.167140	14.420994	18.579901	23.883866	30.633486	39.204493	50.065612	63.80044	81.13423	102.96656	130.41121	164.84466	28
29	1.775845	2.356566	3.118651	4.116136	5.418388	7.114257	9.317275	12.172182	15.863093	20.623691	26.749930	34.615839	44.693122	57.575454	74.00851	94.92705	121.50054	155.18934	197.81359	29
30	1.811362	2.427262	3.243398	4.321942	5.743491	7.612255	10.062657	13.267678	17.449402	22.892297	29.959922	39.115898	50.950159	66.211772	85.84988	111.06465	143.37064	184.67531	237.37631	30
31	1.847589	2.500080	3.373133	4.538039	6.088101	8.145113	10.867669	14.461770	19.194342	25.410449	33.555113	44.200965	58.083181	76.143038	99.58586	129.94564	169.17735	219.76362	284.85158	31
32	1.884541	2.575083	3.508059	4.764941	6.453387	8.715271	11.737083	15.763329	21.113777	28.205599	37.581726	49.947090	66.214826	87.565068	115.51959	152.03640	199.62928	261.51871	341.82189	32
33	1.922231	2.652335	3.648381	5.003189	6.840590	9.325340	12.676050	17.182028	23.225154	31.308214	42.091533	56.440212	75.484902	100.699829	134.00273	177.88259	235.56255	311.20726	410.18627	33
34	1.960676	2.731905	3.794316	5.253348	7.251025	9.978114	13.690134	18.728411	25.547670	34.752118	47.142517	63.777439	86.052788	115.804803	155.44317	208.12263	277.96381	370.33664	492.22352	34
35	1.999890	2.813662	3.946089	5.516015	7.686087	10.676581	14.785344	20.413968	28.102437	38.574851	52.799620	72.068506	98.100178	133.175523	180.31407	243.50347	327.99729	440.70061	590.66823	35
36	2.039887	2.898278	4.103933	5.791816	8.147252	11.423942	15.968172	22.251225	30.912681	42.818085	59.135574	81.437412	111.834203	153.151852	209.16432	284.89906	387.03680	524.43372	708.80187	36
37	2.080685	2.985227	4.268090	6.081407	8.636087	12.223618	17.245626	24.253835	34.003949	47.528074	66.231843	92.024276	127.490992	176.124630	242.63062	333.33191	456.70343	624.07613	850.56225	37
38	2.122299	3.074783	4.438813	6.385477	9.154252	13.079271	18.625276	26.436680	37.404343	52.756152	74.179664	103.987432	145.339731	202.543324	281.45151	389.99833	538.91004	742.65059	1020.67470	38
39	2.164745	3.167027	4.616366	6.704751	9.703507	13.994820	20.115298	28.815982	41.144778	58.559340	83.081224	117.505798	165.687293	232.924823	326.48376	456.29805	635.91385	883.75421	1224.80964	39
40	2.208040	3.262038	4.801021	7.039989	10.285718	14.974458	21.724521	31.409420	45.259256	65.000867	93.050970	132.781552	188.883514	267.863546	378.72116	533.86871	750.37834	1051.66751	1469.77157	40
41	2.252200	3.359899	4.993061	7.391988	10.902861	16.022670	23.462483	34.236268	49.785181	72.150963	104.217087	150.043153	215.327206	308.043078	439.31654	624.62639	885.44645	1251.48433	1763.72588	41
42	2.297244	3.460696	5.192784	7.761588	11.557033	17.144257	25.339482	37.317532	54.763699	80.087569	116.723137	169.548763	245.473015	354.249540	509.60719	730.81288	1044.82681	1489.26636	2116.47106	42
43	2.343189	3.564517	5.400495	8.149667	12.250455	18.344355	27.366640	40.676110	60.240069	88.897201	130.729914	191.590103	279.839237	407.386971	591.14434	855.05107	1232.89563	1772.22696	2539.76527	43
44	2.390053	3.671452	5.616515	8.557150	12.985482	19.628460	29.555972	44.336960	66.264076	98.675893	146.417503	216.496816	319.016730	468.495017	685.72744	1000.40975	1454.81685	2108.95009	3047.71832	44
45	2.437854	3.781596	5.841176	8.985008	13.764611	21.002452	31.920449	48.327286	72.890484	109.530242	163.987604	244.641402	363.679072	538.769269	795.44383	1170.47941	1716.68388	2509.65060	3657.26119	45
46	2.486611	3.895044	6.074823	9.434258	14.590487	22.472623	34.474085	52.676742	80.179532	121.578568	183.666116	276.444784	414.594142	619.584659	922.71434	1369.46091	2025.68698	2986.48422	4388.71439	46
47	2.536344	4.011895	6.317816	9.905971	15.465917	24.045707	37.232012	57.417649	88.197485	134.952211	205.706050	312.382606	472.637322	712.522358	1070.34921	1602.26927	2390.31063	3553.91622	5266.45726	47
48	2.587070	4.132252	6.570528	10.401270	16.393872	25.728907	40.210573	62.585237	97.017234	149.796954	230.390776	352.992345	538.806547	819.400712	1241.60509	1874.65504	2820.56655	4229.16030	6319.74872	48
49	2.638812	4.256219	6.833349	10.921333	17.377504	27.529930	43.427419	68.217908	106.718957	166.274619	258.037669	398.681350	614.239464	942.310819	1440.26190	2193.34640	3328.26863	5032.70076	7583.69846	49
50	2.691588	4.383906	7.106683	11.467400	18.420154	29.457025	46.901613	74.357520	117.390853	184.564827	289.002190	450.735925	700.232988	1083.657442	1670.70380	2566.21528	3927.35686	5988.91290	9100.43815	50

Table I (Continued).—Amount of 1.00 with Interest for a Given Time.

Year	25%	30%	35%	40%	45%	50%	55%
1	1.25	1.30	1.35	1.40	1.45	1.50	1.55
2	1.5625	1.69	1.8225	1.96	2.1025	2.25	2.4025
3	1.953125	2.197	2.460375	2.744	3.048625	3.375	3.723875
4	2.44140625	2.8561	3.32150625	3.8416	4.42050625	5.0625	5.77200625
5	3.0517578125	3.71293	4.4840334375	5.37824	6.4097340625	7.59375	8.9466096875
6	3.814697266	4.826809	6.053445141	7.529536	9.294114390625	11.390625	13.86724501560
7	4.768371582	6.2748517	8.172150940	10.5413504	13.476465866	17.0859375	21.494229774
8	5.960464478	8.15730721	11.032403769	14.75789056	19.540875506289	25.62890625	33.316056150
9	7.450580597	10,604,499373	14.89374509	20.661046784	28.334269484	38.443359375	51.63988703
10	9.313225746	13.785849184	20.10655587	28.9254654976	41.08469075	57.66503906	80.04182490
11	11.641532182	17.921603940	27.143850424	40.49565170	59.57280159	86.49755859	124.06482860
12	14.55191523	23.29808512	36.64419807	56.693 91238	86.38056231	129,7463379	192.30048433
13	18.18989404	30.28751066	49.46966739	79.37147733	125.2618153	194.61950685	298.0657506
14	22.73736754	39.37376386	66.78405098	111.12006826	181.6151322	291.92926035	462.0019134
15	28.42170943	51.185893000	90.15846883	155.5680956	263.3419418	437.8938904	716.1029660
16	35.52713679	66.54166090	121.7139329	217.7953338	381.8458156	656.84083560	1,109.959597
17	44.40892098	86.50415917	164.3138094	304.9134673	553.6764326	985.26125340	1,720.437375
18	55.51115123	112.4554069	221.8236427	426.8788542	802.8308273	1,477.891880	2,666.677931
19	69.38893904	146.1920290	299.4619176	597.6303959	1,164.104700	2,216.83782	4,133.350793
20	86.73617380	190.0496377	404.2735890	836.6825545	1,687.951814	3,325.256730	6,406.693733
21	108.42021725	247.0645290	545.7693451	1,171.355576	2,447.530130	4,987.885095	9,930.375286
22	135.5252716	321.1838877	736.7886159	1,639.897806	3,548.918688	7,481.827643	15,392.08169
23	169.4065895	417.5390540	994.6646315	2,295.856928	5,145.932098	11,222.74146	23,857.72662
24	211.7582369	542.8007702	1,342.797253	3,214.199699	7,461.601542	16,834.11219	36,979.47626
25	264.69779601	705.6410013	1,812.776291	4,499.879582	10,819.32224	25,251.16829	57,318.18822
26	330.8722450	917.33330169	2,447.247993	6,299.831415	15,688.01725	37,876.75244	88,843.19172
27	413.59030625	1,192.5332922	3,303.784791	8,819.763981	22,747.62501	56,815.12866	137,706.9472
28	516.9878828	1,550.293280	4,460.109468	12,347.66957	32,984.05626	85,222.69299	213,445.7682
29	646.2348535	2,015.381277	6,021.147782	17,286.73740	47,826.88158	127,834.0395	330,840.9407
30	807.7935669	2,619.995642	8,128.549504	24,201.43236	69,348.97829	191,751.0592	512,803.4580
31	1,009.741959	3,405.994335	10,973.54183	33,882.00530	100,556.0185	287,626.5888	794,845.3599
32	1,262.177449	4,427.7926355	14,814.28147	47,434.80742	145,806.2268	431,439.8832	1,232,010.308
33	1,577.721811	5,756.1304268	19,999.27998	66,408.73039	211,419.0289	647,159.8248	1,909,615.977
34	1,972.152264	7,482.9695551	26,999.02797	92,972.22255	306,557.5919	970,739.7372	2,959,904.764
35	2,465.190330	9,727.860420	36,448.68777	130,161.1116	444,508.5084	1,456,109.606	4,587,852.385
36	3,081.487911	12,646.21855	49,205.72849	182,225.5562	644,537.3371	2,184,164.409	7,111,171.196
37	3,851.859889	16,440.08412	66,427.73346	255,115.7787	934,579.1388	3,276,246.614	11,022,315.35
38	4,814.824861	21,372.10936	89,677.44017	357,162.0902	1,355,139.751	4,914,369.921	17,084,588.79
39	6,018.531076	27,783.74217	121,064.5442	500,026.9263	1,964,952.639	7,371,554.882	26,481,112.62
40	7,523.163845	36,118.86482	163,437.1347	700,037.6969	2,849,181.327	11,057,332.32	41,045,724.59
41	9,403.954806	46,954.52427	220,640.1318	980,052.7756	4,131,312.924	16,585,998.48	
42	11,754.94351	61,040.88155	297,864.1779	1,372,073.886	5,990,403.740	24,878,997.72	
43	14,693.67939	79,353.14601	402,116.6402	1,920,903.440	8,686,085.423	37,318,496.58	
44	18,367.09924	103,159.0898	542,857.4643	2,689,264.816	12,594,823.86	55,977,744.87	
45	22,958.87404	134,106.8167	732,857.5770	3,764,970.743	18,262,494.600	83,966,617.30	
46	28,698.59255	174,338.8617	989,357.7290	5,270,959.039	26,480,617.17	125,949,926.00	
47	35,873.24069	226,640.5202	1,335,632.934	7,379,342.654	38,396,894.90	188,924,889.00	
48	44,841.55086	294,632.6763	1,803,104.461	10,331,079.72	55,675,497.61	283,387,333.50	
49	56,051.93857	383,022.4792	2,434,191.022	14,463,511.61	80,729,471.53	425,081,000.25	
50	70,064.92321	497,929.2240	3,286,157.881	20,248,916,25	117,057,733.71	637,621,500.00	

Table I (Continued).—Amount of 1.00 with Interest for a Given Time.

Year	60%	65%	70%	75%	80%	85%	90%	95%	100%
1	1.60	1.65	1.70	1.75	1.80	1.85	1.90	1.95	2.00
2	2.56	2.7225	2.89	3.0625	3.24	3.4225	3.61	3.8025	4.00
3	4.096	4.492125	4.913	5.359375	5.832	6.331625	6.859	7.414875	8.00
4	6.5536	7.41200625	8.3521	9.37890625	10.4976	11.71350625	13.0321	14.45900625	16.00
5	10.48576	12.2298103125	14.19857	16.4130859375	18.89568	21.66998656	24.76099	28.1950621875	32.00
6	16.777216	20.1791870156	24.137569	28.722900391	34.012224	40.08947514	47.045881	54.98037127	64.00
7	26.8435456	33.295658576	41.0338673	50.26507568	61.2220032	74.16552901	89.3871739	107.2117240	128.00
8	42.94967296	54.93783665	69.75757441	87.96388244	110.1996058	137.2062287	169.8356304	209.0628618	256.00
9	68.71947674	90.647430489	118.5878765	153.9367943	198.3592904	253.8315230	322.6876978	407.6725805	512.00
10	109.9511628	149.56826028	201.59939004	269.3893900	357.0467227	469.5883175	613.1066258	794.9615319	1,024.00
11	175.9218605	246.7876295	342.7189630	471.4314325	642.6841009	868.7383874	1,164.902589	1,550.174987	2,048.00
12	281.4749768	407.1995887	582.6222371	825.0050069	1,156.831381	1,607.166017	2,213.314919	3,022.841225	4,096.00
13	450.3599629	671.8793214	990.4578031	1,443.758762	2,082.296486	2,973.257131	4,205.298346	5,894.5403888	8,192.00
14	720.5759406	1,108.600880	1,683.778265	2,526.577834	3,748.133675	5,500.525692	7,990.066858	11,494.35376	16,384.00
15	1,152.921505	1,829.191452	2,862.423051	4,421.511209	6,746.640617	10,175.97253	15,181.12703		
16	1,844.674408	3,018.165896	4,866.119187	7,737.644616	12,143.95311	18,825.54918			
17	2,951.479053	4,979.973728	8,272.402618	13,540.87808	21,859.11560	34,827.26598			
18	4,722.366485	8,216.956651	14,063.08445	23,695.53664	39,346.40808				
19	7,555.786376	13,557.97847	23,907.24356	41,468.93912					
20	12,089.25820	22,370.66448	40,642.31406	72,570.64345					
21	19,342.81312	36,911.59639	69,091.93390						
22	30,948.50099	60,904.13404	117,456.2876						
23	49,517.60158	100,491.8212	199,675.6889						
24	79,228.16253	165,811.5050	339,448.6711						
25	126,765.0601	273,588.9831	577,062.7411						
26	202,824.0962								
27	324,518.5539								
28	519,229.6862								
29	830,767.4979								
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Table II.—Present or Discounted Value of 1.00 Due at a Given Future Time.

YEAR	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%	YEAR
1	.970874	.961538	.952381	.943396	.934579	.925926	.917431	.909091	.900901	.892857	.884956	.877193	.869565	.862069	.854701	.847458	.840336	.833333	1
2	.942596	.924556	.907029	.889996	.873439	.857339	.841680	.826446	.811622	.797194	.783147	.769468	.756144	.743163	.730514	.718184	.706165	.694444	2
3	.915142	.888996	.863838	.839619	.816298	.793832	.772183	.751315	.731191	.711780	.693050	.674972	.657516	.640658	.624371	.608631	.593416	.578704	3
4	.888487	.854804	.822702	.792095	.762895	.735030	.708425	.683013	.658731	.635518	.613319	.592080	.571753	.552291	.533650	.515789	.498669	.482253	4
5	.862609	.821927	.783526	.747258	.712986	.680583	.649931	.620921	.593451	.567427	.542760	.519369	.497177	.476113	.456111	.437109	.419049	.401878	5
6	.837484	.790315	.746215	.704961	.666342	.630170	.596267	.564474	.534641	.506631	.480319	.455587	.432328	.410442	.389839	.370432	.352142	.334898	6
7	.813092	.759918	.710681	.665057	.622750	.583490	.547034	.513158	.481658	.452349	.425061	.399637	.375937	.353830	.333195	.313925	.295918	.279082	7
8	.789409	.730690	.676839	.627412	.582009	.540269	.501866	.466507	.433927	.403883	.376160	.350559	.326902	.305025	.284783	.266038	.248671	.232568	8
9	.766417	.702587	.644609	.591898	.543934	.500249	.460428	.424098	.390925	.360610	.332885	.307508	.284262	.262953	.243404	.225456	.208967	.193807	9
10	.744094	.675564	.613913	.558395	.508349	.463193	.422411	.385543	.352184	.321973	.294588	.269744	.247185	.226684	.208038	.191064	.175602	.161506	10
11	.722421	.649581	.584679	.526788	.475093	.428883	.387533	.350494	.317283	.287476	.260698	.236617	.214943	.195417	.177810	.161919	.147565	.134588	11
12	.701380	.624597	.556837	.496969	.444012	.397114	.355535	.318631	.285841	.256675	.230706	.207559	.186907	.168463	.151974	.137220	.124004	.112157	12
13	.680951	.600574	.530321	.468839	.414964	.367698	.326179	.289664	.257514	.229174	.204165	.182069	.162528	.145227	.129892	.116288	.104205	.093464	13
14	.661118	.577475	.505068	.442301	.387817	.340461	.299246	.263331	.231995	.204620	.180677	.159710	.141329	.125195	.111019	.098549	.087567	.077887	14
15	.641862	.555264	.481017	.417265	.362446	.315242	.274538	.239392	.209004	.182696	.159891	.140096	.122894	.107927	.094888	.083516	.073586	.064905	15
16	.623167	.533908	.458112	.393646	.338735	.291890	.251870	.217629	.188292	.163122	.141496	.122892	.106865	.093041	.081101	.070776	.061837	.054088	16
17	.605016	.513373	.436297	.371364	.316574	.270269	.231073	.197845	.169633	.145644	.125218	.107800	.092926	.080207	.069317	.059980	.051964	.045073	17
18	.587395	.493628	.415521	.350344	.295864	.250249	.211994	.179859	.152822	.130040	.110812	.094561	.080805	.069144	.059245	.050830	.043667	.037561	18
19	.570286	.474642	.395734	.330513	.276508	.231712	.194490	.163508	.137678	.116107	.098064	.082948	.070265	.059607	.050637	.043077	.036695	.031301	19
20	.553676	.456387	.376890	.311805	.258419	.214548	.178431	.148644	.124034	.103667	.086782	.072762	.061100	.051385	.043280	.036506	.030836	.026084	20
21	.537549	.438834	.358942	.294155	.241513	.198656	.163698	.135131	.111742	.092560	.076798	.063826	.053131	.044298	.036991	.030937	.025913	.021737	21
22	.521892	.421955	.341850	.277505	.225713	.183941	.150182	.122846	.100669	.082643	.067963	.055988	.046201	.038188	.031616	.026218	.021775	.018114	22
23	.506692	.405726	.325571	.261797	.210947	.170315	.137781	.111678	.090693	.073788	.060144	.049112	.040174	.032920	.027022	.022218	.018299	.015095	23
24	.491934	.390121	.310068	.246979	.197147	.157699	.126405	.101526	.081705	.065882	.053225	.043081	.034934	.028380	.023096	.018829	.015377	.012579	24
25	.477606	.375117	.295303	.232999	.184249	.146018	.115968	.092296	.073608	.058823	.047102	.037790	.030378	.024465	.019740	.015957	.012922	.010483	25
26	.463695	.360689	.281241	.219810	.172195	.135202	.106393	.083905	.066314	.052521	.041683	.033149	.026415	.021091	.016872	.013523	.010859	.008736	26
27	.450189	.346817	.267848	.207368	.160930	.125187	.097608	.076278	.059742	.046894	.036888	.029078	.022970	.018182	.014421	.011460	.009125	.007280	27
28	.437077	.333477	.255094	.195630	.150402	.115914	.089548	.069343	.053822	.041869	.032644	.025507	.019974	.015674	.012325	.009711	.007668	.006066	28
29	.424346	.320651	.242946	.184557	.140563	.107328	.082155	.063039	.048488	.037383	.028889	.022375	.017369	.013512	.010534	.008230	.006444	.005055	29
30	.411987	.308319	.231377	.174110	.131367	.099377	.075371	.057309	.043683	.033378	.025565	.019627	.015103	.011648	.009004	.006975	.005415	.004213	30
31	.399987	.296460	.220359	.164255	.122773	.092016	.069148	.052099	.039354	.029802	.022624	.017217	.013133	.010042	.007696	.005911	.004550	.003511	31
32	.388337	.285058	.209866	.154957	.114741	.085200	.063438	.047362	.035454	.026609	.020021	.015102	.011420	.008657	.006577	.005009	.003824	.002926	32
33	.377026	.274094	.199873	.146186	.107235	.078889	.058200	.043057	.031940	.023758	.017718	.013248	.009931	.007463	.005622	.004245	.003213	.002438	33
34	.366045	.263554	.190355	.137912	.100219	.073045	.053395	.039143	.028775	.021212	.015680	.011621	.008635	.006433	.004805	.003598	.002700	.002032	34
35	.355383	.253415	.181290	.130105	.093663	.067635	.048986	.035584	.025924	.018940	.013876	.010194	.007509	.005546	.004107	.003049	.002269	.001693	35
36	.345032	.243669	.172657	.122741	.087535	.062625	.044941	.032350	.023355	.016910	.012279	.008942	.006529	.004781	.003510	.002584	.001907	.001411	36
37	.334983	.234297	.164436	.115793	.081809	.057986	.041231	.029408	.021040	.015098	.010867	.007844	.005678	.004121	.003000	.002190	.001602	.001176	37
38	.325226	.225285	.156605	.109239	.076457	.053690	.037826	.026735	.018955	.013481	.009617	.006880	.004937	.003553	.002564	.001856	.001347	.000980	38
39	.315754	.216621	.149148	.103056	.071455	.049713	.034703	.024304	.017077	.012036	.008510	.006035	.004293	.003063	.002192	.001573	.001132	.000816	39
40	.306557	.208289	.142046	.097222	.066780	.046031	.031838	.022095	.015384	.010747	.007531	.005294	.003733	.002640	.001873	.001333	.000951	.000680	40
41	.297628	.200278	.135282	.091719	.062412	.042621	.029209	.020086	.013860	.009595	.006665	.004644	.003246	.002276	.001601	.001129	.000799	.000567	41
42	.288959	.192575	.128840	.086527	.058329	.039464	.026797	.018260	.012486	.008567	.005898	.004074	.002823	.001962	.001368	.000957	.000671	.000472	42
43	.280543	.185168	.122704	.081630	.054513	.036541	.024584	.016600	.011249	.007649	.005219	.003573	.002455	.001692	.001170	.000811	.000564	.000394	43
44	.272372	.178046	.116861	.077009	.050946	.033834	.022555	.015091	.010134	.006830	.004619	.003135	.002134	.001458	.001000	.000687	.000474	.000328	44
45	.264439	.171198	.111297	.072650	.047613	.031328	.020692	.013719	.009130	.006098	.004088	.002750	.001856	.001257	.000854	.000583	.000398	.000273	45
46	.256737	.164614	.105997	.068538	.044499	.029007	.018984	.012472	.008225	.005448	.003617	.002412	.001614	.001084	.000730	.000494	.000335	.000228	46
47	.249259	.158283	.100949	.064658	.041587	.026859	.017416	.011338	.007410	.004861	.003201	.002116	.001403	.000934	.000624	.000418	.000281	.000190	47
48	.241999	.152195	.096142	.060998	.038867	.024869	.015978	.010307	.006676	.004340	.002833	.001856	.001220	.000805	.000533	.000355	.000236	.000158	48
49	.234950	.146341	.091564	.057546	.036324	.023027	.014659	.009370	.006014	.003875	.002507	.001628	.001061	.000694	.000456	.000300	.000199	.000132	49
50	.228107	.140713	.087204	.054288	.033948	.021321	.013449	.008519	.005418	.003460	.002219	.001428	.000923	.000599	.000390	.000255	.000167	.000110	50

Table II (Continued).—Present or Discounted Value of 1.00 Due at a Given Future Time.

Year	25%	30%	35%	40%	45%	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%	100%
1	.80000	.76923	.74074	.71429	.68966	.66667	.64516	.62500	.60606	.58824	.57143	.55556	.54054	.52632	.51282	.50000
2	.64000	.59172	.54870	.51020	.47562	.44444	.41623	.39063	.36731	.34602	.32653	.30864	.29218	.27701	.26298	.25000
3	.51200	.45517	.40644	.36443	.32802	.29630	.26854	.24414	.22261	.20354	.18659	.17147	.15794	.14579	.13486	.12500
4	.40960	.35013	.30107	.26031	.22622	.19753	.17325	.15259	.13492	.11973	.10662	.09526	.08537	.07673	.06916	.06250
5	.32768	.26933	.22301	.18593	.15601	.13169	.11177	.09537	.08177	.07043	.06093	.05292	.04615	.04039	.03547	.03125
6	.26214	.20718	.16520	.13281	.10759	.08779	.07211	.05960	.04956	.04143	.03482	.02940	.02494	.02126	.01819	.01563
7	.20972	.15937	.12237	.09486	.07420	.05853	.04652	.03725	.03003	.02437	.01989	.01633	.01348	.01119	.00933	.00781
8	.16777	.12259	.09064	.06776	.05117	.03902	.03002	.02328	.01820	.01434	.01137	.00907	.00729	.00589	.00478	.00391
9	.13422	.09430	.06714	.04840	.03529	.02601	.01936	.01455	.01103	.00843	.00650	.00504	.00394	.00310	.00245	.00195
10	.10737	.07254	.04974	.03457	.02434	.01734	.01249	.00909	.00669	.00496	.00371	.00280	.00213	.00163	.00126	.00098
11	.08590	.05580	.03684	.02469	.01679	.01156	.00806	.00568	.00405	.00292	.00212	.00156	.00115	.00086	.00065	.00049
12	.06872	.04292	.02729	.01764	.01158	.00771	.00520	.00355	.00246	.00172	.00121	.00086	.00062	.00045	.00033	.00024
13	.05498	.03302	.02021	.01260	.00798	.00514	.00335	.00222	.00149	.00101	.00069	.00048	.00034	.00024	.00017	.00012
14	.04398	.02540	.01497	.00900	.00551	.00343	.00216	.00139	.00090	.00059	.00040	.00027	.00018	.00013	.00009	.00006
15	.03518	.01954	.01109	.00643	.00380	.00228	.00140	.00087	.00055	.00035	.00023	.00015	.00010	.00007		
16	.02815	.01503	.00822	.00459	.00262	.00152	.00090	.00054	.00033	.00021	.00013	.00008	.00005			
17	.02252	.01156	.00609	.00328	.00181	.00101	.00058	.00034	.00020	.00012	.00007	.00005	.00003			
18	.01801	.00889	.00451	.00234	.00125	.00068	.00037	.00021	.00012	.00007	.00004	.00003				
19	.01441	.00684	.00334	.00167	.00086	.00045	.00024	.00013	.00007	.00004	.00002					
20	.01153	.00526	.00247	.00120	.00059	.00030	.00016	.00008	.00004	.00002	.00001					
21	.00922	.00405	.00183	.00085	.00041	.00020	.00010	.00005	.00003	.00001						
22	.00738	.00311	.00136	.00061	.00028	.00013	.00006	.00003	.00002	.00001						
23	.00590	.00239	.00101	.00044	.00019	.00009	.00004	.00002	.00001	.00001						
24	.00472	.00184	.00074	.00031	.00013	.00006	.00003	.00001	.00001	.00000						
25	.00378	.00142	.00055	.00022	.00009	.00004	.00002	.00001	.00000							
26	.00302	.00109	.00041	.00016	.00006	.00003	.00001	.00000								
27	.00242	.00084	.00030	.00011	.00004	.00002	.00001									
28	.00193	.00065	.00022	.00008	.00003	.00001	.00000									
29	.00155	.00050	.00017	.00006	.00002	.00001										
30	.00124	.00038	.00012	.00004	.00001	.00001										
31	.00099	.00029	.00009	.00005	.00001	.00000										
32	.00079	.00023	.00007	.00002	.00001											
33	.00063	.00017	.00005	.00002	.00000											
34	.00051	.00013	.00004	.00001												
35	.00041	.00010	.00003	.00001												
36	.00032	.00008	.00002	.00001												
37	.00026	.00006	.00002	.00000												
38	.00021	.00005	.00001													
39	.00017	.00004	.00001													
40	.00013	.00003	.00001													
41	.00011	.00002	.00000													
42	.00009	.00002														
43	.00007	.00001														
44	.00005	.00001														
45	.00004	.00001														
46	.00003	.00001														
47	.00003	.00000														
48	.00002															
49	.00002															
50	.00001															

Table III.—Annual Payment Which, with Interest, Will Amount to 1.00 at the End of a Given Time.

Year	2%	2½%	3%	3½%	4%	4½%	5%	5½%	6%	6½%	7%	8%	9%	10%
1	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
2	.495050	.493827	.492611	.491400	.490196	.488998	.487805	.486618	.485437	.484262	.483092	.480769	.478469	.476190
3	.326755	.325137	.323530	.321934	.320349	.318773	.317209	.315654	.314110	.312576	.311052	.308034	.305055	.302115
4	.242624	.240818	.239027	.237251	.235490	.233744	.232012	.230295	.228591	.226903	.225228	.221921	.218669	.215471
5	.192158	.190247	.188355	.186481	.184627	.182792	.180975	.179176	.177396	.175635	.173891	.170456	.167092	.163797
6	.158526	.156550	.154597	.152668	.150762	.148878	.147017	.145179	.143363	.141568	.139796	.136315	.132920	.129607
7	.134512	.132495	.130506	.128545	.126610	.124701	.122820	.120964	.119135	.117331	.115553	.112072	.108691	.105405
8	.116510	.114467	.112456	.110477	.108528	.106610	.104722	.102864	.101036	.099237	.097468	.094015	.090674	.087444
9	.102515	.100457	.098434	.096446	.094493	.092574	.090690	.088839	.087022	.085238	.083486	.080080	.076799	.073641
10	.091327	.089259	.087231	.085241	.083291	.081379	.079505	.077668	.075868	.074105	.072378	.069029	.065820	.062745
11	.082178	.080106	.078077	.076092	.074149	.072248	.070389	.068571	.066793	.065055	.063357	.060076	.056947	.053963
12	.074560	.072487	.070462	.068484	.066552	.064666	.062825	.061029	.059277	.057568	.055902	.052695	.049651	.046763
13	.068118	.066048	.064030	.062062	.060144	.058275	.056456	.054684	.052960	.051283	.049651	.046522	.043567	.040779
14	.062602	.060537	.058526	.056571	.054669	.052820	.051024	.049279	.047585	.045940	.044345	.041297	.038433	.035746
15	.057825	.055766	.053767	.051825	.049941	.048114	.046342	.044626	.042963	.041353	.039795	.036830	.034059	.031474
16	.053650	.051599	.049611	.047685	.045820	.044015	.042270	.040583	.038952	.037378	.035858	.032977	.030300	.027817
17	.049970	.047928	.045953	.044043	.042199	.040418	.038699	.037042	.035445	.033906	.032425	.029629	.027046	.024664
18	.046702	.044670	.042709	.040817	.038993	.037237	.035546	.033920	.032357	.030855	.029413	.026702	.024212	.021930
19	.043782	.041761	.039814	.037940	.036139	.034407	.032745	.031150	.029621	.028156	.026753	.024128	.021730	.019547
20	.041157	.039147	.037216	.035361	.033582	.031876	.030243	.028679	.027185	.025756	.024393	.021852	.019546	.017460
21	.038785	.036787	.034872	.033037	.031280	.029601	.027996	.026465	.025005	.023613	.022289	.019832	.017617	.015624
22	.036631	.034647	.032747	.030932	.029199	.027546	.025971	.024471	.023046	.021691	.020406	.018032	.015905	.014005
23	.034668	.032696	.030814	.029019	.027309	.025682	.024137	.022670	.021278	.019961	.018714	.016422	.014382	.012572
24	.032871	.030913	.029047	.027273	.025587	.023987	.022471	.021036	.019679	.018398	.017189	.014978	.013023	.011300
25	.031220	.029276	.027428	.025674	.024012	.022439	.020952	.019549	.018227	.016981	.015811	.013679	.011806	.010168
26	.029699	.027769	.025938	.024205	.022567	.021021	.019564	.018193	.016904	.015695	.014561	.012507	.010715	.009159
27	.028293	.026377	.024564	.022852	.021239	.019719	.018292	.016952	.015697	.014523	.013426	.011448	.009735	.008258
28	.026990	.025088	.023293	.021603	.020013	.018521	.017123	.015814	.014593	.013453	.012392	.010489	.008852	.007451
29	.025778	.023891	.022115	.020445	.018880	.017414	.016046	.014679	.013580	.012474	.011449	.009619	.008056	.006728
30	.024650	.022778	.021019	.019371	.017830	.016392	.015051	.013805	.012649	.011577	.010586	.008827	.007336	.006079
31	.023596	.021739	.019999	.018372	.016855	.015443	.014132	.012917	.011792	.010754	.009797	.008107	.006686	.005496
32	.022611	.020768	.019047	.017442	.015949	.014563	.013280	.012095	.011002	.009997	.009073	.007451	.006096	.004972
33	.021687	.019859	.018156	.016572	.015104	.013745	.012490	.011335	.010273	.009299	.008408	.006852	.005562	.004499
34	.020819	.019007	.017322	.015760	.014315	.012982	.011755	.010630	.009598	.008656	.007797	.006304	.005077	.004074
35	.020002	.018206	.016539	.014998	.013577	.012270	.011072	.009975	.008974	.008062	.007234	.005803	.004636	.003690
36	.019233	.017452	.015804	.014284	.012887	.011606	.010434	.009366	.008395	.007513	.006715	.005345	.004235	.003343
37	.018507	.016741	.015112	.013613	.012240	.010984	.009840	.008800	.007857	.007005	.006237	.004924	.003870	.003030
38	.017821	.016070	.014459	.012982	.011632	.010402	.009284	.008272	.007358	.006535	.005795	.004539	.003538	.002747
39	.017171	.015436	.013844	.012388	.011061	.009856	.008765	.007780	.006894	.006099	.005387	.004185	.003236	.002491
40	.016556	.014836	.013262	.011827	.010523	.009343	.008278	.007320	.006462	.005694	.005009	.003860	.002960	.002259
41	.015972	.014268	.012712	.011298	.010017	.008862	.007822	.006891	.006059	.005318	.004660	.003562	.002708	.002050
42	.015417	.013729	.012192	.010798	.009540	.008409	.007395	.006489	.005683	.004968	.004336	.003287	.002478	.001860
43	.014890	.013217	.011698	.010325	.009090	.007982	.006993	.006113	.005333	.004644	.004036	.003034	.002268	.001688
44	.014388	.012730	.011230	.009878	.008665	.007581	.006616	.005761	.005006	.004341	.003758	.002802	.002077	.001532
45	.013910	.012268	.010785	.009453	.008262	.007202	.006262	.005431	.004700	.004060	.003500	.002587	.001902	.001391
46	.013453	.011827	.010363	.009051	.007882	.006845	.005928	.005122	.004415	.003797	.003260	.002390	.001742	.001263
47	.013018	.011407	.009961	.008669	.007522	.006507	.005614	.004831	.004148	.003553	.003037	.002208	.001595	.001147
48	.012602	.011006	.009578	.008306	.007181	.006189	.005318	.004559	.003898	.003325	.002831	.002040	.001461	.001042
49	.012209	.010623	.009213	.007962	.006857	.005887	.005040	.004302	.003674	.003112	.002639	.001886	.001339	.000946
50	.011823	.010258	.008865	.007634	.006550	.005602	.004777	.004061	.003444	.002914	.002460	.001743	.001227	.000859

Table IV.—Present or Discounted Value of Each 1.00 of Total Income Receivable in Equal Annual Amounts, Compound Interest Valuation Premises.

YEAR	FOUR PER CENT.	FIVE PER CENT.	SIX PER CENT.	SEVEN PER CENT.	EIGHT PER CENT.	NINE PER CENT.	TEN PER CENT.	ELEVEN PER CENT.	TWELVE PER CENT.	THIRTEEN PER CENT.	FOURTEEN PER CENT.	FIFTEEN PER CENT.	SIXTEEN PER CENT.	SEVENTEEN PER CENT.	EIGHTEEN PER CENT.	NINETEEN PER CENT.	TWENTY PER CENT.	YEAR
1	0.96154	0.95238	0.94340	0.93458	0.92593	0.91743	0.90909	0.90090	0.89286	0.88495	0.87719	0.86957	0.86207	0.85470	0.84746	0.84034	0.83333	1
2	0.94305	0.92970	0.91669	0.90401	0.89163	0.87956	0.86777	0.85626	0.84502	0.83405	0.82333	0.81285	0.80262	0.79261	0.78282	0.77325	0.76389	2
3	0.92503	0.90775	0.89100	0.87477	0.85903	0.84376	0.82895	0.81457	0.80061	0.78705	0.77388	0.76108	0.74863	0.73653	0.72476	0.71331	0.70216	3
4	0.90747	0.88649	0.86628	0.84680	0.82803	0.80993	0.79247	0.77561	0.75934	0.74362	0.72843	0.71374	0.69953	0.68581	0.67252	0.65965	0.64718	4
5	0.89036	0.86590	0.84247	0.82004	0.79854	0.77793	0.75816	0.73918	0.72096	0.70346	0.68662	0.67043	0.65486	0.63987	0.62543	0.61153	0.59812	5
6	0.87369	0.84595	0.81955	0.79442	0.77048	0.74765	0.72588	0.70509	0.68523	0.66626	0.64811	0.63075	0.61412	0.59820	0.58293	0.56829	0.55425	6
7	0.85744	0.82662	0.79748	0.76990	0.74377	0.71899	0.69549	0.67317	0.65197	0.63180	0.61261	0.59435	0.57694	0.56034	0.54450	0.52939	0.51494	7
8	0.84159	0.80790	0.77622	0.74642	0.71833	0.69185	0.66687	0.64327	0.62095	0.59985	0.57986	0.56092	0.54295	0.52590	0.50970	0.49430	0.47965	8
9	0.82615	0.78976	0.75674	0.72391	0.69410	0.66614	0.63989	0.61523	0.59203	0.57018	0.54960	0.53018	0.51184	0.49451	0.47811	0.46259	0.44789	9
10	0.81109	0.77217	0.73601	0.70236	0.67101	0.64177	0.61446	0.58892	0.56502	0.54262	0.52161	0.50188	0.48332	0.46586	0.44941	0.43389	0.41925	10
11	0.79641	0.75513	0.71699	0.68170	0.64900	0.61865	0.59046	0.56423	0.53979	0.51699	0.49570	0.47579	0.45715	0.43967	0.42324	0.40786	0.39337	11
12	0.78209	0.73860	0.69865	0.66189	0.62801	0.59673	0.56781	0.54105	0.51620	0.49314	0.47169	0.45172	0.43309	0.41570	0.39944	0.38421	0.36993	12
13	0.76813	0.72258	0.68098	0.64290	0.60798	0.57592	0.54641	0.51922	0.49412	0.47091	0.44941	0.42947	0.41095	0.39371	0.37765	0.36267	0.34867	13
14	0.75451	0.70705	0.66393	0.62468	0.58887	0.55615	0.52619	0.49870	0.47344	0.45018	0.42872	0.40889	0.39054	0.37352	0.35772	0.34302	0.32933	14
15	0.74123	0.69198	0.64748	0.60719	0.57063	0.53738	0.50707	0.47939	0.45406	0.43083	0.40948	0.38982	0.37170	0.35495	0.33944	0.32506	0.31170	15
16	0.72827	0.67736	0.63162	0.59042	0.55321	0.51953	0.48898	0.46120	0.43587	0.41273	0.39157	0.37214	0.35428	0.33783	0.32265	0.30861	0.29560	16
17	0.71563	0.66318	0.61631	0.57432	0.53657	0.50256	0.47186	0.44405	0.41880	0.39583	0.37487	0.35572	0.33816	0.32204	0.30720	0.29351	0.28086	17
18	0.70329	0.64942	0.60153	0.55884	0.52066	0.48642	0.45563	0.42787	0.40276	0.38000	0.35930	0.34044	0.32321	0.30744	0.29295	0.27963	0.26734	18
19	0.69126	0.63607	0.58727	0.54398	0.50545	0.47106	0.44026	0.41259	0.38767	0.36516	0.34476	0.32622	0.30934	0.29392	0.27980	0.26684	0.25492	19
20	0.67952	0.62311	0.57349	0.52970	0.49091	0.45643	0.42568	0.39817	0.37347	0.35124	0.33116	0.31297	0.29644	0.28139	0.26764	0.25504	0.24348	20
21	0.66806	0.61053	0.56019	0.51598	0.47699	0.44249	0.41185	0.38453	0.36010	0.33817	0.31843	0.30059	0.28444	0.26975	0.25637	0.24413	0.23292	21
22	0.65687	0.59832	0.54735	0.50278	0.46367	0.42920	0.39871	0.37162	0.34748	0.32589	0.30650	0.28903	0.27324	0.25893	0.24590	0.23403	0.22316	22
23	0.64595	0.58646	0.53493	0.49010	0.45092	0.41653	0.38623	0.35941	0.33558	0.31433	0.29531	0.27821	0.26279	0.24884	0.23618	0.22465	0.21411	23
24	0.63529	0.57494	0.52293	0.47789	0.43870	0.40444	0.37436	0.34784	0.32435	0.30345	0.28480	0.26807	0.25303	0.23944	0.22712	0.21593	0.20571	24
25	0.62488	0.56376	0.51133	0.46614	0.42699	0.39290	0.36308	0.33687	0.31373	0.29320	0.27492	0.25857	0.24388	0.23065	0.21868	0.20781	0.19790	25
26	0.61472	0.55289	0.50012	0.45484	0.41577	0.38188	0.35234	0.32646	0.30368	0.28353	0.26562	0.24964	0.23531	0.22242	0.21079	0.20023	0.19063	26
27	0.60460	0.54223	0.48928	0.44395	0.40501	0.37135	0.34212	0.31659	0.29417	0.27439	0.25686	0.24124	0.22727	0.21472	0.20340	0.19315	0.18394	27
28	0.59511	0.53208	0.47879	0.43347	0.39468	0.36127	0.33238	0.30720	0.28516	0.26576	0.24859	0.23334	0.21972	0.20749	0.19649	0.18653	0.17749	28
29	0.58565	0.52210	0.46865	0.42337	0.38477	0.35166	0.32309	0.29828	0.27661	0.25759	0.24079	0.22589	0.21261	0.20070	0.18999	0.18032	0.17164	29
30	0.57640	0.51241	0.45883	0.41363	0.37526	0.34246	0.31423	0.28979	0.26851	0.24986	0.23342	0.21887	0.20591	0.19431	0.18389	0.17449	0.16596	30
31	0.56737	0.50299	0.44933	0.40425	0.36612	0.33364	0.30577	0.28171	0.26081	0.24253	0.22645	0.21223	0.19959	0.18829	0.17815	0.16901	0.16072	31
32	0.55855	0.49383	0.44013	0.39520	0.35734	0.32519	0.29770	0.27402	0.25349	0.23557	0.21984	0.20595	0.19362	0.18261	0.17274	0.16384	0.15579	32
33	0.54993	0.48493	0.43122	0.38648	0.34891	0.31710	0.28998	0.26668	0.24653	0.22897	0.21358	0.20001	0.18798	0.17725	0.16764	0.15898	0.15115	33
34	0.54151	0.47625	0.42259	0.37806	0.34079	0.30935	0.28261	0.25969	0.23990	0.22270	0.20764	0.19439	0.18264	0.17218	0.16281	0.15438	0.14676	34
35	0.53327	0.46783	0.41424	0.36993	0.33299	0.30191	0.27555	0.25301	0.23359	0.21673	0.20200	0.18905	0.17758	0.16738	0.15825	0.15003	0.14262	35
36	0.52523	0.45963	0.40614	0.36209	0.32548	0.29477	0.26879	0.24663	0.22757	0.21105	0.19664	0.18398	0.17278	0.16283	0.15392	0.14592	0.13869	36
37	0.51737	0.45166	0.39829	0.35451	0.31825	0.28792	0.26232	0.24053	0.22182	0.20564	0.19153	0.17916	0.16822	0.15851	0.14982	0.14202	0.13498	37
38	0.50968	0.44389	0.39068	0.34720	0.31129	0.28134	0.25612	0.23470	0.21634	0.20048	0.18667	0.17457	0.16389	0.15440	0.14593	0.13832	0.13145	38
39	0.50217	0.43633	0.38331	0.34013	0.30458	0.27501	0.25018	0.22912	0.21110	0.19556	0.18205	0.17021	0.15977	0.15050	0.14223	0.13480	0.12810	39
40	0.49482	0.42898	0.37616	0.33329	0.29812	0.26893	0.24448	0.22378	0.20609	0.19086	0.17763	0.16604	0.15584	0.14678	0.13870	0.13145	0.12491	40
41	0.48764	0.42181	0.36922	0.32669	0.29188	0.26309	0.23900	0.21866	0.20130	0.18637	0.17341	0.16207	0.15209	0.14324	0.13536	0.12827	0.12188	41
42	0.48061	0.41484	0.36249	0.32030	0.28587	0.25746	0.23375	0.21375	0.19671	0.18207	0.16938	0.15828	0.14852	0.13986	0.13215	0.12523	0.11899	42
43	0.47374	0.40804	0.35596	0.31412	0.28008	0.25205	0.22870	0.20904	0.19232	0.17796	0.16552	0.15466	0.14510	0.13664	0.12909	0.12233	0.11623	43
44	0.46702	0.40143	0.34962	0.30813	0.27448	0.24683	0.22384	0.20452	0.18810	0.17402	0.16183	0.15119	0.14184	0.13356	0.12618	0.11956	0.11360	44
45	0.46045	0.39498	0.34346	0.30234	0.26908	0.24180	0.21917	0.20018	0.18406	0.17024	0.15829	0.14787	0.13871	0.13061	0.12338	0.11691	0.11108	45
46	0.45401	0.38870	0.33749	0.29674	0.26386	0.23696	0.21468	0.19600	0.18017	0.16662	0.15490	0.14469	0.13572	0.12778	0.12071	0.11438	0.10867	46
47	0.44772	0.38257	0.33168	0.29131	0.25881	0.23229	0.21035	0.19199	0.17644	0.16314	0.15165	0.14164	0.13285	0.12508	0.11815	0.11195	0.10636	47
48	0.44157	0.37661	0.32604	0.28605	0.25394	0.22778	0.20619	0.18813	0.17286	0.15980	0.14853	0.13872	0.13010	0.12248	0.11570	0.10962	0.10415	48
49	0.43554	0.37079	0.32056	0.28095	0.24928	0.22343	0.20217	0.18441	0.16941	0.15659	0.14553	0.13591	0.12746	0.11999	0.11333	0.10739	0.10203	49
50	0.42964	0.36512	0.31624	0.27601	0.24467	0.21923	0.19830	0.18083	0.16609	0.15350	0.14265	0.13321	0.12492	0.11760	0.11108	0.10525	0.09998	50

Table IV (Continued).—Present or Discounted Value of Each 1.00 of Total Income Receivable in Equal Annual Amounts, Compound Interest Valuation Premises.

Year	25%	30%	35%	40%	45%	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%	100%
1	.80000	.76923	.74074	.71429	.68966	.66667	.64516	.62500	.60606	.58824	.57143	.55556	.54054	.52632	.51282	.50000
2	.72000	.68047	.64472	.61224	.58264	.55556	.53070	.50781	.48669	.46713	.44898	.43210	.41636	.40166	.38790	.37500
3	.65067	.60537	.56529	.52964	.49777	.46914	.44331	.41992	.39866	.37927	.36152	.34522	.33022	.31637	.30356	.29167
4	.59040	.54156	.49924	.46231	.42988	.40123	.37580	.35309	.33272	.31458	.29779	.28273	.26901	.25646	.24496	.23438
5	.53786	.48711	.44399	.40703	.37511	.34733	.32299	.30154	.28253	.26559	.25042	.23677	.22444	.21325	.20306	.19375
6	.49190	.44046	.39753	.36133	.33052	.30407	.28118	.26122	.24370	.22823	.21449	.20221	.19119	.18125	.17225	.16406
7	.45159	.40030	.35822	.32326	.29390	.26899	.24766	.22923	.21318	.19911	.18669	.17565	.16580	.15695	.14897	.14174
8	.41611	.36559	.32477	.29132	.26356	.24025	.22045	.20348	.18880	.17601	.16477	.15483	.14599	.13807	.13095	.12451
9	.38479	.33544	.29615	.26433	.23820	.21644	.19811	.18249	.16905	.15739	.14719	.13819	.13020	.12307	.11667	.11089
10	.35705	.30915	.27150	.24136	.21681	.19653	.17966	.16515	.15282	.14215	.13284	.12465	.11740	.11093	.10518	.09990
11	.33240	.28612	.25017	.22166	.19863	.17972	.16396	.15065	.13929	.12949	.12096	.11346	.10683	.10092	.09563	.09086
12	.31043	.26586	.23160	.20466	.18304	.16538	.15073	.13840	.12789	.11884	.11098	.10408	.09798	.09255	.08769	.08331
13	.29078	.24794	.21534	.18988	.16958	.15306	.13939	.12792	.11817	.10978	.10249	.09611	.09047	.08545	.08096	.07691
14	.27315	.23205	.20102	.17696	.15786	.14237	.12959	.11888	.10979	.10198	.09520	.08926	.08402	.07926	.07518	.07142
15	.25728	.21788	.18836	.16560	.14759	.13303	.12104	.11101	.10251	.09520	.08887	.08332	.07842	.07407		
16	.24296	.20520	.17710	.15553	.13853	.12481	.11353	.10411	.09612	.08927	.08332	.07812	.07353			
17	.23000	.19381	.16704	.14668	.13048	.11753	.10689	.09801	.09048	.08402	.07843	.07353	.06920			
18	.21822	.18354	.15801	.13866	.12330	.11104	.10097	.09251	.08546	.07936	.07407	.06944				
19	.20749	.17424	.14987	.13136	.11686	.10522	.09567	.08771	.08097	.07518	.07017					
20	.19769	.16579	.14250	.12485	.11105	.09997	.09089	.08333	.07692	.07143	.06667					
21	.18872	.15809	.13581	.11895	.10578	.09522	.08657	.07936	.07326	.06803						
22	.18048	.15104	.12969	.11357	.10098	.09090	.08264	.07576	.06993	.06493						
23	.17289	.14458	.12410	.10865	.09660	.08695	.07905	.07246	.06689							
24	.16588	.13863	.11896	.10413	.09258	.08333	.07576	.06944	.06410							
25	.15940	.13314	.11422	.09998	.08888	.08000	.07273	.06667	.06154							
26	.15338	.12807	.10985	.09614	.08546	.07692	.06993	.06410								
27	.14779	.12335	.10579	.09258	.08230	.07407	.06734	.06173								
28	.14258	.11897	.10202	.08928	.07936	.07143	.06493	.05952								
29	.13772	.11489	.09851	.08620	.07663	.06896	.06270	.05747								
30	.13317	.11107	.09523	.08333	.07407	.06667	.06061									
31	.12890	.10750	.09216	.08064	.07168	.06452	.05865									
32	.12490	.10414	.08928	.07812	.06944	.06250	.05682									
33	.12114	.10099	.08658	.07576	.06734	.06061	.05510									
34	.11759	.09803	.08403	.07353	.06536	.05882	.05348									
35	.11424	.09523	.08163	.07143	.06349	.05714	.05195									
36	.11108	.09259	.07936	.06944	.06173	.05556	.05051									
37	.10808	.09008	.07722	.06757	.06006	.05405	.04914									
38	.10524	.08772	.07519	.06579	.05848	.05263										
39	.10255	.08547	.07326	.06410	.05698	.05128										
40	.09999	.08333	.07143	.06250	.05556	.05000										
41	.09755	.08130	.06969	.06098	.05420	.04878										
42	.09523	.07936	.06803	.05952	.05291	.04762										
43	.09302	.07752	.06645	.05814	.05168											
44	.09090	.07576	.06493	.05682	.05051											
45	.08889	.07407	.06349	.05556	.04938											
46	.08695	.07246	.06211	.05435	.04831											
47	.08510	.07092	.06079	.05319	.04728											
48	.08333	.06944	.05952	.05208	.04630											
49	.08163	.06803	.05831	.05102	.04535											
50	.08000	.06667	.05714	.05000	.04444											

Table V.—Present or Discounted Value of Each 1.00 of Total Income Receivable in Equal Annual Amounts, Hoskold Sinking Fund Valuation Premises, 3% Sinking Fund Rate of Interest.

Year	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%	Year
1	.97087	.96154	.95238	.94340	.93458	.92593	.91743	.90909	.90090	.89286	.88496	.87719	.86957	.86207	.85470	.84746	.84034	.83333	1
2	.95673	.93877	.92147	.90480	.88871	.87319	.85821	.84372	.82972	.81618	.80307	.79038	.77808	.76615	.75459	.74337	.73248	.72191	2
3	.94287	.91693	.89239	.86912	.84703	.82604	.80607	.78704	.76888	.75155	.73497	.71912	.70393	.68937	.67541	.66199	.64910	.63670	3
4	.92928	.89597	.86497	.83604	.80899	.78363	.75982	.73740	.71628	.69633	.67746	.65958	.64263	.62652	.61121	.59662	.58271	.56944	4
5	.91594	.87583	.83909	.80530	.77413	.74528	.71851	.69359	.67034	.64860	.62823	.60910	.59110	.57413	.55811	.54296	.52860	.51499	5
6	.90287	.85647	.81461	.77665	.74207	.71044	.68139	.65463	.62989	.60695	.58562	.56574	.54717	.52978	.51346	.49811	.48366	.47002	6
7	.89004	.83784	.79142	.74989	.71248	.67864	.64786	.61975	.59398	.57027	.54838	.52811	.50928	.49175	.47539	.46008	.44572	.43224	7
8	.87746	.81991	.76944	.72482	.68510	.64950	.61742	.58836	.56191	.53774	.51556	.49514	.47627	.45879	.44255	.42741	.41328	.40006	8
9	.86512	.80263	.74856	.70131	.65967	.62270	.58966	.55994	.53308	.50867	.48640	.46600	.44725	.42994	.41392	.39906	.38522	.37231	9
10	.85302	.78598	.72870	.67921	.63601	.59798	.56424	.53410	.50702	.48255	.46034	.44008	.42153	.40448	.38876	.37421	.36071	.34815	10
11	.84115	.76991	.70980	.65839	.61393	.57509	.54088	.51050	.48336	.45896	.43690	.41687	.39859	.38185	.36645	.35226	.33912	.32692	11
12	.82950	.75441	.69178	.63876	.59328	.55385	.51933	.48887	.46176	.43753	.41571	.39595	.37799	.36159	.34655	.33272	.31994	.30811	12
13	.81807	.73943	.67459	.62020	.57393	.53408	.49841	.46896	.44201	.41799	.39645	.37702	.35940	.34336	.32869	.31522	.30281	.29134	13
14	.80686	.72497	.65817	.60264	.55575	.51563	.48092	.45058	.42384	.40010	.37888	.35979	.34254	.32686	.31256	.29946	.28741	.27629	14
15	.79586	.71099	.64247	.58600	.53865	.49838	.46371	.43356	.40708	.38366	.36278	.34406	.32717	.31187	.29793	.28518	.27349	.26271	15
16	.78507	.69746	.62744	.57020	.52253	.48221	.44767	.41775	.39158	.36849	.34797	.32962	.31311	.29817	.28459	.27220	.26084	.25039	16
17	.77448	.68437	.61305	.55519	.50731	.46703	.43268	.40303	.37719	.35446	.33431	.31634	.30019	.28562	.27239	.26034	.24930	.23917	17
18	.76408	.67170	.59925	.54090	.49291	.45274	.41863	.38929	.36380	.34144	.32167	.30407	.28829	.27407	.26118	.24945	.23873	.22890	18
19	.75388	.65943	.58601	.52730	.47928	.43928	.40544	.37644	.35131	.32933	.30994	.29270	.27728	.26340	.25085	.23944	.22902	.21947	19
20	.74387	.64754	.57329	.51432	.46635	.42656	.39303	.36439	.33964	.31803	.29901	.28214	.26707	.25353	.24129	.23019	.22006	.21078	20
21	.73405	.63601	.56107	.50193	.45407	.41454	.38134	.35307	.32870	.30747	.28882	.27251	.25758	.24436	.23243	.22162	.21176	.20274	21
22	.72441	.62483	.54932	.49009	.44239	.40315	.37031	.34241	.31843	.29758	.27930	.26313	.24873	.23582	.22419	.21365	.20406	.19530	22
23	.71494	.61398	.53800	.47876	.43127	.39235	.35988	.33237	.30876	.28829	.27056	.25494	.24046	.22786	.21651	.20624	.19690	.18837	23
24	.70565	.60345	.52711	.46792	.42067	.38210	.35000	.32288	.29966	.27955	.26198	.24688	.23271	.22040	.20933	.19932	.19022	.18191	24
25	.69653	.59323	.51661	.45752	.41056	.37234	.34061	.31390	.29106	.27132	.25408	.23891	.22544	.21342	.20261	.19284	.18397	.17588	25
26	.68757	.58330	.50648	.44755	.40090	.36306	.33174	.30540	.28293	.26355	.24665	.23178	.21861	.20685	.19629	.18676	.17811	.17023	26
27	.67878	.57365	.49671	.43798	.39166	.35420	.32329	.29733	.27524	.25620	.23962	.22506	.21217	.20067	.19038	.18105	.17262	.16493	27
28	.67015	.56427	.48728	.42878	.38282	.34576	.31524	.28967	.26794	.24924	.23298	.21871	.20609	.19485	.18477	.17568	.16744	.15994	28
29	.66167	.55515	.47817	.41993	.37435	.33769	.30757	.28238	.26101	.24264	.22659	.21271	.20035	.18935	.17949	.17061	.16257	.15525	29
30	.65335	.54628	.46936	.41142	.36622	.32997	.30025	.27544	.25442	.23637	.22072	.20701	.19491	.18414	.17450	.16582	.15796	.15082	30
31	.64518	.53764	.46084	.40323	.35843	.32258	.29326	.26882	.24814	.23042	.21506	.20161	.18975	.17921	.16978	.16129	.15361	.14663	31
32	.63715	.52924	.45259	.39534	.35094	.31551	.28657	.26250	.24216	.22474	.20967	.19648	.18486	.17454	.16530	.15700	.14949	.14266	32
33	.62927	.52106	.44461	.38772	.34374	.30872	.28018	.25647	.23645	.21934	.20453	.19160	.18021	.17009	.16105	.15293	.14558	.13891	33
34	.62152	.51310	.43688	.38038	.33682	.30221	.27405	.25069	.23100	.21418	.19964	.18695	.17578	.16587	.15701	.14905	.14187	.13534	34
35	.61392	.50534	.42939	.37329	.33016	.29596	.26818	.24517	.22579	.20925	.19497	.18252	.17156	.16184	.15317	.14537	.13833	.13195	35
36	.60645	.49778	.42213	.36644	.32374	.28994	.26254	.23987	.22080	.20454	.19051	.17829	.16753	.15800	.14950	.14187	.13497	.12872	36
37	.59911	.49041	.41509	.35982	.31755	.28416	.25713	.23479	.21602	.20003	.18625	.17424	.16369	.15434	.14600	.13852	.13177	.12564	37
38	.59191	.48322	.40825	.35342	.31158	.27859	.25192	.22991	.21144	.19572	.18217	.17037	.16001	.15084	.14266	.13533	.12871	.12271	38
39	.58483	.47621	.40162	.34723	.30582	.27323	.24692	.22523	.20704	.19157	.17826	.16667	.15650	.14749	.13947	.13228	.12579	.11991	39
40	.57787	.46937	.39518	.34124	.30026	.26806	.24210	.22073	.20282	.18760	.17451	.16312	.15313	.14429	.13642	.12936	.12299	.11727	40
41	.57103	.46270	.38892	.33543	.29488	.26307	.23746	.21639	.19876	.18378	.17090	.15971	.14990	.14122	.13349	.12656	.12032	.11466	41
42	.56432	.45619	.38284	.32981	.28968	.25826	.23299	.21222	.19485	.18011	.16745	.15644	.14680	.13827	.13068	.12388	.11776	.11221	42
43	.55772	.44984	.37693	.32436	.28466	.25361	.22867	.20820	.19109	.17658	.16412	.15330	.14382	.13545	.12799	.12131	.11530	.10985	43
44	.55123	.44363	.37118	.31907	.27979	.24912	.22451	.20433	.18747	.17319	.16092	.15028	.14096	.13273	.12541	.11885	.11294	.10759	44
45	.54486	.43757	.36559	.31394	.27508	.24478	.22049	.20059	.18398	.16991	.15785	.14738	.13821	.13012	.12292	.11648	.11068	.10543	45
46	.53860	.43165	.36014	.30896	.27051	.24058	.21661	.19698	.18061	.16676	.15488	.14458	.13556	.12761	.12053	.11420	.10850	.10334	46
47	.53244	.42587	.35484	.30412	.26609	.23651	.21285	.19349	.17736	.16372	.15202	.14188	.13301	.12519	.11823	.11201	.10640	.10134	47
48	.52639	.42022	.34968	.29943	.26180	.23257	.20922	.19012	.17422	.16078	.14926	.13928	.13055	.12285	.11601	.10989	.10439	.09941	48
49	.52044	.41469	.34466	.29486	.25764	.22876	.20570	.18687	.17119	.15794	.14660	.13677	.12818	.12061	.11388	.10786	.10244	.09755	49
50	.51460	.40929	.33976	.29042	.25360	.22506	.20230	.18371	.16826	.15520	.14402	.13435	.12589	.11844	.11182	.10590	.10057	.09576	50

Table V (Continued).—Present or Discounted Value of Each 1.00 of Total Income Receivable in Equal Annual Amounts, Hoskold Sinking Fund Valuation Premises, 3% Sinking Fund Rate of Interest.

Year	25%	30%	35%	40%	45%	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%	100%
1	.800000	.769231	.740741	.714286	.689655	.666667	.645161	.625000	.606061	.588235	.571429	.555556	.540541	.526316	.512821	.500000
2	.673300	.630827	.593394	.560155	.530442	.503722	.479565	.457619	.437594	.419248	.402379	.386814	.372409	.359038	.346594	.334983
3	.581196	.534591	.494905	.460704	.430925	.404761	.381593	.360934	.342396	.325670	.310502	.296684	.284043	.272436	.261740	.251852
4	.511219	.463799	.424429	.391220	.362830	.338283	.316846	.297964	.281206	.266233	.252774	.240610	.229563	.219486	.210256	.201771
5	.456252	.409539	.371502	.339931	.313306	.290548	.270873	.253693	.238563	.225135	.213139	.202357	.192612	.183764	.175692	.168300
6	.411932	.366625	.330296	.300518	.275665	.254609	.236542	.220868	.207143	.195024	.184244	.174594	.165904	.158038	.150885	.144350
7	.375440	.331835	.297305	.269285	.246091	.226575	.209928	.195559	.183031	.172012	.162244	.153586	.145697	.138628	.132213	.126366
8	.344869	.303062	.270296	.243923	.222239	.204096	.188692	.175449	.163944	.153854	.144935	.136993	.129876	.123462	.117652	.112364
9	.318887	.278870	.247776	.222920	.202597	.185670	.171353	.159086	.148458	.139161	.130960	.123672	.117152	.111285	.105978	.101154
10	.296533	.258244	.228712	.205242	.186140	.170291	.156929	.145512	.135643	.127028	.119441	.112710	.106697	.101293	.096411	.091977
11	.277096	.240451	.212366	.190156	.172151	.157261	.144742	.134069	.124862	.116838	.109783	.103532	.097954	.092947	.088426	.084325
12	.260041	.224944	.198195	.177131	.160114	.146080	.134309	.124292	.115667	.108160	.101569	.095735	.090534	.085870	.081662	
13	.244955	.211310	.185791	.165772	.149647	.136381	.125276	.115843	.107731	.100681	.094497	.089028	.084158	.079793		
14	.231515	.199228	.174844	.155779	.140462	.127888	.117380	.108467	.100813	.094168	.088344	.083199	.078620			
15	.219467	.188448	.165112	.146918	.132336	.120388	.110418	.101973	.094728	.088445	.082943	.078085				
16	.208604	.178770	.156402	.139009	.125097	.113717	.104234	.096211	.089335	.083377	.078163					
17	.198760	.170034	.148562	.131905	.118607	.107745	.098705	.091065	.084522	.078857	.073903					
18	.189798	.162107	.141468	.125490	.112755	.102367	.093732	.086440	.080200	.074801						
19	.181605	.154884	.135017	.119668	.107452	.097499	.089234	.082261	.076298	.071142						
20	.174085	.148273	.129127	.114360	.102624	.093072	.085148	.078466	.072757							
21	.167160	.142201	.123727	.109501	.098210	.089029	.081418	.075006	.069530							
22	.160760	.136604	.118759	.105037	.094158	.085321	.078000	.071837								
23	.154829	.131428	.114172	.100921	.090426	.081909	.074857	.068924								
24	.149318	.126628	.109925	.097114	.086978	.078758	.071957	.066238								
25	.144182	.122164	.105981	.093583	.083782	.075840	.069273									
26	.139385	.118003	.102308	.090298	.080812	.073129	.066781									
27	.134894	.114113	.098880	.087235	.078044	.070605	.064461									
28	.130681	.110470	.095674	.084372	.075459	.068249	.062297									
29	.126721	.107051	.092667	.081691	.073039	.066044										
30	.122992	.103835	.089843	.079173	.070769	.063977										
31	.119475	.100807	.087184	.076805	.068634	.062035										
32	.116151	.097948	.084678	.074574	.066625	.060207										
33	.113005	.095246	.082310	.072468	.064728	.058482										
34	.110024	.092687	.080071	.070477	.062937											
35	.107194	.090262	.077949	.068592	.061241											
36	.104505	.087959	.075936	.066805	.059634											
37	.101946	.085770	.074024	.065108	.058109											
38	.099508	.083686	.072205	.063494	.056659											
39	.097183	.081700	.070473	.061958	.055279											
40	.094962	.079805	.068821	.060494	.053965											
41	.092840	.077996	.067244	.059097	.052711											
42	.090810	.076266	.065737	.057763	.051514											
43	.088865	.074610	.064296	.056488	.050370											
44	.087001	.073024	.062916	.055267	.049275											
45	.085213	.071503	.061594	.054097												
46	.083496	.070044	.060326	.052975												
47	.081845	.068643	.059108	.051899												
48	.080259	.067296	.057938	.050865												
49	.078731	.066000	.056814	.049872												
50	.077260	.064753	.055731	.048916												

Table VI.—Present or Discounted Value of Each 1.00 of Total Income Receivable in Equal Annual Amounts, Hoskold Sinking Fund Valuation Premises, 4% Sinking Fund Rate of Interest.

Year	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%	Years
1	.961538	.952381	.943396	.934579	.925926	.917431	.909091	.900901	.892857	.884956	.877193	.869565	.862069	.854701	.847458	.840336	.833333	1
2	.943047	.925590	.908767	.892545	.876891	.861778	.847176	.833061	.819409	.806197	.793404	.781011	.768999	.757351	.746050	.735082	.724432	2
3	.925030	.900053	.876389	.853938	.832608	.812318	.792993	.774566	.756976	.740167	.724089	.708694	.693941	.679789	.666202	.653148	.640596	3
4	.907474	.875687	.846052	.818357	.792418	.768073	.745179	.723610	.703255	.684013	.665791	.648525	.632127	.616538	.601699	.587568	.574066	4
5	.890364	.852416	.817571	.785462	.755780	.728260	.702674	.678824	.656540	.635673	.616091	.597680	.580337	.563973	.548506	.533864	.519984	5
6	.873689	.830171	.790778	.754961	.722245	.692247	.664641	.639153	.615547	.593623	.573207	.554148	.536316	.519596	.503887	.489100	.475155	6
7	.857436	.808887	.765540	.726603	.691435	.659514	.630411	.603767	.579285	.556710	.535829	.516458	.498438	.481634	.465925	.451209	.437394	7
8	.841593	.788505	.741717	.700171	.663032	.629635	.599440	.572010	.546979	.524048	.502962	.483507	.465601	.448788	.432091	.416787	.405150	8
9	.826148	.768972	.719198	.675476	.636765	.602251	.571286	.543349	.518017	.494942	.473836	.454455	.436598	.420091	.404787	.390558	.377296	9
10	.811090	.750239	.697881	.652354	.612404	.577064	.545581	.517355	.491906	.468843	.447846	.428649	.411031	.394803	.379808	.366910	.352994	10
11	.796407	.732258	.677672	.630660	.589748	.553820	.522019	.493671	.468244	.445307	.424513	.405574	.388253	.372351	.357700	.344158	.331605	11
12	.782089	.714987	.658490	.610267	.568626	.532304	.500344	.472004	.446703	.423976	.403449	.384819	.367833	.352283	.337995	.324820	.312634	12
13	.768127	.698388	.640259	.591062	.548887	.512330	.480338	.452106	.427009	.404552	.384339	.366050	.349422	.334259	.320321	.307516	.295695	13
14	.754509	.682424	.622911	.572946	.530401	.493738	.461816	.433771	.408937	.386792	.366923	.348996	.332738	.317928	.304380	.291940	.280476	14
15	.741226	.667060	.606385	.555828	.513053	.476391	.444619	.416820	.392293	.370492	.350986	.333432	.317549	.303111	.289929	.277846	.266730	15
16	.728268	.652265	.590626	.539630	.496741	.460168	.428611	.401104	.376915	.355477	.336347	.319171	.303663	.289593	.276769	.265033	.254261	16
17	.715628	.638009	.575581	.524281	.481377	.444964	.413672	.386492	.362664	.341603	.322854	.306056	.290920	.277210	.264734	.253333	.242873	17
18	.703294	.624267	.561205	.509715	.466880	.430685	.399699	.372873	.349421	.328744	.310378	.293955	.279183	.265825	.253686	.242608	.232457	18
19	.691260	.611010	.547455	.495876	.453179	.417252	.386603	.360148	.337083	.316793	.298808	.282756	.268339	.255321	.243508	.232740	.222834	19
20	.679516	.598217	.534292	.482711	.440212	.404590	.374303	.348234	.325560	.306658	.288049	.272358	.258289	.245601	.234102	.223632	.214058	20
21	.668055	.585864	.521680	.470172	.427921	.392637	.362729	.337054	.314774	.295257	.278019	.262682	.248949	.236581	.225383	.215198	.205893	21
22	.656869	.573930	.509587	.458217	.416255	.381334	.351819	.326544	.304658	.285521	.268646	.253654	.240248	.228186	.217279	.207366	.198319	22
23	.645950	.562395	.497981	.446806	.405169	.370630	.341517	.316645	.295150	.276388	.259868	.245212	.232120	.220356	.209727	.200076	.191274	23
24	.635290	.551242	.486835	.435904	.394620	.360479	.331776	.307306	.286198	.267803	.251630	.237299	.224513	.213034	.202672	.193271	.184703	24
25	.624883	.540453	.476123	.425478	.384571	.350840	.322550	.298481	.277755	.259720	.243885	.229969	.217377	.206173	.196067	.186905	.178562	25
26	.614722	.530011	.465820	.415498	.374988	.341676	.313799	.290128	.269778	.252095	.236586	.222878	.210670	.199730	.189870	.180938	.172809	26
27	.604799	.519902	.455905	.405936	.365839	.332951	.305489	.282212	.262230	.244899	.229703	.216289	.204355	.193669	.184045	.175333	.167408	27
28	.595109	.510110	.446356	.396768	.357097	.324637	.297587	.274698	.255078	.238075	.223196	.210068	.198398	.187957	.178560	.170058	.162328	28
29	.585645	.500621	.437155	.387970	.348734	.316704	.290064	.267557	.248292	.231615	.217037	.204185	.192770	.182564	.173385	.165084	.157542	29
30	.576401	.491424	.428283	.379521	.340727	.309128	.282893	.260763	.241844	.225484	.211198	.198614	.187445	.177465	.168496	.160387	.153024	30
31	.567371	.482505	.419724	.371400	.333054	.301885	.276061	.254290	.235709	.219659	.205655	.193330	.182398	.172637	.163867	.155945	.148754	31
32	.558548	.473854	.411463	.363589	.325695	.294954	.269516	.248117	.229866	.214116	.200387	.188311	.177609	.168057	.159481	.151737	.144710	32
33	.549929	.465459	.403483	.356072	.318632	.288316	.263258	.242224	.224295	.208837	.195373	.183540	.173058	.163709	.155318	.147745	.140671	33
34	.541506	.457310	.395773	.348833	.311847	.281952	.257288	.236591	.218976	.203803	.190596	.178996	.168728	.159574	.151361	.143953	.137236	34
35	.533275	.449397	.388319	.341856	.305324	.275846	.251559	.231203	.213894	.198997	.186039	.174666	.164603	.155637	.147597	.140347	.133776	35
36	.525230	.441710	.381108	.335129	.299050	.269984	.246067	.226043	.209033	.194404	.181688	.170534	.160670	.151885	.144011	.136913	.130481	36
37	.517367	.434242	.374131	.328638	.293009	.264350	.240798	.221099	.204379	.190011	.177530	.166587	.156915	.148305	.140590	.133639	.127342	37
38	.509681	.426983	.367375	.322371	.287190	.258932	.235737	.216356	.199920	.185804	.173550	.162813	.153287	.144885	.137325	.130514	.124347	38
39	.502166	.419926	.360832	.316318	.281581	.253719	.230874	.211803	.195642	.181773	.169740	.159201	.149894	.141616	.134203	.127529	.121486	39
40	.494819	.413063	.354492	.310468	.276171	.248698	.226196	.207428	.191536	.177906	.166087	.155740	.146607	.138486	.131217	.124674	.118752	40
41	.487635	.406386	.348346	.304812	.270950	.243860	.221694	.203223	.187592	.174194	.162583	.152422	.143457	.135488	.128358	.121941	.116135	41
42	.480610	.399890	.342385	.299339	.265909	.239195	.217359	.199176	.183800	.170628	.159218	.149238	.140436	.132614	.125617	.119322	.113627	42
43	.473739	.393567	.336602	.294043	.261038	.234694	.213180	.195280	.180152	.167200	.155985	.146180	.137535	.129856	.122988	.116811	.111224	43
44	.467019	.387411	.330990	.288914	.256329	.230349	.209148	.191525	.176640	.163901	.152876	.143241	.134748	.127206	.120464	.114400	.108918	44
45	.460445	.381416	.325541	.283945	.251774	.226162	.205262	.187906	.173256	.160725	.149884	.140414	.132069	.124660	.118039	.112085	.106703	45
46	.454014	.375576	.320249	.279129	.247367	.222095	.201508	.184414	.169995	.157665	.147003	.137692	.129491	.122211	.115706	.109859	.104574	46
47	.447720	.369887	.315107	.274459	.243100	.218173	.197882	.181044	.166847	.154714	.144227	.135071	.127008	.119853	.113462	.107718	.102827	47
48	.441565	.364342	.310109	.269929	.238967	.214377	.194376	.177788	.163809	.151868	.141549	.132544	.124615	.117582	.111301	.105656	.100556	48
49	.435540	.358938	.305250	.265534	.234962	.210704	.190986	.174642	.160875	.149120	.138966	.130107	.122309	.115394	.109218	.103670	.098658	49
50	.429644	.353668	.300525	.261266	.231080	.207146	.187705	.171600	.158040	.146466	.136472	.127755	.120084	.113282	.107210	.101755	.096829	50

Table VI (Continued).—Present or Discounted Value of Each 1.00 of Total Income Receivable in Equal Annual Amounts, Hoskold Sinking Fund Valuation Premises, 4% Sinking Fund Rate of Interest.

Year	25%	30%	35%	40%	45%	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%	100%
1	.800000	.769231	.740741	.714286	.689655	.666667	.645161	.625000	.606061	.588235	.571429	.555556	.540541	.526316	.512821	.500000
2	.675497	.632754	.596099	.561674	.531804	.504951	.480679	.458633	.438521	.420099	.403168	.387538	.373080	.359562	.347175	.335526
3	.584438	.537332	.497264	.462759	.432704	.406331	.382988	.362182	.343619	.326686	.311425	.297526	.284815	.273146	.262395	.252459
4	.514944	.466863	.426993	.393398	.364703	.339910	.318275	.299226	.282330	.267240	.253661	.241432	.230311	.220169	.210885	.202349
5	.460165	.412689	.374093	.342098	.315146	.292130	.272247	.254898	.239628	.226084	.213989	.203123	.193206	.184395	.176269	.168820
6	.415875	.369744	.332825	.302511	.277425	.256110	.237835	.221997	.208135	.196903	.185029	.176298	.166540	.158615	.151410	.144858
7	.379324	.334866	.299736	.271277	.247754	.227984	.211157	.196608	.183950	.172823	.162966	.154172	.145879	.139154	.132698	.126903
8	.348648	.305977	.272612	.245808	.222803	.205414	.189817	.176422	.164793	.154602	.145598	.137585	.130408	.125945	.118089	.112762
9	.322535	.281655	.249973	.224697	.204063	.186901	.172401	.159989	.149244	.139852	.131571	.124217	.117641	.111726	.106378	.101618
10	.300038	.260898	.230792	.206915	.187515	.171441	.157905	.146351	.136272	.127666	.120006	.113215	.107148	.101699	.096778	.092311
11	.280455	.242976	.214333	.191751	.173441	.158337	.145653	.134850	.125539	.117431	.110307	.103997	.098371	.093322	.088765	.084634
12	.263263	.227244	.200055	.178615	.161326	.147089	.135160	.125021	.116298	.108712	.102055	.096167	.090920	.086217	.081976	.078133
13	.248024	.213590	.187552	.167172	.150787	.137827	.126074	.116825	.108320	.101195	.094950	.089450	.084518	.080116	.076151	
14	.234446	.201395	.176511	.157100	.141535	.129777	.118128	.109106	.101365	.094649	.088768	.083675	.078955	.074820		
15	.222266	.190508	.166691	.148168	.133549	.121225	.111122	.102575	.095246	.088996	.083539	.078456	.074079			
16	.211277	.180739	.157900	.140191	.126654	.114807	.104897	.096716	.089822	.083800	.078536	.073893	.069768			
17	.201314	.171899	.149984	.133085	.119612	.108491	.099381	.091597	.084981	.079256	.074254	.069845				
18	.192238	.163884	.142819	.126552	.113612	.103075	.094325	.086942	.080633	.075177	.070413	.066219				
19	.183937	.156577	.136308	.120676	.108265	.098168	.089794	.082754	.076707	.071497	.066949					
20	.176316	.149888	.130360	.115319	.103398	.093706	.085678	.078916	.073144	.068159	.063810					
21	.169294	.143743	.124893	.110413	.098942	.089631	.081921	.075435	.069896	.065117						
22	.162804	.138076	.119870	.105906	.094855	.085893	.078478	.072242	.066924	.062335						
23	.156785	.132836	.115232	.101749	.091090	.082453	.075312	.069309	.064193							
24	.151193	.127974	.110938	.097904	.087611	.079276	.072390	.066604	.061675							
25	.145979	.123452	.106948	.094337	.084386	.076334	.069685	.064101	.059346							
26	.141108	.119236	.103234	.091019	.081388	.073601	.067174	.061779	.057186							
27	.136548	.115295	.099766	.087934	.078595	.071056	.064836	.059618								
28	.132269	.111603	.096522	.085031	.075986	.068680	.062655	.057603								
29	.128245	.108157	.093480	.082321	.073543	.066456	.060615	.055718								
30	.124457	.104878	.090622	.079777	.071251	.064371	.058703	.053952								
31	.120882	.101807	.087931	.077384	.069096	.062412	.056907	.052294								
32	.117504	.098908	.085395	.075129	.067067	.060568	.055217									
33	.114306	.096163	.082998	.073001	.065153	.058829	.053624									
34	.111276	.093574	.080732	.070989	.063344	.057186	.052119									
35	.108399	.091114	.078584	.069084	.061632	.055632	.050697									
36	.105664	.088779	.076547	.067277	.060010	.054160	.049349									
37	.103062	.086559	.074611	.065561	.058470	.052762	.048070									
38	.100583	.084445	.072770	.063930	.057006	.051436	.046856									
39	.098219	.082431	.071016	.062378	.055613	.050172										
40	.095961	.080509	.069344	.060898	.054286	.048969										
41	.093802	.078674	.067747	.059486	.053020	.047822										
42	.091737	.076919	.066222	.058137	.051812	.046727										
43	.089760	.075240	.064763	.056848	.050656	.045681										
44	.087864	.073631	.063366	.055614	.049551	.044680										
45	.086045	.072089	.062028	.054431	.048492	.043722										
46	.084299	.070609	.060744	.053298	.047478	.042804										
47	.082621	.069187	.059511	.052210	.046504	.041923										
48	.081007	.067821	.058327	.051165	.045569	.041077										
49	.079453	.066507	.057189	.050161	.044671	.040264										
50	.077957	.065242	.056093	.049194	.043807	.039483										

Table VII.—Present or Discounted Value of Each 1.00 of Total Income Receivable in Equal Annual Amounts, Hoskold Sinking Fund Valuation Premises, 5% Sinking Fund Rate of Interest.

Year	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%	Year
1	.95238	.94339	.93457	.92592	.91743	.90909	.90090	.89285	.88495	.87719	.86956	.86206	.85470	.84745	.84033	.83333	1
2	.92970	.91273	.89637	.88058	.86534	.85062	.83639	.82263	.80932	.79642	.78394	.77184	.76010	.74872	.73767	.72695	2
3	.90775	.88366	.86086	.83918	.81858	.79896	.78026	.76241	.74536	.72906	.71346	.69851	.68417	.67041	.65719	.64449	3
4	.88649	.85612	.82778	.80125	.77637	.75298	.73097	.71020	.69059	.67202	.65443	.63773	.62187	.60678	.59240	.57869	4
5	.86590	.82996	.79689	.76636	.73808	.71161	.68734	.66451	.64314	.62310	.60427	.58655	.56984	.55406	.53918	.52497	5
6	.84595	.80508	.76798	.73415	.70318	.67471	.64846	.62418	.60165	.58068	.56113	.54285	.52573	.50966	.49453	.48028	6
7	.82662	.78141	.74088	.70435	.67126	.64113	.61359	.58832	.56505	.54355	.52363	.50512	.48787	.47176	.45668	.44253	7
8	.80790	.75885	.71542	.67569	.64194	.61058	.58214	.55624	.53255	.51078	.49073	.47219	.45501	.43903	.42413	.41021	8
9	.78976	.73735	.69146	.65095	.61493	.58268	.55364	.52737	.50347	.48165	.46164	.44322	.42622	.41047	.39585	.38225	9
10	.77217	.71682	.66888	.62694	.58995	.55709	.52769	.50124	.47732	.45557	.43572	.41753	.40079	.38535	.37105	.35778	10
11	.75513	.69721	.64755	.60449	.56680	.53354	.50396	.47749	.45366	.43210	.41249	.39459	.37818	.36307	.34913	.33622	11
12	.73860	.67847	.62739	.58346	.54528	.51180	.48218	.45581	.43217	.41086	.39156	.37398	.35792	.34318	.32961	.31707	12
13	.72258	.66053	.60830	.56372	.52523	.49166	.46212	.43593	.41255	.39155	.37259	.35538	.33968	.32522	.31212	.29995	13
14	.70705	.64336	.59020	.54516	.50650	.47296	.44359	.41765	.39458	.37392	.35532	.33849	.32317	.30918	.29635	.28455	14
15	.69198	.62691	.57302	.52767	.48897	.45555	.42641	.40078	.37805	.35776	.33954	.32309	.30815	.29454	.28208	.27063	15
16	.67736	.61113	.55669	.51116	.47252	.43931	.41046	.38516	.36280	.34290	.32506	.30899	.29444	.28119	.26908	.25798	16
17	.66318	.59599	.54116	.49557	.45706	.42411	.39563	.37066	.34869	.32918	.31173	.29604	.28186	.26897	.25721	.24643	17
18	.64942	.58145	.52636	.48081	.44251	.40986	.38170	.35716	.33558	.31647	.29941	.28410	.27028	.25774	.24632	.23586	18
19	.63607	.56749	.51225	.46682	.42878	.39649	.36871	.34457	.32339	.30467	.28801	.27306	.25959	.24739	.23629	.22613	19
20	.62311	.55406	.49879	.45355	.41582	.38390	.35652	.33279	.31203	.29369	.27740	.26282	.24970	.23782	.22702	.21716	20
21	.61053	.54115	.48592	.44093	.40356	.37204	.34507	.32175	.30139	.28345	.26752	.25330	.24050	.22894	.21844	.20886	21
22	.59832	.52872	.47363	.42893	.39194	.36083	.33429	.31139	.29143	.27387	.25830	.24442	.23194	.22068	.21047	.20115	22
23	.58646	.51675	.46186	.41751	.38093	.35024	.32412	.30164	.28207	.26489	.24967	.23612	.22396	.21298	.20304	.19398	23
24	.57494	.50523	.45059	.40662	.37047	.34022	.31453	.29245	.27327	.25645	.24158	.22835	.21648	.20579	.19611	.18720	24
25	.56376	.49412	.43979	.39623	.36051	.33071	.30545	.28378	.26498	.24852	.23398	.22105	.20948	.19905	.18962	.18103	25
26	.55289	.48340	.42943	.38630	.35104	.32168	.29686	.27558	.25716	.24104	.22682	.21419	.20289	.19273	.18353	.17517	26
27	.54233	.47306	.41948	.37681	.34201	.31310	.28869	.26782	.24976	.23398	.22007	.20773	.19670	.18678	.17782	.16967	27
28	.53208	.46308	.40993	.36772	.33340	.30493	.28094	.26046	.24275	.22730	.21370	.20164	.19086	.18118	.17243	.16449	28
29	.52211	.45345	.40075	.35902	.32517	.29714	.27357	.25346	.23611	.22098	.20767	.19588	.18535	.17589	.16735	.15961	29
30	.51242	.44414	.39191	.35068	.31730	.28972	.26656	.24682	.22980	.21498	.20196	.19042	.18013	.17090	.16256	.15500	30
31	.50299	.43514	.38342	.34269	.30978	.28263	.25987	.24049	.22381	.20929	.19654	.18525	.17519	.16617	.15803	.15065	31
32	.49383	.42644	.37523	.33501	.30257	.27586	.25349	.23447	.21810	.20387	.19139	.18034	.17050	.16168	.15373	.14652	32
33	.48493	.41803	.36735	.32763	.29566	.26938	.24739	.22872	.21267	.19872	.18649	.17568	.16605	.15743	.14965	.14261	33
34	.47626	.40989	.35975	.32054	.28904	.26318	.24156	.22323	.20748	.19381	.18183	.17124	.16182	.15338	.14578	.13889	34
35	.46783	.40201	.35242	.31372	.28268	.25723	.23599	.21798	.20263	.18912	.17738	.16701	.15779	.14953	.14210	.13536	35
36	.45963	.39438	.34534	.30715	.27657	.25153	.23065	.21296	.19780	.18465	.17314	.16298	.15395	.14587	.13859	.13200	36
37	.45166	.38699	.33851	.30084	.27070	.24606	.22553	.20816	.19327	.18037	.16909	.15913	.15028	.14237	.13524	.12880	37
38	.44389	.37982	.33192	.29474	.26505	.24080	.22061	.20355	.18894	.17628	.16521	.15545	.14678	.13903	.13205	.12574	38
39	.43633	.37288	.32554	.28887	.25962	.23575	.21590	.19913	.18478	.17236	.16150	.15193	.14343	.13584	.12900	.12282	39
40	.42898	.36615	.31937	.28320	.25438	.23089	.21137	.19489	.18079	.16860	.15794	.14856	.14023	.13278	.12608	.12003	40
41	.42181	.35962	.31341	.27772	.24933	.22621	.20701	.19081	.17697	.16499	.15454	.14533	.13716	.12986	.12329	.11736	41
42	.41484	.35328	.30764	.27244	.24446	.22170	.20281	.18690	.17329	.16153	.15127	.14224	.13422	.12706	.12062	.11480	42
43	.40804	.34714	.30205	.26733	.23977	.21736	.19877	.18313	.16976	.15821	.14813	.13926	.13139	.12437	.11805	.11235	43
44	.40143	.34117	.29663	.26239	.23523	.21317	.19489	.17950	.16636	.15501	.14511	.13640	.12868	.12179	.11559	.11000	44
45	.39498	.33537	.29139	.25761	.23085	.20913	.19114	.17600	.16308	.15193	.14221	.13366	.12608	.11931	.11323	.10774	45
46	.38870	.32974	.28631	.25299	.22662	.20523	.18752	.17263	.15993	.14897	.13941	.13102	.12357	.11692	.11095	.10557	46
47	.38257	.32427	.28138	.24852	.22253	.20146	.18403	.16938	.15689	.14612	.13672	.12847	.12116	.11463	.10876	.10348	47
48	.37661	.31895	.27660	.24418	.21856	.19781	.18066	.16624	.15396	.14336	.13413	.12601	.11883	.11242	.10666	.10147	48
49	.37079	.31378	.27196	.23998	.21473	.19429	.17740	.16321	.15113	.14071	.13163	.12365	.11659	.11029	.10463	.09953	49
50	.36512	.30875	.26746	.23591	.21102	.19088	.17425	.16029	.14839	.13814	.12921	.12137	.11443	.10824	.10268	.09767	50

Table VIII.—Present or Discounted Value of Each 1.00 of Total Income Receivable in Equal Annual Amounts, Hoskold Sinking Fund Valuation Premises, 6% Sinking Fund Rate of Interest.

Year	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%	Year
1	.94340	.93458	.92593	.91743	.90909	.90090	.89286	.88496	.87719	.86957	.86207	.85470	.84746	.84034	.83333	1
2	.91670	.90019	.88427	.86891	.85406	.83972	.82585	.81243	.79944	.78686	.77467	.76285	.75139	.74026	.72946	2
3	.89100	.86781	.84579	.82486	.80494	.78595	.76785	.75057	.73404	.71822	.70307	.68855	.67461	.66123	.64837	3
4	.86628	.83726	.81013	.78470	.76082	.73835	.71717	.69717	.67825	.66034	.64335	.62721	.61186	.59724	.58331	4
5	.84247	.80842	.77701	.74795	.72099	.69590	.67250	.65063	.63013	.61088	.59277	.57571	.55960	.54437	.52995	5
6	.81955	.78114	.74617	.71420	.68485	.65782	.63284	.60969	.58817	.56813	.54940	.53187	.51542	.49996	.48540	6
7	.79748	.75532	.71739	.68309	.65191	.62346	.59739	.57341	.55128	.53081	.51179	.49408	.47757	.46212	.44764	7
8	.77622	.73084	.69047	.65433	.62178	.59232	.56552	.54104	.51859	.49794	.47886	.46119	.44478	.42950	.41523	8
9	.75574	.70761	.66525	.62767	.59411	.56395	.53671	.51198	.48943	.46878	.44980	.43230	.41611	.40109	.38712	9
10	.73601	.68555	.64157	.60289	.56861	.53802	.51055	.48575	.46325	.44274	.42397	.40672	.39083	.37613	.36249	10
11	.71699	.66457	.61930	.57980	.54504	.51421	.48668	.46195	.43961	.41934	.40085	.38392	.36836	.35402	.34075	11
12	.69865	.64461	.59833	.55825	.52320	.49229	.46483	.44027	.41818	.39820	.38004	.36346	.34827	.33430	.32141	12
13	.68097	.62559	.57854	.53807	.50290	.47204	.44474	.42044	.39865	.37901	.36121	.34501	.33020	.31661	.30409	13
14	.66393	.60746	.55985	.51916	.48398	.45327	.42622	.40222	.38078	.36151	.34410	.32828	.31385	.30064	.28850	14
15	.64748	.59016	.54217	.50139	.46632	.43584	.40909	.38544	.36437	.34549	.32847	.31304	.29900	.28617	.27439	15
16	.63162	.57365	.52542	.48468	.44980	.41960	.39320	.36993	.34926	.33077	.31415	.29911	.28545	.27298	.26156	16
17	.61631	.55786	.50954	.46892	.43429	.40444	.37842	.35555	.33528	.31720	.30097	.28632	.27303	.26092	.24984	17
18	.60153	.54277	.49446	.45405	.41974	.39026	.36464	.34218	.32233	.30465	.28882	.27454	.26161	.24985	.23910	18
19	.58727	.52832	.48012	.43999	.40604	.37696	.35177	.32973	.31029	.29301	.27756	.26365	.25108	.23965	.22921	19
20	.57350	.51449	.46649	.42668	.39313	.36447	.33971	.31810	.29907	.28219	.26712	.25357	.24173	.23022	.22009	20
21	.56019	.50123	.45350	.41406	.38094	.35272	.32840	.30721	.28859	.27210	.25739	.24419	.23228	.22148	.21164	21
22	.54734	.48852	.44111	.40209	.36941	.34165	.31776	.29700	.27878	.26267	.24832	.23546	.22386	.21336	.20379	22
23	.53493	.47633	.42929	.39072	.35850	.33119	.30775	.28741	.26959	.25385	.23984	.22730	.21601	.20579	.19649	23
24	.52293	.46462	.41801	.37990	.34815	.32131	.29830	.27837	.26094	.24556	.23190	.21967	.20867	.19872	.18967	24
25	.51133	.45338	.40722	.36959	.33833	.31195	.28938	.26986	.25280	.23777	.22443	.21251	.20179	.19210	.18350	25
26	.50012	.44257	.39690	.35978	.32900	.30308	.28094	.26181	.24513	.23044	.21741	.20578	.19533	.18589	.17732	26
27	.48928	.43218	.38702	.35041	.32012	.29465	.27294	.25421	.23788	.22352	.21080	.19945	.18926	.18006	.17171	27
28	.47879	.42219	.37756	.34146	.31166	.28665	.26535	.24700	.23102	.21699	.20456	.19348	.18353	.17456	.16643	28
29	.46865	.41257	.36849	.33291	.30360	.27903	.25814	.24016	.22453	.21080	.19866	.18784	.17813	.16938	.16145	29
30	.45883	.40331	.35978	.32473	.29590	.27178	.25129	.23367	.21837	.20494	.19307	.18250	.17303	.16449	.15675	30
31	.44933	.39439	.35142	.31690	.28855	.26486	.24476	.22750	.21251	.19938	.18777	.17744	.16819	.15986	.15231	31
32	.44013	.38579	.34340	.30940	.28153	.25826	.23855	.22163	.20695	.19410	.18275	.17265	.16361	.15547	.14810	32
33	.43122	.37750	.33568	.30221	.27480	.25195	.23261	.21603	.20165	.18907	.17797	.16810	.15926	.15131	.14411	33
34	.42259	.36950	.32826	.29530	.26836	.24592	.22695	.21069	.19660	.18428	.17342	.16376	.15513	.14735	.14032	34
35	.41424	.36178	.32112	.28868	.26219	.24015	.22153	.20559	.19179	.17972	.16909	.15964	.15119	.14359	.13672	35
36	.40614	.35433	.31425	.28231	.25626	.23462	.21636	.20071	.18719	.17537	.16496	.15571	.14744	.14001	.13329	36
37	.39829	.34713	.30762	.27619	.25058	.22932	.21138	.19606	.18279	.17121	.16103	.15196	.14387	.13660	.13003	37
38	.39068	.34018	.30124	.27030	.24512	.22423	.20663	.19159	.17858	.16724	.15724	.14838	.14046	.13334	.12691	38
39	.38331	.33346	.29508	.26463	.23987	.21935	.20207	.18731	.17455	.16343	.15364	.14495	.13720	.13023	.12393	39
40	.37616	.32696	.28915	.25917	.23483	.21466	.19769	.18320	.17069	.15978	.15018	.14167	.13408	.12725	.12109	40
41	.36922	.32068	.28341	.25391	.22997	.21015	.19348	.17926	.16699	.15629	.14688	.13853	.13109	.12440	.11837	41
42	.36249	.31459	.27788	.24884	.22529	.20582	.18944	.17548	.16343	.15294	.14370	.13553	.12823	.12167	.11576	42
43	.35596	.30871	.27253	.24394	.22078	.20164	.18555	.17184	.16002	.14972	.14066	.13264	.12548	.11906	.11326	43
44	.34962	.30301	.26736	.23922	.21644	.19762	.18181	.16834	.15673	.14662	.13774	.12987	.12285	.11655	.11086	44
45	.34346	.29748	.26236	.23466	.21225	.19374	.17820	.16498	.15357	.14365	.13493	.12720	.12031	.11414	.10856	45
46	.33749	.29213	.25753	.23025	.20820	.19000	.17473	.16173	.15053	.14078	.13222	.12464	.11788	.11182	.10635	46
47	.33168	.28695	.25285	.22599	.20429	.18640	.17138	.15861	.14760	.13803	.12962	.12218	.11554	.10959	.10422	47
48	.32604	.28192	.24832	.22187	.20052	.18291	.16815	.15559	.14478	.13537	.12711	.11980	.11329	.10745	.10218	48
49	.32056	.27705	.24393	.21789	.19687	.17955	.16503	.15268	.14206	.13281	.12470	.11752	.11112	.10538	.10021	49
50	.31524	.27232	.23968	.21403	.19334	.17630	.16202	.14988	.13943	.13034	.12237	.11531	.10903	.10339	.09831	50

Table IX.—Years Purchase Valuation Factors, Compound Interest Valuation Premises.

Year	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%	Year
1	.962	.952	.943	.935	.926	.917	.909	.901	.893	.885	.877	.870	.862	.855	.847	.840	.833	1
2	1.886	1.859	1.833	1.808	1.783	1.759	1.736	1.713	1.690	1.668	1.647	1.626	1.605	1.585	1.566	1.547	1.528	2
3	2.775	2.723	2.678	2.624	2.577	2.531	2.487	2.444	2.402	2.361	2.322	2.283	2.246	2.210	2.174	2.140	2.106	3
4	3.630	3.546	3.465	3.387	3.312	3.240	3.170	3.102	3.037	2.974	2.914	2.855	2.798	2.743	2.690	2.639	2.589	4
5	4.452	4.330	4.212	4.100	3.993	3.890	3.791	3.696	3.605	3.517	3.433	3.352	3.274	3.199	3.127	3.058	2.991	5
6	5.242	5.076	4.917	4.767	4.623	4.486	4.355	4.231	4.111	3.998	3.889	3.785	3.685	3.589	3.498	3.410	3.326	6
7	6.002	5.786	5.582	5.389	5.206	5.033	4.868	4.712	4.564	4.423	4.288	4.160	4.039	3.922	3.812	3.706	3.605	7
8	6.733	6.463	6.210	5.971	5.747	5.535	5.335	5.146	4.968	4.799	4.639	4.487	4.344	4.207	4.078	3.954	3.837	8
9	7.435	7.108	6.802	6.515	6.247	5.995	5.759	5.537	5.328	5.132	4.946	4.772	4.607	4.451	4.303	4.163	4.031	9
10	8.111	7.722	7.360	7.024	6.710	6.418	6.145	5.889	5.650	5.426	5.216	5.019	4.833	4.659	4.494	4.339	4.193	10
11	8.761	8.306	7.887	7.499	7.139	6.805	6.495	6.207	5.938	5.687	5.453	5.234	5.029	4.836	4.656	4.486	4.327	11
12	9.385	8.863	8.384	7.943	7.536	7.161	6.814	6.492	6.194	5.918	5.660	5.421	5.197	4.988	4.793	4.611	4.439	12
13	9.986	9.394	8.853	8.358	7.904	7.487	7.103	6.750	6.424	6.122	5.842	5.583	5.342	5.118	4.909	4.715	4.533	13
14	10.563	9.899	9.295	8.746	8.244	7.786	7.367	6.982	6.628	6.303	6.002	5.724	5.468	5.229	5.008	4.802	4.611	14
15	11.118	10.380	9.712	9.108	8.559	8.061	7.606	7.191	6.811	6.462	6.142	5.847	5.576	5.324	5.092	4.876	4.676	15
16	11.652	10.838	10.106	9.447	8.851	8.312	7.824	7.379	6.974	6.604	6.265	5.954	5.668	5.405	5.162	4.938	4.730	16
17	12.166	11.274	10.477	9.763	9.122	8.544	8.022	7.549	7.120	6.729	6.373	6.047	5.749	5.475	5.222	4.990	4.775	17
18	12.659	11.690	10.828	10.059	9.372	8.756	8.201	7.702	7.250	6.840	6.467	6.128	5.818	5.534	5.273	5.033	4.812	18
19	13.134	12.085	11.158	10.336	9.604	8.950	8.365	7.839	7.366	6.938	6.550	6.198	5.877	5.584	5.316	5.070	4.843	19
20	13.590	12.462	11.470	10.594	9.818	9.129	8.514	7.963	7.469	7.025	6.623	6.259	5.929	5.628	5.353	5.101	4.870	20
21	14.029	12.821	11.764	10.836	10.017	9.292	8.649	8.075	7.562	7.102	6.687	6.312	5.973	5.665	5.384	5.127	4.891	21
22	14.451	13.163	12.042	11.061	10.201	9.442	8.772	8.176	7.645	7.170	6.743	6.359	6.011	5.696	5.410	5.149	4.910	22
23	14.857	13.489	12.303	11.272	10.371	9.580	8.883	8.266	7.718	7.230	6.792	6.399	6.044	5.723	5.432	5.167	4.925	23
24	15.247	13.799	12.550	11.469	10.529	9.707	8.985	8.348	7.784	7.283	6.835	6.434	6.073	5.747	5.451	5.182	4.937	24
25	15.622	14.094	12.783	11.654	10.675	9.823	9.077	8.422	7.843	7.330	6.873	6.464	6.097	5.766	5.467	5.195	4.948	25
26	15.983	14.375	13.003	11.826	10.810	9.929	9.161	8.488	7.896	7.372	6.906	6.491	6.118	5.783	5.481	5.206	4.956	26
27	16.330	14.643	13.211	11.987	10.935	10.026	9.237	8.548	7.943	7.409	6.935	6.513	6.136	5.797	5.492	5.215	4.964	27
28	16.663	14.898	13.406	12.137	11.051	10.116	9.307	8.602	7.984	7.441	6.961	6.534	6.152	5.810	5.502	5.223	4.970	28
29	16.984	15.141	13.591	12.278	11.158	10.198	9.370	8.650	8.022	7.470	6.983	6.551	6.166	5.820	5.510	5.229	4.975	29
30	17.292	15.372	13.765	12.409	11.258	10.274	9.427	8.694	8.055	7.496	7.003	6.566	6.177	5.829	5.517	5.235	4.979	30
31	17.588	15.593	13.929	12.532	11.350	10.343	9.479	8.733	8.085	7.518	7.020	6.579	6.187	5.837	5.523	5.239	4.982	31
32	17.874	15.803	14.084	12.646	11.435	10.406	9.526	8.769	8.112	7.538	7.035	6.590	6.196	5.844	5.528	5.243	4.985	32
33	18.148	16.003	14.230	12.754	11.514	10.464	9.569	8.800	8.135	7.556	7.048	6.600	6.203	5.849	5.532	5.246	4.988	33
34	18.411	16.193	14.368	12.854	11.587	10.518	9.609	8.829	8.157	7.572	7.060	6.609	6.210	5.854	5.536	5.249	4.990	34
35	18.664	16.374	14.498	12.948	11.655	10.567	9.644	8.855	8.176	7.586	7.070	6.617	6.215	5.858	5.539	5.251	4.992	35
36	18.908	16.547	14.621	13.035	11.717	10.612	9.676	8.879	8.193	7.598	7.079	6.623	6.220	5.862	5.541	5.253	4.993	36
37	19.143	16.711	14.737	13.117	11.775	10.653	9.706	8.900	8.207	7.609	7.087	6.629	6.224	5.865	5.543	5.255	4.994	37
38	19.368	16.868	14.846	13.194	11.829	10.691	9.733	8.919	8.221	7.618	7.093	6.634	6.228	5.867	5.545	5.256	4.995	38
39	19.585	17.017	14.949	13.265	11.879	10.725	9.757	8.936	8.233	7.627	7.100	6.638	6.231	5.870	5.547	5.257	4.996	39
40	19.793	17.159	15.046	13.332	11.925	10.757	9.779	8.951	8.244	7.634	7.105	6.642	6.234	5.871	5.548	5.258	4.996	40
41	19.993	17.294	15.138	13.394	11.967	10.787	9.799	8.965	8.253	7.641	7.110	6.645	6.236	5.873	5.549	5.259	4.997	41
42	20.186	17.423	15.225	13.453	12.007	10.813	9.818	8.978	8.262	7.647	7.114	6.648	6.238	5.874	5.550	5.260	4.998	42
43	20.371	17.546	15.306	13.507	12.043	10.838	9.834	8.989	8.270	7.652	7.117	6.650	6.239	5.876	5.551	5.260	4.998	43
44	20.549	17.663	15.383	13.558	12.077	10.861	9.849	8.999	8.276	7.657	7.121	6.652	6.241	5.877	5.552	5.261	4.998	44
45	20.720	17.774	15.456	13.605	12.109	10.881	9.863	9.008	8.283	7.661	7.123	6.654	6.242	5.877	5.552	5.261	4.999	45
46	20.884	17.880	15.525	13.650	12.138	10.900	9.875	9.016	8.288	7.665	7.125	6.656	6.243	5.878	5.553	5.261	4.999	46
47	21.043	17.981	15.589	13.692	12.164	10.918	9.886	9.024	8.293	7.668	7.128	6.657	6.244	5.879	5.553	5.262	4.999	47
48	21.195	18.077	15.650	13.730	12.189	10.933	9.897	9.030	8.297	7.670	7.129	6.659	6.245	5.879	5.554	5.262	4.999	48
49	21.341	18.169	15.707	13.767	12.212	10.948	9.906	9.036	8.301	7.673	7.131	6.660	6.246	5.880	5.554	5.262	4.999	49
50	21.482	18.256	15.762	13.801	12.234	10.962	9.915	9.042	8.305	7.675	7.133	6.661	6.246	5.880	5.554	5.263	4.999	50

Table IX (Continued).—Years Purchase Valuation Factors, Compound Interest Valuation Premises.

Year	25%	30%	35%	40%	45%	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%	100%
1	0.80000	0.76923	0.74074	0.71429	0.68966	0.66667	0.64516	0.62500	0.60606	0.58824	0.57143	0.55556	0.54054	0.52632	0.51282	0.50000
2	1.44000	1.36095	1.28944	1.22449	1.16528	1.11111	1.06139	1.01563	0.97337	0.93426	0.89796	0.86420	0.83272	0.80332	0.77581	0.75000
3	1.95200	1.81611	1.69588	1.58892	1.49330	1.40741	1.32993	1.25977	1.19598	1.13780	1.08455	1.03557	0.99066	0.94912	0.91067	0.87500
4	2.36160	2.16624	1.99695	1.84923	1.71951	1.60494	1.50318	1.41235	1.33090	1.25753	1.19117	1.13093	1.07603	1.02585	0.97983	0.93750
5	2.68928	2.43557	2.21996	2.03516	1.87553	1.73663	1.61496	1.50772	1.41267	1.32796	1.25210	1.18385	1.12218	1.06624	1.01530	0.96875
6	2.95142	2.64275	2.38516	2.16797	1.98312	1.82442	1.68707	1.56733	1.46222	1.36939	1.28691	1.21325	1.14712	1.08749	1.03349	0.98438
7	3.16114	2.80211	2.50752	2.26284	2.05733	1.88294	1.73359	1.60458	1.49226	1.39376	1.30681	1.22958	1.16061	1.09868	1.04281	0.99219
8	3.32891	2.92470	2.59817	2.33060	2.10850	1.92196	1.76361	1.62786	1.51046	1.40809	1.31818	1.23866	1.16790	1.10457	1.04760	0.99609
9	3.46313	3.01900	2.66531	2.37900	2.14379	1.94798	1.78297	1.64241	1.52149	1.41652	1.32467	1.24370	1.17184	1.10767	1.05005	0.99805
10	3.57050	3.09154	2.71504	2.41357	2.16813	1.96532	1.79547	1.65151	1.52818	1.42149	1.32838	1.24650	1.17397	1.10930	1.05131	0.99902
11	3.65640	3.14734	2.75188	2.43826	2.18492	1.97688	1.80353	1.65719	1.53223	1.42440	1.33051	1.24806	1.17512	1.11016	1.05195	0.99951
12	3.72512	3.19026	2.77917	2.45590	2.19650	1.98459	1.80873	1.66075	1.53468	1.42612	1.33172	1.24892	1.17574	1.11061	1.05228	0.99976
13	3.78010	3.22328	2.79939	2.46850	2.20448	1.98972	1.81208	1.66297	1.53617	1.42713	1.33241	1.24940	1.17607	1.11085	1.05245	0.99988
14	3.82408	3.24867	2.81436	2.47750	2.20999	1.99315	1.81425	1.66435	1.53707	1.42772	1.33281	1.24967	1.17626	1.11097	1.05254	0.99994
15	3.85926	3.26821	2.82545	2.48393	2.21378	1.99543	1.81564	1.66522	1.53762	1.42807	1.33303	1.24981	1.17635	1.11104		
16	3.88741	3.28324	2.83367	2.48852	2.21640	1.99696	1.81654	1.66576	1.53795	1.42828	1.33316	1.24990	1.17641			
17	3.90993	3.29480	2.83975	2.49180	2.21821	1.99797	1.81713	1.66610	1.53815	1.42840	1.33323	1.24994	1.17644			
18	3.92794	3.30369	2.84426	2.49414	2.21945	1.99865	1.81750	1.66631	1.53827	1.42847	1.33328	1.24997				
19	3.94235	3.31053	2.84760	2.49582	2.22031	1.99910	1.81774	1.66645	1.53835	1.42851	1.33330					
20	3.95388	3.31579	2.85008	2.49701	2.22091	1.99940	1.81790	1.66653	1.53839	1.42854	1.33331					
21	3.96311	3.31984	2.85191	2.49787	2.22131	1.99960	1.81800	1.66658	1.53842	1.42855						
22	3.97049	3.32296	2.85327	2.49848	2.22160	1.99973	1.81806	1.66661	1.53844	1.42856						
23	3.97639	3.32535	2.85427	2.49891	2.22179	1.99982	1.81811	1.66663	1.53845							
24	3.98111	3.32719	2.85502	2.49922	2.22192	1.99988	1.81813	1.66665	1.53845							
25	3.98489	3.32861	2.85557	2.49944	2.22202	1.99992	1.81815	1.66665	1.53846							
26	3.98791	3.32970	2.85598	2.49960	2.22208	1.99995	1.81816	1.66666								
27	3.99033	3.33054	2.85628	2.49972	2.22212	1.99996	1.81817	1.66666								
28	3.99226	3.33118	2.85650	2.49980	2.22215	1.99998	1.81817	1.66666								
29	3.99381	3.33168	2.85667	2.49986	2.22218	1.99998	1.81818	1.66666								
30	3.99505	3.33206	2.85679	2.49990	2.22219	1.99999	1.81818									
31	3.99604	3.33235	2.85688	2.49993	2.22220	1.99999	1.81818									
32	3.99683	3.33258	2.85695	2.49995	2.22221	2.00000	1.81818									
33	3.99746	3.33275	2.85700	2.49996	2.22221	2.00000	1.81818									
34	3.99797	3.33289	2.85704	2.49997	2.22221	2.00000	1.81818									
35	3.99838	3.33299	2.85706	2.49998	2.22222	2.00000	1.81818									
36	3.99870	3.33307	2.85708	2.49999	2.22222	2.00000	1.81818									
37	3.99896	3.33313	2.85710	2.49999	2.22222	2.00000	1.81818									
38	3.99917	3.33318	2.85711	2.49999	2.22222	2.00000										
39	3.99934	3.33321	2.85712	2.50000	2.22222	2.00000										
40	3.99947	3.33324	2.85713	2.50000	2.22222	2.00000										
41	3.99957	3.33326	2.85713	2.50000	2.22222	2.00000										
42	3.99966	3.33328	2.85713	2.50000	2.22222	2.00000										
43	3.99973	3.33329	2.85714	2.50000	2.22222											
44	3.99978	3.33330	2.85714	2.50000	2.22222											
45	3.99983	3.33331	2.85714	2.50000	2.22222											
46	3.99986	3.33331	2.85714	2.50000	2.22222											
47	3.99989	3.33332	2.85714	2.50000	2.22222											
48	3.99991	3.33332	2.85714	2.50000	2.22222											
49	3.99993	3.33332	2.85714	2.50000	2.22222											
50	3.99994	3.33333	2.85714	2.50000	2.22222											

Table. X—Years Purchase Valuation Factors, Hoskold Sinking Fund Valuation Premises, 3% Sinking Fund Rate of Interest.

Year	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%	Year
1	0.97087	0.961538	0.952381	0.943396	0.934579	0.925926	0.917431	0.909091	0.900901	0.892857	0.884956	0.877193	0.869565	0.862069	0.854701	0.847458	0.840336	0.833333	1
2	1.91347	1.877543	1.842941	1.809692	1.777428	1.746387	1.716412	1.687448	1.659446	1.632358	1.606140	1.580751	1.556152	1.532307	1.509181	1.486744	1.464964	1.443812	2
3	2.82861	2.750802	2.677159	2.607355	2.541100	2.478128	2.418202	2.361106	2.306644	2.254687	2.204924	2.157356	2.111797	2.068122	2.026218	1.985978	1.947305	1.910109	3
4	3.71710	3.583882	3.459884	3.344179	3.235963	3.134530	3.039264	2.949617	2.865107	2.785305	2.709829	2.638334	2.570515	2.506096	2.444826	2.386481	2.330855	2.277764	4
5	4.57971	4.379156	4.195430	4.026501	3.870550	3.726413	3.592540	3.467953	3.351717	3.243020	3.141152	3.045488	2.955480	2.870636	2.790532	2.714776	2.643023	2.574966	5
6	5.41719	5.138812	4.887645	4.659887	4.452409	4.262620	4.088549	3.927768	3.779325	3.641694	3.513734	3.394462	3.283021	3.178655	3.080728	2.988658	2.901936	2.820099	6
7	6.23028	5.864884	5.539971	5.249169	4.987373	4.750450	4.535017	4.338275	4.157894	3.991915	3.838678	3.696771	3.564982	3.442266	3.327717	3.220546	3.120063	3.025660	7
8	7.01969	6.559253	6.155498	5.798567	5.480762	5.195982	4.939338	4.706848	4.495263	4.301882	4.124453	3.961060	3.810157	3.670812	3.540870	3.419813	3.306262	3.200447	8
9	7.78611	7.223666	6.737007	6.311782	5.937049	5.604318	5.306902	5.039463	4.797685	4.578045	4.377636	4.194035	4.025816	3.869462	3.725812	3.591317	3.466999	3.350826	9
10	8.53020	7.859750	7.287009	6.792071	6.360089	5.979770	5.642369	5.341010	5.070210	4.825544	4.603405	4.400818	4.218809	4.044808	3.887564	3.742088	3.607107	3.481524	10
11	9.25262	8.469018	7.807776	7.242312	6.753223	6.326013	5.949638	5.615534	5.316959	5.048830	4.805903	4.585527	4.384476	4.200314	4.030999	3.874806	3.730265	3.596120	11
12	9.95400	9.052880	8.301367	7.665062	7.119369	6.646193	6.232002	5.866407	5.541330	5.250389	4.988474	4.751450	4.535927	4.339109	4.158660	3.992420	3.839330	3.697376	12
13	10.63496	9.612654	8.769657	8.062595	7.461042	6.943020	6.492261	6.096465	5.746181	5.433910	5.153854	4.901251	4.672382	4.465697	4.272965	4.097864	3.936550	3.787455	13
14	11.29607	10.149570	9.214353	8.436943	7.780506	7.218844	6.732813	6.308100	5.933791	5.601414	5.304299	5.037115	4.795557	4.576107	4.375868	4.192409	4.023718	3.868078	14
15	11.93794	10.664781	9.637014	8.789928	8.079726	7.475709	6.955719	6.503364	6.106252	5.754847	5.441686	5.160849	4.907576	4.678000	4.468943	4.277771	4.102285	3.940629	15
16	12.56110	11.159363	10.039067	9.123184	8.360446	7.715404	7.162767	6.684007	6.265238	5.895849	5.567592	5.273960	5.009748	4.770745	4.553509	4.355196	4.173454	4.006236	16
17	13.16612	11.634329	10.421820	9.438189	8.624219	7.939499	7.355509	6.861543	6.412208	6.025820	5.683351	5.377717	5.103277	4.855488	4.630647	4.425708	4.238141	4.065825	17
18	13.75351	12.090627	10.786475	9.736274	8.872430	8.149382	7.535301	7.007281	6.548416	6.145953	5.790096	5.473193	5.189179	4.933187	4.701265	4.490170	4.297218	4.120166	18
19	14.32380	12.529149	11.134136	10.018647	9.106317	8.346278	7.703336	7.152366	6.674949	6.257279	5.888800	5.561306	5.268319	5.004657	4.766129	4.549303	4.351347	4.169900	19
20	14.87748	12.950733	11.465825	10.286404	9.326992	8.531280	7.860665	7.292753	6.792753	6.360688	5.980299	5.642841	5.341432	5.070590	4.825889	4.603719	4.401104	4.215672	20
21	15.41502	13.356168	11.782480	10.540543	9.535454	8.705359	8.008215	7.414450	6.902656	6.456954	6.065319	5.718476	5.409154	5.131579	4.881102	4.653938	4.446979	4.257642	21
22	15.93692	13.746197	12.084973	10.781974	9.732607	8.869385	8.146812	7.533105	7.005382	6.546757	6.144492	5.786799	5.472054	5.188138	4.932246	4.700410	4.489390	4.296504	22
23	16.44361	14.121521	12.374109	11.011530	9.919267	9.024138	8.277193	7.644447	7.101572	6.630688	6.218368	5.854324	5.530548	5.240708	4.979735	4.742520	4.528700	4.332495	23
24	16.93554	14.482801	12.650635	11.229972	10.096175	9.170323	8.400014	7.749090	7.191791	6.709274	6.287433	5.915500	5.585113	5.289678	5.023929	4.783604	4.565222	4.365908	24
25	17.41315	14.830663	12.915246	11.438000	10.264003	9.308571	8.518866	7.845777	7.275544	6.782978	6.352115	5.972721	5.636093	5.333586	5.065141	4.820953	4.599226	4.396998	25
26	17.87684	15.165695	13.168587	11.636257	10.423367	9.439458	8.625278	7.940397	7.356279	6.852211	6.412793	6.026337	5.683811	5.376128	5.103648	4.855824	4.630953	4.425987	26
27	18.32703	15.488457	13.411260	11.825334	10.574825	9.563502	8.728729	8.027988	7.431396	6.917341	6.469803	6.076655	6.728551	5.418159	5.139691	4.888441	4.660610	4.453069	27
28	18.76412	15.799477	13.643824	12.005777	10.718891	9.681176	8.826652	8.110745	7.502256	6.976697	6.523445	6.123952	5.770566	5.456739	5.173487	4.919003	4.688361	4.478416	28
29	19.18856	16.099256	13.866804	12.178092	10.856034	9.792912	8.919439	8.189024	7.569182	7.031571	6.573988	6.168473	5.810080	5.491046	5.206225	4.947686	4.714431	4.502179	29
30	19.60044	16.388268	14.080688	12.342744	10.986686	9.899102	9.007446	8.263148	7.632456	7.091230	6.621672	6.210437	5.847295	5.524274	5.236074	4.974648	4.738904	4.524493	30
31	20.00043	16.668964	14.285933	12.500167	11.111243	10.000107	9.090998	8.333408	7.692371	7.142912	6.666714	6.250042	5.882390	5.555589	5.258188	5.000027	4.761929	4.545477	31
32	20.38877	16.935771	14.482969	12.650763	11.230073	10.096256	9.170390	8.400071	7.749138	7.191833	6.709310	6.287465	5.915528	5.585138	5.287701	5.023949	4.783422	4.565238	32
33	20.76579	17.195094	14.672196	12.794903	11.343512	10.187852	9.245894	8.463379	7.802983	7.238188	6.749637	6.322866	5.946855	5.613054	5.314735	5.046525	4.804084	4.583873	33
34	21.13184	17.445320	14.853993	12.932936	11.451873	10.275173	9.317757	8.523553	7.854104	7.282156	6.787854	6.356392	5.976502	5.639459	5.339402	5.067860	4.823416	4.601468	34
35	21.48722	17.686815	15.028714	13.065186	11.555445	10.358477	9.386208	8.580797	7.902684	7.323899	6.824106	6.389173	6.004589	5.664461	5.360801	5.088041	4.841694	4.618100	35
36	21.83225	17.919928	15.196692	13.191952	11.654496	10.438000	9.451457	8.632296	7.948886	7.363865	6.856832	6.418329	6.031225	5.686159	5.382021	5.107155	4.858997	4.633839	36
37	22.16724	18.144992	15.358241	13.313516	11.749276	10.513962	9.513696	8.677220	7.992862	7.401288	6.891247	6.446970	6.056509	5.716643	5.402146	5.125271	4.875394	4.648749	37
38	22.49246	18.362323	15.513656	13.430149	11.840017	10.586566	9.573103	8.736727	8.034763	7.437193	6.922363	6.474196	6.080530	5.731995	5.421249	5.142463	4.890948	4.662889	38
39	22.80822	18.572223	15.663215	13.542069	11.926933	10.655999	9.629843	8.789363	8.074684	7.471393	6.951983	6.500097	6.103372	5.752289	5.439399	5.158791	4.905716	4.676309	39
40	23.11477	18.774979	15.807183	13.649571	12.010226	10.722437	9.684069	8.829057	8.112776	7.503993	6.980200	6.524758	6.125110	5.771593	5.456657	5.174313	4.919750	4.689060	40
41	23.41240	18.970855	15.945808	13.752811	12.090084	10.786043	9.735922	8.872138	8.149135	7.535090	7.007099	6.548256	6.145813	5.789972	5.473082	5.189079	4.932097	4.701183	41
42	23.70136	19.160145	16.079323	13.852013	12.166683	10.846967	9.785533	8.913317	8.183864	7.564773	7.032761	6.570662	6.165545	5.807482	5.488725	5.203139	4.945802	4.712720	42
43	23.98190	19.343067	16.207952	13.947369	12.240185	10.905350	9.833024	8.952703	8.217054	7.593123	7.057257	6.592040	6.184364	5.824176	5.503635	5.216556	4.957905	4.723708	43
44	24.25427	19.519871	16.331904	14.039059	12.310746	10.961325	9.878509	8.990393	8.248794	7.620218	7.080656	6.612451	6.202326	5.840103	5.517855	5.229309	4.969442	4.734179	44
45	24.51871	19.690785	16.451380	14.127252	12.378509	11.015014	9.922094	9.026478	8.279162	7.646127	7.103021	6.631952	6.219479	5.855309	5.531427	5.241497	4.980447	4.744167	45
46	24.77545	19.855029	16.566567	14.212108	12.443609	11.066533	9.963877	9.061046	8.308233	7.670915	7.124408	6.650595	6.235870	5.869835	5.544369	5.253134	4.990953	4.753596	46
47	25.02471	20.015810	16.677644	14.293779	12.506174	11.115989	10.003951	9.094174	8.336077	7.694645	7.144873	6.668422	6.251543	5.883720	5.556775	5.264252	5.000988	4.762800	47
48	25.26671	20.170329	16.784783	14.372406	12.566323	11.163484	10.042402	9.125938	8.362758	7.717373	7.164464	6.685485	6.266537	5.896999	5.568618	5.274880	5.010578	4.771498	48
49	25.50166	20.319777	16.888144	14.448124	12.624168	11.209111	10.079310	9.156408	8.386337	7.739151	7.183230	6.701823	6.280889	5.909707	5.579948	5.285045	5.019749	4.779615	49
50	25.72976	20.464338	16.987881	14.521060	12.679817	11.252962	10.114752	9.185647	8.412870	7.760029	7.201213	6.717473	6.294633	5.921873	5.590793	5.294773	5.028524	4.787770	50

Table X (Continued).—Years Purchase Valuation Factors, Hoskold Sinking Fund Valuation Premises, 3% Sinking Fund Rate of Interest.

Year	25%	30%	35%	40%	45%	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%	100%
1	.800	.769	.741	.714	.690	.667	.645	.625	.606	.588	.571	.556	.541	.526	.513	.500
2	1.347	1.262	1.187	1.120	1.061	1.007	.959	.915	.875	.839	.805	.774	.745	.718	.693	.670
3	1.744	1.604	1.485	1.382	1.293	1.214	1.145	1.083	1.027	.977	.932	.890	.852	.817	.785	.755
4	2.045	1.855	1.698	1.565	1.451	1.353	1.267	1.192	1.125	1.065	1.011	.962	.918	.878	.841	.807
5	2.281	2.048	1.858	1.700	1.567	1.453	1.354	1.268	1.193	1.126	1.066	1.012	.963	.919	.878	.842
6	2.472	2.200	1.982	1.803	1.654	1.528	1.419	1.325	1.243	1.170	1.105	1.048	.995	.948	.905	.866
7	2.628	2.323	2.081	1.885	1.723	1.586	1.469	1.369	1.281	1.204	1.136	1.075	1.020	.970	.925	.885
8	2.759	2.424	2.162	1.951	1.778	1.633	1.510	1.404	1.312	1.231	1.159	1.096	1.039	.988	.941	.899
9	2.870	2.510	2.230	2.006	1.823	1.671	1.542	1.432	1.336	1.252	1.179	1.113	1.054	1.002	.954	.910
10	2.965	2.582	2.287	2.052	1.861	1.703	1.569	1.455	1.356	1.270	1.194	1.127	1.067	1.013	.964	.920
11	3.048	2.645	2.336	2.092	1.894	1.730	1.592	1.475	1.373	1.285	1.208	1.139	1.077	1.022	.973	.928
12	3.120	2.699	2.378	2.126	1.921	1.753	1.612	1.492	1.388	1.298	1.219	1.149	1.086	1.030	.980	
13	3.184	2.747	2.415	2.155	1.945	1.773	1.629	1.506	1.401	1.309	1.228	1.157	1.094	1.037		
14	3.241	2.789	2.448	2.181	1.966	1.790	1.643	1.519	1.411	1.318	1.237	1.165	1.101			
15	3.292	2.827	2.477	2.204	1.985	1.806	1.656	1.530	1.421	1.327	1.244	1.171				
16	3.338	2.860	2.502	2.224	2.002	1.819	1.668	1.539	1.429	1.334	1.251					
17	3.379	2.891	2.526	2.242	2.016	1.832	1.678	1.548	1.437	1.341	1.256					
18	3.416	2.918	2.546	2.259	2.030	1.843	1.687	1.556	1.444	1.346						
19	3.450	2.943	2.565	2.274	2.042	1.852	1.695	1.563	1.450	1.352						
20	3.482	2.965	2.583	2.287	2.052	1.861	1.703	1.569	1.455							
21	3.510	2.986	2.598	2.300	2.062	1.870	1.710	1.575	1.460							
22	3.537	3.005	2.613	2.311	2.071	1.877	1.716	1.580								
23	3.561	3.023	2.626	2.321	2.080	1.884	1.722	1.585								
24	3.584	3.039	2.638	2.331	2.087	1.890	1.727	1.590								
25	3.605	3.054	2.650	2.340	2.095	1.896	1.732									
26	3.624	3.068	2.660	2.348	2.101	1.901	1.736									
27	3.642	3.081	2.670	2.355	2.107	1.906	1.740									
28	3.659	3.093	2.679	2.362	2.113	1.911	1.744									
29	3.675	3.104	2.687	2.369	2.118	1.915										
30	3.690	3.115	2.695	2.375	2.123	1.919										
31	3.704	3.125	2.703	2.381	2.128	1.923										
32	3.717	3.134	2.710	2.386	2.132	1.927										
33	3.729	3.143	2.716	2.391	2.136	1.930										
34	3.741	3.151	2.722	2.396	2.140											
35	3.752	3.159	2.728	2.401	2.143											
36	3.762	3.167	2.734	2.405	2.147											
37	3.772	3.173	2.739	2.409	2.150											
38	3.781	3.180	2.744	2.413	2.153											
39	3.790	3.186	2.748	2.416	2.156											
40	3.798	3.192	2.753	2.420	2.159											
41	3.806	3.198	2.757	2.423	2.161											
42	3.814	3.203	2.761	2.426	2.164											
43	3.821	3.208	2.765	2.429	2.166											
44	3.828	3.213	2.768	2.432	2.168											
45	3.835	3.218	2.772	2.434												
46	3.841	3.222	2.775	2.437												
47	3.847	3.226	2.778	2.439												
48	3.852	3.230	2.781	2.442												
49	3.858	3.234	2.784	2.444												

Table XI.—Years Purchase Valuation Factors, Hoskold Sinking Fund Valuation Premises, 4% Sinking Fund Rate of Interest.

Year	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%	Year
1	.962	.952	.943	.935	.926	.917	.909	.901	.893	.885	.877	.870	862	.855	.845	.840	.833	1
2	1,886	1,851	1,818	1,786	1,754	1,724	1,694	1,666	1,639	1,612	1,587	1,562	1,538	1,515	1,492	1,470	1,449	2
3	2,775	2,700	2,629	2,562	2,498	2,437	2,379	2,324	2,271	2,220	2,172	2,126	2,082	2,039	1,999	1,959	1,922	3
4	3,630	3,503	3,384	3,273	3,170	3,072	2,981	2,894	2,813	2,736	2,663	2,594	2,529	2,466	2,407	2,350	2,296	4
5	4,452	4,262	4,088	3,927	3,779	3,641	3,513	3,394	3,283	3,178	3,080	2,989	2,902	2,820	2,743	2,669	2,600	5
6	5,242	4,991	4,745	4,530	4,333	4,153	3,988	3,835	3,693	3,561	3,439	3,325	3,218	3,118	3,023	2,935	2,851	6
7	6,002	5,662	5,359	5,086	4,840	4,617	4,413	4,226	4,055	3,897	3,751	3,615	3,489	3,371	3,261	3,158	3,062	7
8	6,733	6,308	5,934	5,601	5,304	5,037	4,796	4,576	4,376	4,192	4,024	3,868	3,724	3,590	3,466	3,350	3,241	8
9	7,435	6,921	6,473	6,079	5,731	5,420	5,142	4,890	4,662	4,454	4,265	4,090	3,929	3,781	3,643	3,515	3,396	9
10	8,111	7,502	6,979	6,524	6,124	5,771	5,456	5,174	4,919	4,688	4,478	4,286	4,110	3,948	3,798	3,659	3,530	10
11	8,760	8,055	7,454	6,938	6,497	6,092	5,742	5,430	5,151	4,898	4,670	4,461	4,271	4,096	3,935	3,786	3,648	11
12	9,385	8,580	7,902	7,323	6,823	6,387	6,004	5,664	5,360	5,088	4,841	4,618	4,414	4,227	4,056	3,898	3,752	12
13	9,986	9,079	8,323	7,684	7,136	6,660	6,244	5,877	5,551	5,259	4,996	4,759	4,542	4,345	4,164	3,998	3,844	13
14	10,563	9,554	8,721	8,021	7,426	6,912	6,465	6,073	5,725	5,415	5,137	4,886	4,658	4,451	4,261	4,087	3,927	14
15	11,118	10,006	9,096	8,337	7,696	7,146	6,669	6,252	5,884	5,557	5,265	5,001	4,763	4,547	4,349	4,168	4,001	15
16	11,652	10,436	9,450	8,634	7,948	7,363	6,858	6,418	6,031	5,688	5,382	5,107	4,859	4,634	4,428	4,241	4,068	16
17	12,166	10,846	9,785	8,913	8,183	7,564	7,032	6,570	6,165	5,807	5,489	5,203	4,946	4,713	4,500	4,307	4,129	17
18	12,659	11,237	10,102	9,175	8,404	7,752	7,195	6,712	6,290	5,917	5,587	5,291	5,025	4,786	4,566	4,367	4,184	18
19	13,134	11,609	10,402	9,422	8,610	7,928	7,345	6,843	6,405	6,019	5,677	5,372	5,098	4,851	4,627	4,422	4,235	19
20	13,590	11,964	10,686	9,654	8,804	8,092	7,486	6,965	6,511	6,113	5,761	5,447	5,166	4,912	4,682	4,473	4,281	20
21	14,029	12,303	10,955	9,874	8,985	8,245	7,617	7,078	6,610	6,200	5,838	5,516	5,228	4,968	4,733	4,519	4,324	21
22	14,451	12,626	11,211	10,081	9,158	8,389	7,740	7,184	6,702	6,281	5,910	5,580	5,285	5,020	4,780	4,562	4,363	22
23	14,857	12,935	11,454	10,277	9,319	8,524	7,855	7,283	6,788	6,357	5,977	5,640	5,339	5,068	4,824	4,602	4,399	23
24	15,247	13,230	11,684	10,462	9,471	8,651	7,963	7,375	6,869	6,427	6,039	5,695	5,388	5,113	4,864	4,639	4,433	24
25	15,622	13,511	11,903	10,637	9,614	8,771	8,064	7,462	6,944	6,493	6,097	5,747	5,434	5,154	4,902	4,673	4,464	25
26	15,983	13,780	12,111	10,803	9,750	8,884	8,159	7,543	7,014	6,554	6,151	5,795	5,477	5,193	4,937	4,704	4,492	26
27	16,330	14,037	12,309	10,960	9,878	8,990	8,248	7,620	7,080	6,612	6,202	5,840	5,518	5,229	4,969	4,734	4,520	27
28	16,663	14,283	12,498	11,109	9,999	9,090	8,332	7,692	7,142	6,666	6,249	5,882	5,555	5,263	5,000	4,762	4,545	28
29	16,984	14,518	12,677	11,251	10,113	9,184	8,412	7,759	7,200	6,717	6,294	5,921	5,590	5,294	5,028	4,787	4,569	29
30	17,292	14,743	12,848	11,386	10,222	9,274	8,487	7,823	7,255	6,765	6,336	5,958	5,623	5,324	5,055	4,812	4,591	30
31	17,589	14,958	13,011	11,513	10,325	9,358	8,558	7,883	7,307	6,809	6,375	5,993	5,654	5,352	5,080	4,834	4,611	31
32	17,874	15,163	13,167	11,635	10,422	9,439	8,625	7,940	7,356	6,852	6,412	6,026	5,683	5,378	5,103	4,856	4,631	32
33	18,148	15,360	13,315	11,750	10,515	9,514	8,688	7,993	7,402	6,892	6,447	6,057	5,711	5,402	5,125	4,876	4,649	33
34	18,411	15,549	13,456	11,860	10,603	9,586	8,748	8,044	7,445	6,929	6,480	6,086	5,737	5,426	5,146	4,894	4,666	34
35	18,665	15,729	13,591	11,965	10,686	9,655	8,805	8,092	7,486	6,965	6,511	6,113	5,761	5,447	5,166	4,912	4,682	35
36	18,908	15,902	13,720	12,065	10,766	9,719	8,858	8,138	7,525	6,999	6,541	6,139	5,784	5,468	5,184	4,929	4,697	36
37	19,143	16,067	13,843	12,160	10,841	9,781	8,910	8,181	7,562	7,030	6,569	6,164	5,806	5,487	5,202	4,945	4,712	37
38	19,368	16,225	13,960	12,250	10,913	9,839	8,958	8,222	7,597	7,061	6,595	6,187	5,826	5,506	5,218	4,960	4,725	38
39	19,584	16,377	14,072	12,336	10,982	9,895	9,004	8,260	7,630	7,089	6,620	6,209	5,846	5,523	5,234	4,974	4,738	39
40	19,793	16,523	14,180	12,419	11,047	9,948	9,049	8,297	7,661	7,116	6,643	6,230	5,864	5,539	5,249	4,987	4,750	40
41	19,993	16,662	14,282	12,497	11,109	9,998	9,089	8,332	7,691	7,142	6,666	6,249	5,882	5,555	5,263	5,000	4,761	41
42	20,186	16,795	14,380	12,572	11,168	10,046	9,129	8,365	7,720	7,166	6,687	6,268	5,898	5,570	5,276	5,012	4,772	42
43	20,371	16,923	14,474	12,644	11,225	10,092	9,167	8,397	7,747	7,190	6,707	6,286	5,914	5,584	5,288	5,023	4,783	43
44	20,549	17,046	14,564	12,712	11,278	10,135	9,203	8,427	7,772	7,212	6,727	6,303	5,929	5,597	5,300	5,034	4,792	44
45	20,720	17,164	14,649	12,778	11,330	10,177	9,237	8,456	7,797	7,233	6,745	6,319	5,943	5,610	5,312	5,044	4,802	45
46	20,885	17,276	14,731	12,840	11,379	10,216	9,269	8,483	7,820	7,253	6,762	6,334	5,957	5,622	5,322	5,054	4,809	46
47	21,043	17,385	14,810	12,900	11,426	10,254	9,300	8,509	7,842	7,272	6,779	6,348	5,969	5,633	5,333	5,063	4,819	47
48	21,195	17,488	14,885	12,957	11,470	10,288	9,330	8,534	7,863	7,290	6,794	6,362	5,982	5,644	5,342	5,072	4,827	48
49	21,341	17,588	14,957	13,011	11,513	10,324	9,358	8,557	7,883	7,307	6,809	6,375	5,993	5,654	5,352	5,080	4,834	49
50	21,482	17,683	15,026	13,063	11,554	10,357	9,385	8,580	7,902	7,323	6,824	6,388	6,004	5,664	5,361	5,088	4,841	50

Table XI (Continued).—Years Purchase Valuation Factors, Hoskold Sinking Fund Valuation Premises, 4% Sinking Fund Rate of Interest.

Year	25%	30%	35%	40%	45%	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%	100%
1	.800	.769	.741	.714	.690	.667	.645	.625	.606	.588	.571	.556	.541	.526	.513	.500
2	1.351	1.266	1.190	1.123	1.064	1.010	.961	.917	.877	.840	.806	.775	.746	.719	.694	.671
3	1.753	1.612	1.492	1.388	1.298	1.219	1.149	1.087	1.031	.980	..934	.893	.854	.819	.787	.757
4	2.060	1.867	1.708	1.574	1.459	1.360	1.273	1.197	1.129	1.069	1.015	.966	.921	.881	.844	.809
5	2.301	2.063	1.870	1.710	1.578	1.461	1.361	1.274	1.198	1.130	1.070	1.016	.967	.922	.881	.844
6	2.495	2.218	1.997	1.816	1.665	1.537	1.427	1.332	1.249	1.175	1.110	1.052	.999	.952	.908	.869
7	2.655	2.344	2.098	1.899	1.734	1.596	1.478	1.376	1.288	1.210	1.141	1.079	1.024	.974	.929	.888
8	2.789	2.448	2.181	1.966	1.790	1.643	1.519	1.411	1.318	1.237	1.165	1.101	1.043	.992	.945	.902
9	2.903	2.535	2.250	2.022	1.837	1.682	1.552	1.440	1.343	1.259	1.184	1.118	1.059	1.006	.957	.914
10	3.000	2.609	2.308	2.069	1.875	1.714	1.579	1.464	1.364	1.277	1.200	1.132	1.071	1.017	.968	.923
11	3.085	2.673	2.358	2.109	1.908	1.742	1.602	1.485	1.381	1.292	1.213	1.144	1.082	1.027	.976	.931
12	3.159	2.728	2.401	2.143	1.936	1.765	1.622	1.500	1.396	1.305	1.225	1.154	1.091	1.035	.984	.938
13	3.224	2.777	2.438	2.173	1.960	1.785	1.639	1.515	1.408	1.316	1.234	1.163	1.099	1.042	.990	
14	3.282	2.820	2.471	2.199	1.981	1.803	1.654	1.527	1.419	1.325	1.243	1.170	1.105	1.047		
15	3.334	2.858	2.500	2.223	2.000	1.818	1.667	1.539	1.429	1.333	1.250	1.177	1.111			
16	3.380	2.892	2.526	2.243	2.017	1.832	1.678	1.548	1.437	1.341	1.257	1.182	1.116			
17	3.422	2.922	2.550	2.261	2.032	1.844	1.689	1.557	1.445	1.347	1.262	1.187				
18	3.460	2.950	2.571	2.278	2.045	1.855	1.698	1.565	1.451	1.353	1.267	1.192				
19	3.495	2.975	2.590	2.293	2.057	1.865	1.706	1.572	1.457	1.358	1.272					
20	3.526	2.998	2.607	2.306	2.068	1.874	1.714	1.578	1.463	1.363	1.276					
21	3.555	3.019	2.623	2.319	2.078	1.882	1.720	1.584	1.468	1.367						
22	3.582	3.038	2.637	2.330	2.087	1.890	1.727	1.589	1.472	1.371						
23	3.606	3.055	2.650	2.340	2.095	1.896	1.732	1.594	1.476							
24	3.629	3.071	2.663	2.350	2.103	1.903	1.737	1.598	1.480							
25	3.649	3.086	2.674	2.358	2.110	1.908	1.742	1.603	1.484							
26	3.669	3.100	2.684	2.366	2.116	1.914	1.747	1.606	1.487							
27	3.687	3.113	2.694	2.374	2.122	1.919	1.751	1.610								
28	3.704	3.125	2.703	2.381	2.128	1.923	1.754	1.613								
29	3.719	3.136	2.711	2.387	2.133	1.927	1.758	1.616								
30	3.734	3.146	2.719	2.393	2.138	1.931	1.761	1.619								
31	3.747	3.156	2.726	2.399	2.142	1.935	1.764	1.621								
32	3.760	3.165	2.733	2.404	2.146	1.938	1.767									
33	3.772	3.174	2.739	2.409	2.150	1.941	1.770									
34	3.783	3.182	2.745	2.414	2.154	1.944	1.772									
35	3.794	3.189	2.750	2.418	2.157	1.947	1.774									
36	3.804	3.196	2.756	2.422	2.160	1.950	1.777									
37	3.813	3.203	2.761	2.426	2.163	1.952	1.779									
38	3.822	3.209	2.765	2.429	2.166	1.955	1.781									
39	3.831	3.215	2.770	2.433	2.169	1.957										
40	3.838	3.220	2.774	2.436	2.171	1.959										
41	3.846	3.226	2.778	2.439	2.174	1.961										
42	3.853	3.231	2.781	2.442	2.176	1.963										
43	3.860	3.235	2.785	2.444	2.178	1.964										
44	3.866	3.240	2.788	2.447	2.180	1.966										
45	3.872	3.244	2.791	2.449	2.182	1.967										
46	3.878	3.248	2.794	2.452	2.184	1.969										
47	3.883	3.252	2.797	2.454	2.186	1.970										
48	3.888	3.255	2.800	2.456	2.187	1.972										
49	3.893	3.259	2.802	2.458	2.189	1.973										
50	3.898	3.262	2.805	2.460	2.190	1.974										

Table XII.—Years Purchase Valuation Factors, Hoskold Sinking Fund Valuation Premises, 5% Sinking Fund Rate of Interest.

Year	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%	Year
1	.952	.943	.935	.925	.917	.909	.901	.893	.885	.877	.870	.862	.855	.847	.840	.833	1
2	1.859	1.825	1.793	1.761	1.731	1.701	1.673	1.645	1.619	1.593	1.568	1.544	1.520	1.497	1.475	1.454	2
3	2.723	2.651	2.583	2.518	2.456	2.397	2.341	2.287	2.236	2.187	2.140	2.096	2.053	2.011	1.972	1.933	3
4	3.546	3.424	3.311	3.205	3.105	3.012	2.924	2.841	2.762	2.688	2.618	2.551	2.487	2.427	2.370	2.315	4
5	4.330	4.150	3.984	3.832	3.690	3.559	3.437	3.323	3.216	3.116	3.021	2.933	2.849	2.770	2.696	2.625	5
6	5.076	4.830	4.608	4.405	4.219	4.048	3.891	3.745	3.610	3.484	3.367	3.257	3.154	3.058	2.967	2.882	6
7	5.786	5.470	5.186	4.930	4.699	4.488	4.295	4.118	3.955	3.805	3.665	3.536	3.415	3.302	3.197	3.098	7
8	6.463	6.071	5.723	5.414	5.136	4.885	4.657	4.450	4.260	4.086	3.926	3.778	3.640	3.512	3.393	3.282	8
9	7.108	6.636	6.223	5.859	5.534	5.244	4.983	4.746	4.531	4.335	4.155	3.989	3.836	3.694	3.563	3.440	9
10	7.722	7.168	6.689	6.269	5.900	5.571	5.277	5.012	4.773	4.556	4.357	4.175	4.008	3.854	3.711	3.578	10
11	8.306	7.669	7.125	6.649	6.235	5.869	5.544	5.252	4.990	4.753	4.537	4.340	4.160	3.994	3.840	3.698	11
12	8.863	8.142	7.529	7.002	6.543	6.142	5.786	5.470	5.186	4.930	4.699	4.488	4.295	4.118	3.955	3.805	12
13	9.394	8.587	7.908	7.328	6.828	6.392	6.008	5.667	5.363	5.090	4.844	4.620	4.416	4.229	4.058	3.899	13
14	9.899	9.007	8.263	7.632	7.091	6.621	6.210	5.847	5.524	5.235	4.974	4.739	4.524	4.329	4.149	3.984	14
15	10.380	9.404	8.595	7.915	7.335	6.833	6.396	6.012	5.671	5.366	5.093	4.846	4.622	4.418	4.231	4.059	15
16	10.838	9.778	8.907	8.179	7.560	7.029	6.567	6.163	5.805	5.486	5.201	4.944	4.711	4.499	4.305	4.128	16
17	11.274	10.132	9.200	8.425	7.770	7.210	6.726	6.301	5.928	5.596	5.299	5.033	4.792	4.572	4.373	4.189	17
18	11.690	10.466	9.474	8.655	7.965	7.377	6.871	6.429	6.040	5.696	5.389	5.114	4.865	4.639	4.434	4.245	18
19	12.085	10.782	9.733	8.870	8.147	7.533	7.005	6.547	6.144	5.789	5.472	5.188	4.932	4.700	4.490	4.296	19
20	12.462	11.081	9.976	9.071	8.316	7.678	7.130	6.656	6.241	5.874	5.548	5.256	4.994	4.756	4.540	4.343	20
21	12.821	11.364	10.204	9.260	8.475	7.813	7.246	6.757	6.329	5.952	5.618	5.319	5.051	4.808	4.587	4.385	21
22	13.163	11.632	10.420	9.436	8.623	7.938	7.354	6.851	6.411	6.025	5.683	5.377	5.103	4.855	4.630	4.425	22
23	13.489	11.885	10.623	9.603	8.761	8.056	7.455	6.938	6.488	6.092	5.742	5.431	5.151	4.899	4.670	4.462	23
24	13.799	12.126	10.814	9.759	8.891	8.165	7.549	7.019	6.558	6.155	5.798	5.480	5.196	4.939	4.707	4.493	24
25	14.094	12.353	10.995	9.906	9.013	8.268	7.636	7.095	6.625	6.213	5.850	5.526	5.237	4.976	4.741	4.526	25
26	14.375	12.568	11.165	10.044	9.127	8.364	7.718	7.165	6.686	6.267	5.897	5.569	5.275	5.011	4.772	4.554	26
27	14.643	12.773	11.326	10.174	9.234	8.454	7.795	7.231	6.744	6.317	5.942	5.609	5.311	5.043	4.801	4.581	27
28	14.898	12.966	11.478	10.296	9.335	8.538	7.866	7.293	6.797	6.364	5.984	5.646	5.344	5.073	4.828	4.606	28
29	15.141	13.150	11.622	10.412	9.430	8.617	7.934	7.350	6.847	6.408	6.022	5.681	5.375	5.101	4.853	4.629	29
30	15.373	13.324	11.757	10.520	9.519	8.692	7.997	7.405	6.894	6.449	6.059	5.713	5.404	5.127	4.877	4.650	30
31	15.593	13.489	11.886	10.623	9.603	8.762	8.056	7.455	6.938	6.488	6.093	5.743	5.431	5.151	4.899	4.670	31
32	15.803	13.646	12.007	10.720	9.682	8.828	8.112	7.503	6.979	6.524	6.124	5.771	5.456	5.174	4.919	4.689	32
33	16.003	13.795	12.123	10.812	9.757	8.890	8.164	7.548	7.018	6.558	6.154	5.797	5.480	5.195	4.938	4.706	33
34	16.193	13.936	12.232	10.898	9.827	8.948	8.213	7.590	7.054	6.590	6.182	5.822	5.502	5.215	4.957	4.722	34
35	16.374	14.070	12.335	10.980	9.894	9.003	8.260	7.629	7.089	6.619	6.208	5.845	5.523	5.234	4.974	4.738	35
36	16.547	14.198	12.432	11.057	9.957	9.055	8.303	7.667	7.121	6.647	6.233	5.867	5.542	5.251	4.989	4.752	36
37	16.711	14.319	12.525	11.131	10.016	9.104	8.345	7.702	7.151	6.674	6.256	5.888	5.560	5.268	5.004	4.766	37
38	16.868	14.433	12.613	11.200	10.072	9.150	8.383	7.735	7.180	6.699	6.278	5.907	5.578	5.283	5.018	4.778	38
39	17.017	14.542	12.696	11.266	10.125	9.194	8.420	7.766	7.206	6.722	6.299	5.925	5.594	5.298	5.031	4.790	39
40	17.159	14.646	12.775	11.328	10.175	9.236	8.455	7.796	7.232	6.744	6.318	5.942	5.609	5.311	5.043	4.801	40
41	17.294	14.744	12.850	11.387	10.223	9.275	8.487	7.823	7.256	6.765	6.336	5.959	5.624	5.324	5.055	4.812	41
42	17.423	14.838	12.921	11.442	10.267	9.311	8.518	7.850	7.278	6.784	6.353	5.974	5.637	5.337	5.066	4.822	42
43	17.546	14.927	12.988	11.495	10.310	9.346	8.547	7.875	7.300	6.803	6.370	5.988	5.650	5.348	5.076	4.831	43
44	17.663	15.011	13.052	11.545	10.350	9.379	8.575	7.898	7.320	6.820	6.385	6.002	5.662	5.359	5.086	4.840	44
45	17.774	15.092	13.113	11.592	10.388	9.411	8.601	7.920	7.339	6.837	6.399	6.015	5.674	5.369	5.095	4.848	45
46	17.880	15.168	13.170	11.638	10.425	9.441	8.626	7.941	7.357	6.853	6.413	6.027	5.684	5.378	5.104	4.856	46
47	17.981	15.241	13.225	11.680	10.459	9.469	8.649	7.961	7.374	6.868	6.426	6.038	5.695	5.388	5.112	4.864	47
48	18.077	15.310	13.277	11.721	10.491	9.495	8.672	7.980	7.390	6.881	6.438	6.048	5.704	5.396	5.120	4.871	48
49	18.169	15.375	13.326	11.759	10.522	9.520	8.693	7.997	7.405	6.895	6.450	6.059	5.713	5.404	5.127	4.877	49
50	18.256	15.438	13.373	11.796	10.551	9.544	8.713	8.015	7.420	6.907	6.461	6.069	5.722	5.412	5.134	4.884	50

Table XIII.—Years Purchase Valuation Factors, Hoskold Sinking Fund Valuation Premises, 6% Sinking Fund Rate of Interest.

Year	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%	Year
1	.943	.935	.926	.917	.909	.901	.893	.885	.877	.870	.862	.855	.847	.840	.833	1
2	1.833	1.800	1.769	1.738	1.708	1.679	1.652	1.625	1.599	1.574	1.549	1.526	1.503	1.481	1.459	2
3	2.673	2.603	2.537	2.475	2.415	2.358	2.304	2.252	2.202	2.155	2.109	2.066	2.024	1.984	1.945	3
4	3.465	3.349	3.241	3.139	3.043	2.953	2.869	2.789	2.713	2.641	2.573	2.509	2.447	2.389	2.333	4
5	4.212	4.042	3.885	3.740	3.605	3.480	3.363	3.253	3.151	3.054	2.964	2.879	2.798	2.722	2.650	5
6	4.917	4.687	4.477	4.286	4.109	3.947	3.797	3.658	3.529	3.409	3.296	3.191	3.093	3.000	2.912	6
7	5.582	5.287	5.022	4.782	4.563	4.364	4.182	4.014	3.859	3.716	3.583	3.459	3.343	3.235	3.133	7
8	6.210	5.847	5.524	5.235	4.974	4.739	4.524	4.328	4.149	3.984	3.831	3.690	3.558	3.436	3.322	8
9	6.802	6.368	5.987	5.647	5.346	5.076	4.830	4.608	4.405	4.219	4.048	3.891	3.745	3.610	3.484	9
10	7.360	6.856	6.416	6.029	5.686	5.360	5.106	4.868	4.633	4.427	4.240	4.067	3.908	3.761	3.625	10
11	7.887	7.310	6.812	6.378	5.995	5.656	5.353	5.081	4.836	4.613	4.409	4.223	4.052	3.894	3.748	11
12	8.384	7.735	7.180	6.699	6.278	5.907	5.578	5.283	5.018	4.778	4.560	4.362	4.179	4.012	3.857	12
13	8.853	8.133	7.521	6.995	6.538	6.137	5.782	5.466	5.182	4.927	4.696	4.485	4.293	4.116	3.953	13
14	9.295	8.504	7.838	7.268	6.776	6.346	5.967	5.631	5.331	5.061	4.817	4.596	4.394	4.209	4.039	14
15	9.712	8.852	8.133	7.521	6.995	6.538	6.136	5.782	5.466	5.182	4.927	4.696	4.485	4.293	4.116	15
16	10.105	9.178	8.407	7.755	7.197	6.714	6.291	5.919	5.588	5.292	5.026	4.786	4.567	4.368	4.185	16
17	10.477	9.484	8.662	7.972	7.383	6.875	6.433	6.044	5.700	5.392	5.116	4.867	4.642	4.436	4.247	17
18	10.828	9.770	8.900	8.173	7.555	7.025	6.564	6.159	5.802	5.484	5.199	4.942	4.709	4.497	4.304	18
19	11.158	10.038	9.122	8.360	7.715	7.162	6.684	6.265	5.896	5.567	5.274	5.010	4.771	4.553	4.355	19
20	11.470	10.290	9.330	8.534	7.863	7.289	6.794	6.362	5.981	5.644	5.342	5.071	4.826	4.604	4.402	20
21	11.764	10.526	9.524	8.695	8.000	7.407	6.896	6.451	6.060	5.714	5.405	5.128	4.878	4.651	4.444	21
22	12.041	10.747	9.704	8.846	8.127	7.516	6.991	6.534	6.133	5.779	5.463	5.180	4.925	4.694	4.483	22
23	12.303	10.956	9.874	8.987	8.246	7.617	7.078	6.610	6.201	5.839	5.516	5.228	4.968	4.733	4.519	23
24	12.550	11.151	10.032	9.118	8.356	7.711	7.159	6.681	6.263	5.893	5.566	5.272	5.008	4.769	4.552	24
25	12.783	11.335	10.181	9.240	8.458	7.799	7.235	6.747	6.320	5.944	5.611	5.313	5.045	4.803	4.583	25
26	13.003	11.507	10.319	9.354	8.554	7.880	7.304	6.807	6.373	5.991	5.653	5.350	5.079	4.833	4.610	26
27	13.211	11.669	10.450	9.461	8.643	7.956	7.369	6.864	6.423	6.035	5.692	5.385	5.110	4.862	4.636	27
28	13.406	11.821	10.572	9.561	8.726	8.026	7.430	6.916	6.469	6.076	5.728	5.417	5.140	4.888	4.660	28
29	13.591	11.965	10.686	9.654	8.804	8.092	7.486	6.965	6.511	6.113	5.761	5.447	5.166	4.912	4.682	29
30	13.765	12.099	10.793	9.742	8.877	8.153	7.539	7.010	6.551	6.148	5.792	5.475	5.191	4.935	4.703	30
31	13.929	12.225	10.894	9.824	8.945	8.211	7.588	7.053	6.588	6.181	5.821	5.501	5.214	4.956	4.722	31
32	14.084	12.345	10.989	9.901	9.009	8.264	7.634	7.092	6.622	6.211	5.848	5.526	5.236	4.975	4.739	32
33	14.230	12.458	11.077	9.973	9.068	8.314	7.676	7.129	6.654	6.239	5.873	5.547	5.256	4.993	4.756	33
34	14.368	12.563	11.161	10.040	9.124	8.361	7.716	7.163	6.684	6.266	5.896	5.568	5.274	5.010	4.771	34
35	14.498	12.662	11.239	10.104	9.177	8.405	7.754	7.196	6.713	6.290	5.918	5.587	5.292	5.026	4.785	35
36	14.621	12.756	11.313	10.163	9.225	8.446	7.789	7.226	6.739	6.313	5.939	5.606	5.308	5.040	4.798	36
37	14.737	12.844	11.382	10.219	9.271	8.485	7.821	7.254	6.763	6.335	5.958	5.623	5.323	5.054	4.811	37
38	14.846	12.927	11.447	10.271	9.315	8.521	7.852	7.280	6.786	6.355	5.975	5.638	5.337	5.067	4.823	38
39	14.949	13.005	11.508	10.321	9.356	8.555	7.881	7.305	6.807	6.374	5.992	5.653	5.351	5.079	4.833	39
40	15.046	13.078	11.566	10.367	9.393	8.586	7.908	7.328	6.828	6.391	6.007	5.667	5.363	5.090	4.844	40
41	15.138	13.148	11.620	10.410	9.429	8.616	7.933	7.350	6.847	6.408	6.022	5.680	5.375	5.100	4.853	41
42	15.225	13.213	11.671	10.451	9.462	8.644	7.956	7.370	6.864	6.423	6.035	5.692	5.386	5.110	4.862	42
43	15.306	13.275	11.719	10.489	9.494	8.671	7.979	7.389	6.881	6.438	6.048	5.704	5.396	5.120	4.870	43
44	15.383	13.332	11.764	10.526	9.523	8.695	8.000	7.407	6.896	6.451	6.061	5.714	5.405	5.128	4.878	44
45	15.456	13.387	11.806	10.560	9.551	8.718	8.019	7.424	6.910	6.464	6.072	5.724	5.414	5.136	4.885	45
46	15.525	13.438	11.846	10.592	9.577	8.740	8.038	7.440	6.924	6.476	6.082	5.733	5.422	5.144	4.892	46
47	15.589	13.487	11.884	10.622	9.602	8.761	8.055	7.455	6.937	6.487	6.092	5.742	5.430	5.151	4.898	47
48	15.650	13.532	11.919	10.650	9.625	8.780	8.071	7.468	6.949	6.498	6.101	5.750	5.439	5.158	4.905	48
49	15.707	13.575	11.953	10.677	9.647	8.798	8.086	7.481	6.961	6.508	6.110	5.758	5.445	5.164	4.910	49
50	15.762	13.616	11.984	10.702	9.667	8.815	8.101	7.494	6.972	6.517	6.119	5.766	5.452	5.170	4.916	50

Table XIV.—Factors of Safety at Speculative Rates of Interest for Valuing 1.00 Due at a Given Future Time, Assuming 3% to be a Safe Rate of Interest.

Year	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%	Year
1	1.00	.990	.981	.972	.963	.954	.945	.936	.928	.920	.912	.904	.896	.888	.880	.873	.866	.858	1
2	1.00	.981	.962	.944	.927	.910	.893	.878	.861	.846	.831	.816	.802	.788	.775	.762	.749	.737	2
3	1.00	.971	.944	.917	.892	.867	.844	.821	.799	.778	.757	.738	.718	.700	.682	.665	.648	.632	3
4	1.00	.962	.926	.892	.859	.827	.797	.769	.741	.715	.690	.666	.644	.622	.601	.581	.561	.543	4
5	1.00	.953	.908	.866	.827	.789	.753	.720	.688	.658	.629	.602	.576	.552	.529	.507	.486	.466	5
6	1.00	.944	.891	.842	.796	.752	.712	.674	.638	.605	.574	.544	.516	.490	.465	.442	.420	.400	6
7	1.00	.935	.874	.818	.766	.718	.673	.631	.592	.556	.523	.492	.462	.435	.410	.386	.364	.343	7
8	1.00	.926	.857	.795	.737	.684	.636	.591	.550	.512	.477	.444	.414	.386	.361	.337	.315	.295	8
9	1.00	.917	.841	.772	.710	.653	.601	.553	.510	.471	.434	.401	.371	.343	.318	.294	.273	.253	9
10	1.00	.908	.825	.750	.683	.622	.568	.518	.473	.433	.396	.363	.332	.306	.280	.257	.236	.217	10
11	1.00	.899	.809	.729	.658	.594	.536	.485	.439	.398	.361	.328	.298	.271	.246	.224	.204	.186	11
12	1.00	.891	.794	.709	.633	.566	.507	.454	.408	.366	.329	.296	.266	.240	.217	.196	.177	.160	12
13	1.00	.882	.779	.689	.609	.540	.479	.425	.378	.337	.300	.267	.239	.213	.191	.171	.153	.137	13
14	1.00	.873	.764	.669	.587	.515	.453	.398	.351	.310	.273	.242	.214	.189	.168	.149	.132	.118	14
15	1.00	.865	.749	.650	.565	.491	.428	.373	.326	.285	.249	.218	.191	.168	.148	.130	.115	.101	15
16	1.00	.857	.735	.632	.544	.468	.404	.349	.302	.262	.227	.197	.171	.149	.130	.114	.099	.087	16
17	1.00	.849	.721	.614	.523	.447	.382	.327	.280	.241	.207	.178	.154	.133	.115	.099	.086	.074	17
18	1.00	.840	.707	.596	.504	.426	.361	.306	.260	.221	.189	.161	.138	.118	.101	.087	.074	.064	18
19	1.00	.832	.694	.580	.485	.406	.341	.287	.241	.204	.172	.145	.123	.105	.089	.076	.064	.055	19
20	1.00	.824	.681	.563	.467	.387	.322	.268	.224	.187	.157	.131	.110	.093	.078	.066	.056	.047	20
21	1.00	.816	.668	.547	.449	.370	.305	.251	.208	.172	.143	.119	.099	.082	.069	.058	.048	.040	21
22	1.00	.809	.655	.532	.432	.352	.288	.235	.193	.158	.130	.107	.089	.073	.061	.050	.042	.035	22
23	1.00	.801	.643	.517	.416	.336	.272	.220	.179	.146	.119	.097	.079	.065	.053	.044	.036	.030	23
24	1.00	.793	.630	.502	.401	.321	.257	.206	.166	.134	.108	.088	.071	.058	.047	.038	.031	.026	24
25	1.00	.785	.618	.488	.386	.306	.243	.193	.154	.123	.099	.079	.064	.051	.041	.033	.027	.022	25
26	1.00	.778	.607	.474	.371	.292	.229	.181	.143	.113	.090	.071	.057	.045	.036	.029	.023	.019	26
27	1.00	.770	.595	.461	.357	.278	.217	.169	.133	.104	.083	.065	.051	.040	.032	.025	.020	.016	27
28	1.00	.763	.584	.448	.344	.265	.205	.159	.123	.096	.075	.058	.046	.036	.028	.022	.018	.014	28
29	1.00	.756	.573	.435	.331	.253	.194	.149	.114	.088	.068	.053	.041	.032	.025	.019	.015	.012	29
30	1.00	.748	.562	.423	.319	.241	.183	.139	.106	.081	.062	.048	.037	.028	.022	.017	.013	.010	30
31	1.00	.741	.551	.411	.307	.230	.173	.130	.098	.075	.057	.043	.033	.025	.019	.015	.011	.009	31
32	1.00	.734	.540	.399	.295	.219	.163	.122	.091	.069	.052	.039	.029	.022	.017	.013	.010	.008	32
33	1.00	.727	.530	.388	.284	.209	.154	.114	.085	.063	.047	.035	.026	.020	.015	.011	.009	.006	33
34	1.00	.720	.520	.377	.274	.200	.146	.107	.079	.058	.043	.032	.024	.018	.013	.010	.007	.006	34
35	1.00	.713	.510	.366	.264	.190	.138	.100	.073	.053	.039	.029	.021	.016	.012	.009	.006	.005	35
36	1.00	.706	.500	.356	.254	.182	.130	.094	.068	.049	.036	.026	.019	.014	.010	.007	.006	.004	36
37	1.00	.699	.491	.345	.244	.173	.123	.088	.063	.045	.032	.023	.017	.012	.009	.007	.005	.004	37
38	1.00	.693	.482	.336	.235	.165	.116	.082	.058	.041	.030	.021	.015	.011	.008	.006	.004	.003	38
39	1.00	.686	.472	.326	.226	.157	.110	.077	.054	.038	.027	.019	.014	.010	.007	.005	.004	.003	39
40	1.00	.679	.463	.317	.218	.150	.104	.072	.050	.035	.025	.017	.012	.009	.006	.004	.003	.002	40
41	1.00	.673	.455	.308	.210	.143	.098	.067	.047	.032	.022	.016	.011	.008	.005	.004	.003	.002	41
42	1.00	.666	.446	.299	.202	.137	.093	.063	.043	.030	.020	.014	.010	.007	.005	.003	.002	.002	42
43	1.00	.660	.437	.291	.194	.130	.088	.059	.040	.027	.019	.013	.009	.006	.004	.003	.002	.001	43
44	1.00	.654	.429	.283	.187	.124	.083	.055	.037	.025	.017	.012	.008	.005	.004	.003	.002	.001	44
45	1.00	.647	.421	.275	.180	.118	.078	.052	.035	.023	.015	.010	.007	.005	.003	.002	.002	.001	45
46	1.00	.641	.413	.267	.173	.113	.074	.049	.032	.021	.014	.009	.006	.004	.003	.002	.001	.001	46
47	1.00	.635	.405	.259	.167	.108	.070	.045	.030	.020	.013	.008	.006	.004	.003	.002	.001	.001	47
48	1.00	.629	.397	.252	.161	.103	.066	.043	.028	.018	.012	.008	.005	.003	.002	.001	.001	.001	48
49	1.00	.623	.390	.245	.155	.098	.062	.040	.026	.016	.011	.007	.005	.003	.002	.001	.001	.001	49
50	1.00	.617	.382	.238	.149	.093	.059	.037	.024	.015	.010	.006	.004	.003	.002	.001	.001	.001	50

Table XIV (Continued).—Factors of Safety at Speculative Rates of Interest for Valuing 1.00 Due at a Given Future Time, Assuming 3% to be a Safe Rate of Interest.

Year	25%	30%	35%	40%	45%	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%	100%
1	.824	.792	.763	.736	.710	.687	.665	.644	.624	.606	.589	.572	.557	.542	.528	.515
2	.679	.628	.582	.541	.505	.472	.442	.414	.390	.367	.346	.327	.310	.294	.279	.265
3	.559	.497	.444	.398	.358	.324	.293	.267	.243	.222	.204	.187	.173	.159	.147	.137
4	.461	.394	.339	.293	.255	.222	.195	.172	.152	.135	.120	.107	.096	.086	.078	.070
5	.380	.312	.259	.216	.181	.153	.130	.111	.095	.082	.071	.061	.054	.047	.041	.036
6	.313	.247	.197	.159	.128	.105	.086	.071	.059	.049	.042	.035	.030	.025	.022	.019
7	.258	.196	.150	.117	.091	.072	.057	.046	.037	.030	.024	.020	.017	.014	.011	.010
8	.213	.155	.115	.086	.065	.049	.038	.029	.023	.018	.014	.011	.009	.007	.006	.005
9	.175	.123	.088	.063	.046	.034	.025	.019	.014	.011	.008	.007	.005	.004	.003	.003
10	.144	.097	.067	.046	.033	.023	.017	.012	.009	.007	.005	.004	.003	.002	.002	.001
11	.119	.077	.051	.034	.023	.016	.011	.008	.006	.004	.003	.002	.002	.001	.001	.001
12	.098	.061	.039	.025	.017	.011	.007	.005	.004	.002	.002	.001	.001	.001	.000	.000
13	.081	.048	.030	.019	.012	.008	.005	.003	.002	.001	.001	.001	.000	.000		
14	.067	.038	.023	.014	.008	.005	.003	.002	.001	.001	.001	.000				
15	.055	.030	.017	.010	.006	.004	.002	.001	.001	.001	.000					
16	.045	.024	.013	.007	.004	.002	.001	.001	.001	.000						
17	.037	.019	.010	.005	.003	.002	.001	.001	.000							
18	.031	.015	.008	.004	.002	.001	.001	.000								
19	.025	.012	.006	.003	.002	.001	.000									
20	.021	.010	.004	.002	.001	.001										
21	.017	.008	.003	.002	.001	.000										
22	.014	.006	.003	.001	.001											
23	.012	.005	.002	.001	.000											
24	.010	.004	.002	.001												
25	.008	.003	.001	.000												
26	.007	.002	.001													
27	.005	.002	.001													
28	.004	.001	.001													
29	.004	.001	.000													
30	.003	.001														
31	.002	.001														
32	.002	.001														
33	.002	.000														
34	.001															
35	.001															
36	.001															
37	.001															
38	.001															
39	.001															
40	.000															

Table XV.—Factors of Safety at Speculative Rates of Interest for Valuing 1.00 Due at a Given Future Time, Assuming 4% to be a Safe Rate of Interest.

Year	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%	Year
1	1.00	.990	.981	.972	.963	.954	.945	.937	.929	.920	.912	.904	.897	.889	.881	.874	.867	1
2	1.00	.981	.963	.945	.927	.910	.894	.878	.862	.847	.832	.818	.804	.790	.777	.764	.751	2
3	1.00	.972	.944	.918	.893	.869	.845	.822	.801	.780	.759	.740	.721	.702	.688	.668	.651	3
4	1.00	.962	.927	.892	.860	.829	.799	.771	.743	.717	.693	.669	.646	.624	.603	.583	.564	4
5	1.00	.953	.909	.867	.828	.791	.755	.722	.690	.660	.632	.605	.579	.555	.532	.510	.489	5
6	1.00	.944	.892	.843	.797	.754	.714	.676	.641	.608	.576	.547	.519	.493	.469	.446	.424	6
7	1.00	.935	.875	.819	.768	.720	.675	.634	.595	.559	.526	.495	.466	.438	.413	.389	.367	7
8	1.00	.926	.859	.797	.739	.687	.638	.594	.553	.515	.480	.447	.417	.390	.364	.340	.318	8
9	1.00	.917	.843	.774	.712	.655	.604	.556	.513	.474	.438	.405	.374	.346	.321	.297	.276	9
10	1.00	.909	.827	.752	.686	.625	.571	.521	.477	.436	.399	.366	.336	.308	.283	.260	.239	10
11	1.00	.900	.811	.731	.660	.597	.540	.488	.443	.401	.364	.331	.301	.274	.249	.227	.207	11
12	1.00	.892	.796	.711	.636	.569	.510	.458	.411	.369	.332	.299	.270	.243	.220	.199	.180	12
13	1.00	.883	.781	.691	.612	.543	.482	.429	.382	.340	.303	.271	.242	.216	.194	.174	.156	13
14	1.00	.875	.766	.672	.590	.518	.456	.402	.354	.313	.277	.245	.217	.192	.171	.152	.135	14
15	1.00	.866	.751	.653	.568	.494	.431	.376	.329	.288	.252	.221	.194	.171	.150	.133	.117	15
16	1.00	.858	.737	.634	.547	.472	.408	.353	.306	.265	.230	.200	.174	.152	.133	.116	.101	16
17	1.00	.850	.723	.617	.526	.450	.385	.330	.284	.244	.210	.181	.156	.135	.117	.101	.088	17
18	1.00	.842	.710	.599	.507	.429	.364	.310	.263	.224	.192	.164	.140	.120	.103	.088	.076	18
19	1.00	.834	.696	.583	.488	.410	.344	.290	.245	.207	.175	.148	.126	.107	.091	.077	.066	19
20	1.00	.826	.683	.566	.470	.391	.326	.272	.227	.190	.159	.134	.113	.095	.080	.068	.057	20
21	1.00	.818	.670	.550	.453	.373	.308	.255	.211	.174	.144	.120	.100	.083	.069	.058	.049	21
22	1.00	.810	.658	.535	.436	.356	.291	.239	.196	.161	.133	.109	.091	.075	.062	.052	.043	22
23	1.00	.802	.645	.520	.420	.340	.275	.224	.182	.148	.121	.099	.081	.067	.056	.045	.037	23
24	1.00	.795	.633	.505	.404	.324	.260	.209	.169	.136	.110	.090	.073	.059	.048	.039	.032	24
25	1.00	.787	.621	.491	.389	.309	.246	.196	.157	.126	.101	.081	.065	.053	.043	.034	.028	25
26	1.00	.780	.609	.477	.375	.295	.233	.184	.146	.116	.092	.073	.058	.047	.037	.030	.024	26
27	1.00	.772	.598	.464	.361	.281	.220	.172	.135	.106	.084	.066	.052	.042	.033	.026	.021	27
28	1.00	.765	.587	.451	.348	.269	.208	.161	.126	.098	.076	.060	.047	.037	.029	.023	.018	28
29	1.00	.758	.576	.438	.335	.256	.197	.151	.117	.090	.070	.054	.042	.033	.026	.020	.016	29
30	1.00	.750	.565	.426	.322	.244	.186	.142	.108	.083	.064	.049	.038	.029	.023	.018	.014	30
31	1.00	.743	.554	.414	.310	.233	.176	.133	.101	.076	.058	.044	.034	.026	.020	.015	.012	31
32	1.00	.736	.544	.403	.299	.223	.166	.124	.093	.070	.053	.040	.030	.023	.018	.013	.010	32
33	1.00	.729	.533	.391	.288	.212	.157	.117	.087	.065	.048	.036	.027	.021	.015	.012	.009	33
34	1.00	.722	.523	.380	.277	.203	.149	.109	.080	.059	.044	.033	.024	.018	.014	.010	.008	34
35	1.00	.715	.513	.370	.267	.193	.140	.102	.075	.055	.040	.030	.022	.016	.012	.009	.007	35
36	1.00	.709	.504	.359	.257	.184	.133	.096	.069	.050	.037	.027	.020	.014	.011	.008	.006	36
37	1.00	.702	.494	.349	.247	.176	.126	.090	.064	.046	.033	.024	.018	.013	.009	.007	.005	37
38	1.00	.695	.485	.339	.238	.168	.119	.084	.060	.043	.031	.022	.016	.011	.008	.006	.004	38
39	1.00	.689	.476	.330	.229	.160	.112	.079	.056	.039	.028	.020	.014	.010	.007	.005	.004	39
40	1.00	.682	.467	.321	.221	.153	.106	.074	.052	.036	.025	.018	.013	.009	.006	.005	.003	40
41	1.00	.676	.458	.312	.213	.146	.100	.069	.048	.033	.023	.016	.011	.009	.006	.004	.003	41
42	1.00	.669	.449	.303	.205	.139	.095	.065	.044	.031	.021	.015	.010	.007	.005	.003	.002	42
43	1.00	.663	.441	.294	.197	.133	.090	.061	.041	.028	.019	.013	.009	.006	.004	.003	.002	43
44	1.00	.656	.433	.286	.190	.127	.085	.057	.038	.026	.018	.012	.008	.006	.004	.003	.002	44
45	1.00	.650	.424	.278	.183	.121	.080	.053	.036	.024	.016	.011	.007	.005	.003	.002	.002	45
46	1.00	.644	.416	.270	.176	.115	.076	.050	.033	.022	.015	.010	.007	.004	.003	.002	.001	46
47	1.00	.638	.408	.263	.170	.110	.072	.047	.031	.020	.013	.009	.006	.004	.003	.002	.001	47
48	1.00	.632	.401	.255	.163	.105	.068	.044	.029	.019-	.012	.008	.005	.004	.002	.002	.001	48
49	1.00	.626	.393	.248	.157	.100	.064	.041	.026	.017	.011	.007	.005	.003	.002	.001	.001	49
50	1.00	.620	.386	.241	.152	.096	.061	.039	.025	.016-	.010	.007	.004	.003	.002	.001	.001	50

Table XV (Continued).—Factors of Safety at Speculative Rates of Interest for Valuing 1.00 Due at a Given Future Time, Assuming 4% to be a Safe Rate of Interest.

Year	25%	30%	35%	40%	45%	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%	100%
1	.832	.800	.770	.743	.717	.693	.671	.650	.630	.612	.594	.578	.562	.547	.533	.520
2	.679	.627	.582	.541	.504	.471	.441	.414	.389	.367	.346	.327	.310	.294	.279	.265
3	.553	.492	.439	.394	.355	.320	.290	.264	.241	.220	.202	.185	.171	.158	.146	.135
4	.451	.386	.332	.287	.249	.218	.191	.168	.149	.132	.117	.105	.094	.085	.076	.069
5	.368	.302	.250	.209	.175	.148	.126	.107	.092	.079	.068	.059	.052	.045	.040	.035
6	.300	.237	.189	.152	.123	.100	.083	.068	.057	.047	.040	.034	.029	.024	.021	.018
7	.245	.186	.143	.111	.087	.068	.054	.043	.035	.028	.023	.019	.016	.013	.011	.009
8	.199	.146	.108	.081	.061	.046	.036	.028	.022	.017	.014	.011	.009	.007	.006	.005
9	.162	.114	.081	.059	.043	.031	.023	.018	.013	.010	.008	.006	.005	.004	.003	.002
10	.132	.089	.061	.043	.030	.021	.015	.011	.008	.006	.005	.003	.003	.002	.002	.001
11	.108	.070	.046	.031	.021	.015	.010	.007	.005	.004	.003	.002	.001	.001	.001	.001
12	.088	.055	.035	.023	.015	.010	.007	.005	.003	.002	.002	.001	.001	.001	.000	
13	.072	.043	.026	.016	.010	.007	.004	.003	.002	.001	.001	.001	.000			
14	.058	.034	.020	.012	.007	.005	.003	.002	.001	.001	.001	.000				
15	.047	.026	.015	.009	.005	.003	.002	.001	.001	.000	.000					
16	.039	.021	.011	.006	.004	.002	.001	.001	.000							
17	.031	.016	.009	.005	.003	.001	.001	.000								
18	.026	.013	.006	.003	.002	.001	.001									
19	.021	.010	.005	.002	.001	.001	.000									
20	.017	.008	.004	.002	.001	.000										
21	.014	.006	.003	.001	.001											
22	.011	.005	.002	.001	.000											
23	.009	.004	.002	.001												
24	.007	.003	.001	.000												
25	.006	.002	.001													
26	.005	.002	.001													
27	.004	.001	.000													
28	.003	.001														
29	.003	.001														
30	.002	.001														
31	.002	.001														
32	.001	.000														
33	.001															
34	.001															
35	.001															
36	.001															
37	.001															
38	.000															

Table XVI.—Factors of Safety at Speculative Rates of Interest for Valuing 1.00 Due at a Given Future Time, Assuming 5% to be a Safe Rate of Interest.

Year	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%	Year
1	1.000	.991	.981	.972	.963	.955	.946	.937	.929	.921	.913	.905	.897	.890	.882	.875	1
2	1.000	.981	.963	.945	.928	.911	.895	.879	.863	.848	.834	.819	.805	.792	.779	.766	2
3	1.000	.972	.945	.919	.894	.870	.846	.824	.802	.781	.761	.742	.723	.705	.687	.670	3
4	1.000	.963	.927	.893	.861	.830	.801	.772	.745	.720	.695	.671	.649	.627	.606	.586	4
5	1.000	.954	.910	.869	.829	.792	.757	.724	.693	.663	.635	.608	.582	.558	.535	.513	5
6	1.000	.945	.893	.844	.799	.756	.716	.679	.644	.611	.579	.550	.522	.496	.472	.448	6
7	1.000	.936	.876	.821	.770	.722	.678	.637	.598	.562	.529	.498	.469	.442	.416	.393	7
8	1.000	.927	.860	.798	.741	.689	.641	.597	.556	.518	.483	.451	.421	.393	.367	.344	8
9	1.000	.918	.844	.776	.714	.658	.606	.559	.516	.477	.441	.408	.378	.350	.324	.301	9
10	1.000	.910	.828	.754	.688	.628	.574	.524	.480	.439	.403	.369	.339	.311	.286	.263	10
11	1.000	.901	.813	.734	.663	.599	.543	.492	.446	.405	.368	.334	.304	.277	.252	.230	11
12	1.000	.892	.797	.713	.638	.572	.513	.461	.414	.373	.336	.303	.273	.246	.223	.201	12
13	1.000	.884	.782	.693	.615	.546	.486	.432	.385	.343	.306	.274	.245	.219	.196	.176	13
14	1.000	.876	.768	.674	.592	.521	.459	.405	.358	.316	.280	.248	.220	.195	.173	.154	14
15	1.000	.867	.753	.655	.571	.498	.436	.380	.332	.291	.255	.224	.197	.174	.153	.133	15
16	1.000	.859	.739	.637	.550	.475	.411	.356	.309	.268	.233	.203	.177	.154	.135	.118	16
17	1.000	.851	.726	.619	.530	.453	.389	.334	.287	.247	.213	.184	.159	.137	.119	.103	17
18	1.000	.843	.712	.602	.510	.433	.368	.313	.267	.228	.194	.166	.143	.122	.105	.090	18
19	1.000	.835	.699	.586	.491	.413	.348	.293	.248	.210	.178	.151	.128	.109	.093	.079	19
20	1.000	.827	.686	.569	.473	.394	.329	.275	.230	.193	.162	.136	.115	.097	.082	.069	20
21	1.000	.820	.673	.553	.456	.376	.311	.258	.214	.178	.148	.123	.103	.086	.072	.061	21
22	1.000	.812	.660	.538	.439	.359	.294	.242	.199	.164	.135	.112	.092	.077	.064	.053	22
23	1.000	.804	.648	.523	.423	.343	.279	.227	.185	.151	.123	.101	.083	.068	.056	.046	23
24	1.000	.797	.636	.509	.408	.327	.264	.212	.172	.139	.113	.092	.074	.061	.050	.041	24
25	1.000	.789	.624	.494	.393	.313	.249	.199	.160	.128	.103	.083	.067	.054	.044	.035	25
26	1.000	.782	.612	.481	.378	.298	.236	.187	.148	.118	.094	.075	.060	.048	.039	.031	26
27	1.000	.774	.601	.467	.364	.286	.223	.175	.138	.109	.086	.068	.054	.043	.034	.027	27
28	1.000	.767	.590	.454	.351	.272	.211	.164	.128	.100	.078	.061	.048	.038	.030	.024	28
29	1.000	.760	.579	.442	.338	.259	.200	.154	.119	.092	.071	.056	.043	.034	.027	.021	29
30	1.000	.752	.568	.430	.326	.248	.189	.144	.110	.085	.065	.050	.039	.030	.023	.018	30
31	1.000	.745	.557	.418	.314	.236	.179	.135	.103	.078	.060	.046	.035	.027	.021	.016	31
32	1.000	.738	.547	.406	.302	.226	.169	.127	.095	.072	.054	.041	.031	.024	.018	.014	32
33	1.000	.731	.537	.395	.291	.215	.160	.119	.089	.066	.050	.037	.028	.021	.016	.012	33
34	1.000	.724	.526	.384	.281	.206	.151	.111	.082	.061	.045	.034	.025	.019	.014	.011	34
35	1.000	.718	.517	.373	.270	.196	.143	.104	.077	.056	.041	.031	.022	.017	.013	.009	35
36	1.000	.711	.507	.363	.260	.187	.135	.098	.071	.052	.038	.028	.020	.015	.011	.008	36
37	1.000	.704	.498	.353	.251	.179	.128	.092	.066	.048	.035	.025	.018	.013	.010	.007	37
38	1.000	.698	.488	.343	.242	.171	.121	.086	.061	.044	.032	.023	.016	.012	.009	.006	38
39	1.000	.691	.479	.333	.233	.163	.114	.081	.057	.040	.029	.021	.015	.011	.008	.005	39
40	1.000	.684	.470	.324	.224	.156	.108	.076	.053	.037	.026	.019	.013	.009	.007	.005	40
41	1.000	.678	.461	.315	.216	.148	.102	.071	.049	.034	.024	.017	.012	.008	.006	.004	41
42	1.000	.672	.453	.306	.208	.142	.097	.066	.046	.032	.022	.015	.011	.007	.005	.004	42
43	1.000	.665	.444	.298	.200	.135	.092	.062	.043	.029	.020	.014	.010	.007	.005	.003	43
44	1.000	.659	.436	.290	.193	.129	.087	.058	.040	.027	.018	.012	.009	.006	.004	.003	44
45	1.000	.653	.428	.281	.186	.123	.082	.055	.037	.025	.017	.011	.008	.005	.004	.002	45
46	1.000	.647	.420	.274	.179	.118	.078	.051	.034	.023	.015	.010	.007	.005	.003	.002	46
47	1.000	.641	.412	.266	.173	.112	.073	.048	.032	.021	.014	.009	.006	.004	.003	.002	47
48	1.000	.634	.404	.259	.166	.107	.069	.045	.029	.019	.013	.008	.006	.004	.002	.002	48
49	1.000	.628	.397	.251	.160	.102	.066	.042	.027	.018	.012	.008	.005	.003	.002	.001	49
50	1.000	.623	.389	.244	.154	.098	.062	.040	.025	.016	.011	.007	.004	.003	.002	.001	50

Table XVII.—Factors of Safety at Speculative Rates of Interest for Valuing 1.00 Due at a Given Future Time, Assuming 6% to be a Safe Rate of Interest.

Year	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%	Year
1	1.000	.991	.981	.972	.964	.955	.945	.938	.930	.922	.914	.906	.898	.891	.883	1
2	1.000	.981	.963	.946	.929	.912	.896	.880	.865	.850	.835	.821	.807	.793	.780	2
3	1.000	.972	.945	.920	.895	.871	.848	.825	.804	.783	.763	.744	.725	.707	.689	3
4	1.000	.963	.928	.894	.862	.832	.802	.774	.747	.722	.697	.674	.651	.630	.609	4
5	1.000	.954	.911	.870	.831	.794	.759	.726	.695	.665	.637	.610	.585	.561	.538	5
6	1.000	.945	.894	.846	.801	.758	.719	.681	.646	.613	.582	.553	.525	.500	.476	6
7	1.000	.936	.877	.823	.772	.724	.680	.639	.601	.565	.532	.501	.472	.445	.420	7
8	1.000	.928	.861	.800	.744	.692	.644	.600	.559	.521	.486	.454	.424	.396	.371	8
9	1.000	.919	.845	.778	.717	.660	.609	.562	.520	.480	.444	.411	.381	.353	.327	9
10	1.000	.910	.830	.756	.690	.631	.577	.528	.482	.443	.406	.373	.342	.314	.289	10
11	1.000	.902	.814	.736	.665	.602	.546	.495	.449	.408	.371	.338	.307	.280	.255	11
12	1.000	.893	.799	.715	.641	.575	.516	.464	.418	.376	.339	.306	.276	.250	.226	12
13	1.000	.885	.784	.696	.618	.549	.489	.436	.388	.347	.310	.277	.248	.222	.199	13
14	1.000	.877	.770	.677	.595	.525	.463	.408	.361	.320	.283	.251	.223	.195	.176	14
15	1.000	.869	.755	.658	.574	.501	.438	.383	.336	.295	.259	.227	.200	.176	.156	15
16	1.000	.861	.742	.640	.553	.478	.414	.359	.312	.271	.236	.205	.180	.157	.137	16
17	1.000	.852	.728	.622	.533	.457	.392	.337	.290	.250	.216	.187	.162	.140	.121	17
18	1.000	.844	.714	.605	.513	.436	.371	.316	.270	.231	.197	.169	.145	.125	.107	18
19	1.000	.837	.701	.588	.495	.417	.351	.297	.251	.213	.181	.153	.130	.111	.095	19
20	1.000	.829	.688	.572	.477	.398	.332	.278	.233	.196	.165	.139	.117	.099	.084	20
21	1.000	.821	.675	.557	.459	.380	.315	.261	.217	.181	.151	.126	.105	.088	.074	21
22	1.000	.813	.663	.541	.443	.363	.298	.245	.202	.166	.138	.114	.094	.078	.065	22
23	1.000	.806	.651	.526	.427	.346	.282	.230	.188	.153	.126	.103	.085	.070	.058	23
24	1.000	.798	.639	.512	.411	.331	.267	.216	.174	.141	.115	.094	.076	.062	.051	24
25	1.000	.791	.627	.498	.396	.316	.252	.202	.162	.130	.105	.085	.068	.055	.045	25
26	1.000	.783	.615	.484	.382	.302	.239	.190	.151	.120	.096	.077	.062	.049	.040	26
27	1.000	.776	.604	.471	.368	.288	.226	.178	.140	.111	.088	.070	.055	.044	.036	27
28	1.000	.769	.593	.458	.354	.275	.214	.167	.130	.102	.080	.063	.050	.039	.031	28
29	1.000	.762	.582	.445	.342	.263	.203	.157	.121	.094	.073	.057	.045	.035	.027	29
30	1.000	.755	.571	.433	.329	.251	.192	.147	.113	.087	.067	.052	.040	.031	.024	30
31	1.000	.747	.560	.421	.317	.240	.181	.138	.105	.080	.061	.047	.036	.028	.021	31
32	1.000	.740	.550	.409	.306	.229	.172	.129	.097	.074	.056	.042	.032	.025	.019	32
33	1.000	.734	.540	.398	.295	.218	.163	.121	.091	.068	.051	.038	.029	.022	.017	33
34	1.000	.727	.530	.387	.284	.209	.154	.114	.084	.063	.047	.035	.026	.020	.015	34
35	1.000	.720	.520	.377	.274	.199	.146	.107	.078	.058	.043	.032	.023	.017	.013	35
36	1.000	.713	.510	.366	.264	.190	.138	.100	.073	.053	.039	.029	.021	.015	.011	36
37	1.000	.707	.501	.356	.254	.182	.130	.094	.068	.049	.036	.026	.019	.014	.010	37
38	1.000	.700	.491	.346	.245	.174	.123	.088	.063	.045	.033	.023	.017	.012	.009	38
39	1.000	.693	.482	.337	.236	.166	.117	.083	.059	.042	.030	.021	.015	.011	.008	39
40	1.000	.687	.473	.327	.227	.158	.111	.077	.054	.038	.027	.019	.014	.010	.007	40
41	1.000	.680	.465	.318	.219	.151	.105	.073	.051	.035	.025	.017	.012	.009	.006	41
42	1.000	.674	.456	.310	.211	.144	.099	.068	.047	.033	.023	.016	.011	.008	.005	42
43	1.000	.668	.448	.301	.203	.138	.094	.064	.044	.030	.021	.014	.010	.007	.005	43
44	1.000	.662	.439	.293	.196	.132	.089	.060	.041	.028	.019	.013	.009	.006	.004	44
45	1.000	.655	.431	.285	.189	.126	.084	.056	.038	.026	.017	.012	.008	.005	.004	45
46	1.000	.649	.423	.277	.182	.120	.079	.053	.035	.024	.016	.011	.007	.005	.003	46
47	1.000	.643	.415	.269	.175	.115	.075	.050	.033	.022	.014	.010	.006	.004	.003	47
48	1.000	.637	.408	.262	.169	.109	.071	.046	.030	.020	.013	.009	.006	.004	.003	48
49	1.000	.631	.400	.255	.163	.105	.067	.044	.028	.018	.012	.008	.005	.003	.002	49
50	1.000	.625	.393	.248	.157	.100	.064	.041	.026	.017	.011	.007	.005	.003	.002	50

Table XVIII.—Factors of Safety at Speculative Rates of Interest for Valuing Equal Annual Income by Compound Interest Premises, Assuming 3% to be a Safe Rate of Interest.

Year	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%	Year
1	1.00	.990	.981	.972	.963	.954	.945	.936	.928	.920	.911	.904	.896	.888	.880	.873	.866	.858	1
2	1.00	.986	.972	.958	.945	.932	.919	.907	.895	.883	.872	.861	.850	.839	.828	.818	.808	.798	2
3	1.00	.981	.963	.945	.928	.911	.895	.879	.864	.849	.835	.821	.807	.794	.781	.769	.757	.745	3
4	1.00	.977	.954	.932	.911	.891	.872	.853	.835	.817	.800	.784	.768	.753	.738	.724	.710	.696	4
5	1.00	.972	.945	.920	.895	.872	.849	.828	.807	.787	.768	.750	.732	.715	.699	.683	.668	.653	5
6	1.00	.968	.937	.908	.880	.853	.828	.804	.781	.759	.738	.718	.699	.680	.663	.646	.629	.614	6
7	1.00	.963	.929	.896	.865	.836	.808	.781	.756	.733	.710	.688	.668	.648	.630	.612	.595	.579	7
8	1.00	.959	.921	.885	.851	.819	.788	.760	.733	.708	.684	.661	.639	.619	.599	.581	.563	.547	8
9	1.00	.955	.913	.874	.837	.802	.770	.740	.711	.684	.659	.635	.613	.592	.572	.553	.535	.518	9
10	1.00	.951	.905	.863	.823	.787	.752	.720	.690	.662	.636	.611	.588	.567	.546	.527	.509	.491	10
11	1.00	.947	.898	.852	.810	.772	.735	.702	.671	.642	.615	.589	.566	.543	.523	.503	.485	.468	11
12	1.00	.943	.890	.842	.798	.757	.719	.685	.652	.622	.595	.569	.545	.522	.501	.482	.463	.446	12
13	1.00	.939	.883	.832	.786	.743	.704	.668	.636	.604	.576	.549	.525	.502	.481	.462	.443	.426	13
14	1.00	.935	.876	.823	.774	.730	.689	.652	.618	.587	.558	.531	.507	.484	.463	.443	.425	.408	14
15	1.00	.931	.869	.814	.763	.717	.675	.637	.602	.571	.541	.515	.490	.467	.446	.427	.408	.392	15
16	1.00	.928	.863	.805	.752	.705	.662	.623	.587	.555	.526	.499	.474	.451	.430	.411	.393	.377	16
17	1.00	.924	.856	.796	.742	.693	.649	.609	.573	.541	.511	.484	.459	.437	.416	.397	.379	.363	17
18	1.00	.920	.850	.787	.731	.681	.637	.596	.560	.527	.497	.470	.446	.423	.402	.383	.366	.350	18
19	1.00	.917	.844	.779	.722	.670	.625	.584	.547	.514	.484	.457	.433	.410	.390	.371	.354	.338	19
20	1.00	.913	.858	.771	.712	.660	.613	.572	.535	.502	.472	.445	.421	.399	.378	.360	.343	.327	20
21	1.00	.910	.832	.763	.703	.650	.603	.561	.524	.491	.461	.434	.409	.387	.367	.349	.333	.317	21
22	1.00	.907	.826	.756	.694	.640	.592	.550	.513	.480	.450	.423	.399	.377	.357	.339	.323	.308	22
23	1.00	.904	.820	.748	.686	.631	.583	.540	.503	.469	.440	.413	.389	.368	.348	.330	.314	.299	23
24	1.00	.900	.815	.741	.677	.622	.573	.531	.493	.460	.430	.404	.380	.359	.339	.322	.306	.292	24
25	1.00	.897	.809	.734	.669	.613	.564	.521	.484	.450	.421	.395	.371	.350	.331	.314	.298	.284	25
26	1.00	.894	.804	.727	.662	.605	.555	.512	.475	.442	.412	.386	.363	.342	.323	.307	.291	.277	26
27	1.00	.891	.799	.721	.654	.597	.547	.504	.466	.433	.404	.378	.355	.335	.316	.300	.285	.271	27
28	1.00	.888	.794	.714	.647	.589	.539	.496	.458	.426	.397	.371	.348	.328	.310	.293	.278	.265	28
29	1.00	.885	.789	.708	.640	.582	.531	.488	.451	.418	.389	.364	.341	.321	.303	.287	.273	.259	29
30	1.00	.882	.784	.702	.633	.574	.524	.481	.444	.411	.382	.357	.335	.315	.297	.281	.267	.254	30
31	1.00	.879	.780	.696	.627	.567	.517	.474	.437	.404	.376	.351	.329	.309	.292	.276	.262	.249	31
32	1.00	.877	.775	.691	.620	.561	.510	.467	.430	.398	.370	.345	.323	.304	.287	.271	.257	.243	32
33	1.00	.874	.771	.685	.614	.554	.504	.461	.424	.392	.364	.339	.318	.299	.282	.266	.253	.240	33
34	1.00	.871	.766	.680	.608	.548	.498	.455	.418	.386	.358	.334	.313	.294	.277	.262	.248	.236	34
35	1.00	.869	.762	.675	.603	.542	.492	.449	.412	.380	.353	.329	.309	.289	.273	.258	.244	.232	35
36	1.00	.866	.758	.670	.597	.537	.486	.443	.407	.375	.348	.324	.303	.285	.268	.254	.241	.229	36
37	1.00	.864	.754	.665	.592	.531	.481	.438	.401	.370	.343	.320	.299	.281	.265	.250	.237	.225	37
38	1.00	.861	.750	.660	.587	.526	.475	.433	.397	.365	.339	.315	.296	.277	.261	.247	.234	.222	38
39	1.00	.859	.746	.655	.582	.521	.470	.428	.392	.361	.334	.311	.291	.273	.257	.243	.230	.219	39
40	1.00	.856	.742	.651	.577	.516	.465	.423	.387	.357	.330	.307	.287	.270	.254	.240	.227	.216	40
41	1.00	.854	.739	.647	.572	.511	.461	.419	.383	.353	.326	.304	.284	.266	.251	.237	.225	.213	41
42	1.00	.852	.735	.642	.568	.507	.456	.414	.379	.349	.323	.300	.280	.263	.248	.234	.222	.211	42
43	1.00	.849	.732	.638	.563	.502	.452	.410	.375	.345	.319	.297	.277	.260	.245	.231	.219	.208	43
44	1.00	.847	.728	.634	.559	.498	.448	.406	.371	.341	.316	.294	.274	.257	.242	.229	.217	.206	44
45	1.00	.845	.725	.630	.555	.494	.444	.402	.367	.338	.312	.291	.271	.255	.240	.226	.215	.204	45
46	1.00	.843	.722	.627	.551	.490	.440	.399	.364	.335	.309	.288	.269	.252	.237	.224	.212	.202	46
47	1.00	.841	.719	.623	.547	.486	.436	.395	.361	.331	.306	.285	.266	.250	.234	.222	.210	.200	47
48	1.00	.839	.715	.619	.543	.482	.433	.392	.357	.328	.304	.282	.264	.247	.233	.220	.208	.198	48
49	1.00	.837	.712	.616	.540	.479	.429	.388	.354	.326	.301	.280	.261	.245	.231	.218	.206	.196	49
50	1.00	.835	.710	.613	.536	.475	.426	.385	.351	.323	.298	.277	.259	.243	.229	.216	.205	.194	50

Table XVIII (Continued).—Factors of Safety at Speculative Rates of Interest for Valuing Equal Annual Income by Compound Interest Premises, Assuming 3% to be a Safe Rate of Interest.

Year	25%	30%	35%	40%	45%	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%	100%
1	.824	.792	.763	.736	.710	.687	.665	.644	.624	.606	.589	.572	.557	.542	.528	.515
2	.753	.711	.674	.640	.609	.581	.555	.531	.509	.488	.469	.452	.435	.420	.405	.392
3	.690	.642	.600	.562	.528	.498	.470	.445	.423	.402	.383	.366	.350	.336	.322	.309
4	.635	.583	.537	.497	.463	.432	.404	.380	.358	.338	.320	.304	.289	.276	.264	.252
5	.587	.532	.485	.444	.410	.379	.353	.329	.308	.290	.273	.258	.245	.233	.222	.212
6	.545	.488	.440	.400	.366	.337	.311	.289	.270	.253	.238	.224	.212	.201	.191	.182
7	.507	.450	.402	.363	.330	.302	.278	.258	.240	.224	.210	.197	.186	.176	.167	.159
8	.474	.417	.370	.332	.300	.274	.251	.232	.215	.201	.188	.176	.166	.157	.149	.142
9	.445	.388	.342	.306	.275	.250	.229	.211	.195	.182	.170	.160	.151	.142	.135	.128
10	.419	.362	.318	.283	.254	.230	.210	.194	.179	.167	.156	.146	.138	.130	.123	.117
11	.395	.340	.297	.264	.236	.214	.195	.179	.166	.154	.144	.135	.127	.120	.114	.108
12	.374	.321	.279	.247	.221	.199	.182	.167	.154	.143	.134	.125	.118	.112	.106	.100
13	.355	.303	.263	.232	.207	.187	.170	.156	.144	.134	.125	.117	.111	.104	.099	.094
14	.339	.288	.249	.219	.196	.176	.161	.147	.136	.126	.118	.111	.104	.098	.093	.089
15	.323	.274	.237	.208	.185	.167	.152	.139	.129	.120	.112	.105	.099	.093		
16	.309	.261	.226	.198	.176	.159	.144	.133	.122	.114	.106	.100	.094			
17	.297	.250	.216	.189	.168	.152	.138	.127	.117	.108	.101	.095	.089			
18	.286	.240	.207	.181	.161	.145	.132	.121	.112	.104	.097	.091				
19	.275	.231	.199	.174	.155	.140	.127	.116	.107	.100	.093					
20	.266	.223	.192	.168	.149	.134	.123	.112	.103	.096	.090					
21	.257	.215	.185	.162	.144	.130	.118	.108	.100	.093						
22	.249	.209	.179	.157	.139	.125	.114	.105	.097	.090						
23	.242	.202	.174	.152	.135	.122	.111	.101	.094							
24	.235	.196	.169	.148	.131	.118	.107	.098	.091							
25	.229	.191	.164	.144	.128	.115	.104	.096	.088							
26	.223	.186	.160	.140	.124	.112	.102	.093								
27	.218	.182	.156	.136	.121	.109	.099	.091								
28	.213	.178	.152	.133	.118	.107	.097	.089								
29	.208	.174	.149	.130	.116	.104	.095	.087								
30	.204	.170	.146	.128	.113	.102	.093									
31	.200	.167	.143	.125	.111	.100	.091									
32	.196	.163	.140	.123	.109	.098	.089									
33	.193	.160	.138	.120	.107	.096	.088									
34	.189	.158	.135	.118	.105	.095	.086									
35	.186	.155	.133	.116	.103	.093	.085									
36	.183	.153	.131	.115	.102	.092	.083									
37	.180	.150	.129	.113	.100	.090	.082									
38	.178	.148	.127	.111	.099	.089										
39	.175	.146	.125	.110	.097	.088										
40	.173	.144	.124	.108	.096	.087										
41	.171	.142	.122	.107	.095	.085										
42	.169	.141	.121	.105	.094	.084										
43	.167	.139	.119	.104	.093											
44	.165	.137	.118	.103	.092											
45	.163	.136	.117	.102	.091											
46	.161	.135	.115	.101	.090											
47	.160	.133	.114	.100	.089											
48	.158	.132	.113	.099	.088											
49	.157	.131	.112	.098	.087											
50	.155	.130	.111	.097	.086											

Table XIX.—Factors of Safety at Speculative Rates of Interest for Valuing Equal Annual Income by Compound Interest Premises, Assuming 4% to be a Safe Rate of Interest.

Year	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%	Year
1	1.00	.990	.981	.972	.963	.954	.945	.937	.929	.921	.912	.904	.897	.889	.881	.874	.867	1
2	1.00	.986	.972	.959	.945	.933	.920	.908	.896	.884	.873	.862	.851	.840	.830	.820	.810	2
3	1.00	.981	.963	.946	.929	.912	.896	.881	.865	.851	.837	.823	.809	.796	.783	.771	.759	3
4	1.00	.977	.955	.933	.912	.893	.873	.855	.837	.819	.803	.787	.771	.756	.741	.727	.713	4
5	1.00	.973	.946	.921	.897	.874	.852	.830	.810	.790	.771	.753	.735	.719	.702	.687	.672	5
6	1.00	.968	.938	.909	.882	.856	.831	.807	.784	.763	.742	.722	.703	.685	.667	.650	.634	6
7	1.00	.964	.930	.898	.867	.839	.811	.785	.760	.737	.714	.693	.673	.654	.635	.617	.601	7
8	1.00	.960	.922	.887	.854	.822	.792	.764	.738	.713	.689	.666	.645	.625	.606	.587	.570	8
9	1.00	.956	.915	.876	.840	.806	.775	.745	.717	.690	.665	.642	.620	.599	.579	.560	.542	9
10	1.00	.952	.907	.866	.827	.791	.758	.726	.697	.669	.643	.619	.596	.574	.554	.535	.517	10
11	1.00	.948	.900	.856	.815	.777	.741	.708	.678	.649	.622	.597	.574	.552	.531	.512	.494	11
12	1.00	.944	.893	.846	.803	.763	.726	.692	.660	.631	.603	.578	.554	.532	.511	.491	.473	12
13	1.00	.941	.887	.837	.792	.750	.711	.676	.643	.613	.585	.559	.535	.513	.492	.472	.454	13
14	1.00	.937	.880	.828	.780	.737	.697	.661	.627	.597	.568	.542	.518	.495	.474	.455	.436	14
15	1.00	.934	.874	.819	.770	.725	.684	.647	.613	.581	.552	.526	.501	.479	.458	.439	.421	15
16	1.00	.930	.867	.811	.760	.713	.671	.633	.599	.567	.538	.511	.486	.464	.443	.424	.406	16
17	1.00	.927	.861	.803	.750	.702	.659	.621	.585	.553	.524	.497	.473	.450	.429	.410	.392	17
18	1.00	.923	.855	.795	.740	.692	.648	.608	.573	.540	.511	.484	.460	.437	.417	.398	.380	18
19	1.00	.920	.850	.787	.731	.681	.637	.597	.561	.528	.499	.472	.448	.425	.405	.386	.369	19
20	1.00	.917	.844	.780	.722	.672	.626	.586	.550	.517	.487	.461	.436	.414	.394	.376	.358	20
21	1.00	.914	.839	.772	.714	.662	.616	.576	.539	.506	.477	.450	.426	.404	.384	.365	.349	21
22	1.00	.911	.833	.765	.706	.653	.607	.566	.529	.496	.467	.440	.416	.394	.374	.356	.340	22
23	1.00	.908	.828	.759	.698	.645	.598	.556	.520	.487	.457	.431	.407	.385	.366	.348	.331	23
24	1.00	.905	.823	.752	.691	.637	.589	.548	.511	.478	.448	.422	.398	.377	.358	.340	.324	24
25	1.00	.902	.818	.746	.683	.629	.581	.539	.502	.469	.440	.414	.390	.369	.350	.333	.317	25
26	1.00	.899	.814	.740	.676	.621	.573	.531	.494	.461	.432	.406	.383	.362	.343	.326	.310	26
27	1.00	.897	.809	.734	.670	.614	.566	.523	.486	.454	.425	.399	.376	.355	.336	.319	.304	27
28	1.00	.894	.805	.728	.663	.607	.559	.516	.479	.447	.418	.392	.369	.349	.330	.313	.298	28
29	1.00	.891	.800	.723	.657	.600	.552	.509	.472	.440	.411	.386	.363	.343	.324	.308	.293	29
30	1.00	.889	.796	.718	.651	.594	.545	.503	.466	.433	.405	.380	.357	.337	.319	.303	.288	30
31	1.00	.887	.792	.712	.645	.588	.539	.497	.460	.427	.399	.374	.352	.332	.314	.298	.283	31
32	1.00	.884	.788	.708	.640	.582	.533	.491	.454	.422	.394	.369	.347	.327	.309	.293	.279	32
33	1.00	.883	.784	.703	.634	.577	.527	.485	.448	.416	.388	.364	.342	.322	.305	.289	.275	33
34	1.00	.880	.780	.698	.629	.571	.522	.480	.443	.411	.383	.359	.337	.318	.301	.285	.271	34
35	1.00	.877	.777	.694	.624	.566	.517	.474	.438	.406	.379	.355	.333	.314	.297	.281	.267	35
36	1.00	.875	.773	.689	.620	.561	.512	.470	.433	.402	.374	.350	.329	.310	.293	.278	.264	36
37	1.00	.873	.770	.685	.615	.557	.507	.465	.429	.397	.370	.346	.325	.306	.290	.275	.261	37
38	1.00	.871	.767	.681	.611	.552	.503	.460	.424	.393	.366	.343	.322	.303	.286	.271	.258	38
39	1.00	.869	.763	.677	.607	.548	.498	.456	.420	.389	.363	.339	.318	.300	.283	.268	.255	39
40	1.00	.867	.760	.674	.602	.543	.494	.452	.416	.386	.359	.336	.315	.297	.280	.266	.252	40
41	1.00	.865	.757	.670	.599	.540	.490	.448	.413	.382	.356	.332	.312	.294	.278	.263	.250	41
42	1.00	.863	.754	.666	.595	.536	.486	.445	.409	.379	.352	.329	.309	.291	.275	.261	.248	42
43	1.00	.861	.751	.663	.591	.532	.483	.441	.406	.376	.349	.326	.306	.288	.272	.258	.245	43
44	1.00	.860	.749	.660	.588	.529	.479	.438	.403	.373	.347	.324	.304	.286	.270	.256	.243	44
45	1.00	.858	.746	.657	.584	.525	.476	.435	.400	.370	.344	.321	.301	.284	.268	.254	.241	45
46	1.00	.856	.743	.654	.581	.522	.473	.432	.397	.367	.341	.319	.299	.281	.266	.252	.239	46
47	1.00	.854	.741	.651	.578	.519	.470	.429	.394	.364	.339	.316	.297	.279	.264	.250	.238	47
48	1.00	.853	.738	.648	.575	.516	.467	.426	.391	.362	.336	.314	.295	.277	.262	.248	.236	48
49	1.00	.851	.736	.645	.572	.513	.464	.423	.389	.360	.334	.312	.293	.275	.260	.247	.234	49
50	1.00	.850	.734	.642	.569	.510	.462	.421	.387	.357	.332	.310	.291	.274	.259	.245	.233	50

Table XIX (Continued).—Factors of Safety at Speculative Rates of Interest for Valuing Equal Annual Income by Compound Interest Premises, Assuming 4% to be a Safe Rate of Interest.

Year	25%	30%	35%	40%	45%	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%	100%
1	.832	.800	.770	.743	.717	.695	.671	.650	.630	.612	.594	.578	.562	.547	.533	.520
2	.763	.722	.684	.649	.618	.589	.563	.538	.516	.495	.476	.458	.442	.426	.411	.398
3	.703	.654	.611	.573	.538	.507	.479	.454	.431	.410	.391	.373	.357	.342	.328	.315
4	.651	.597	.550	.509	.474	.442	.414	.389	.367	.346	.328	.312	.296	.283	.270	.258
5	.604	.547	.499	.457	.421	.390	.363	.339	.317	.298	.281	.266	.252	.240	.228	.218
6	.563	.504	.455	.414	.378	.348	.322	.299	.279	.261	.245	.231	.219	.207	.197	.188
7	.527	.467	.418	.377	.345	.314	.289	.267	.249	.232	.218	.205	.193	.183	.174	.165
8	.494	.434	.386	.346	.313	.285	.262	.242	.224	.209	.196	.184	.173	.164	.156	.148
9	.466	.406	.358	.320	.288	.262	.240	.221	.205	.191	.178	.167	.158	.149	.141	.134
10	.440	.381	.335	.298	.267	.242	.221	.204	.188	.175	.164	.154	.145	.137	.130	.123
11	.417	.359	.314	.278	.249	.226	.206	.189	.175	.163	.152	.142	.134	.127	.120	.114
12	.397	.340	.296	.262	.234	.211	.193	.177	.164	.152	.142	.133	.125	.118	.112	.107
13	.379	.323	.280	.247	.221	.199	.181	.167	.154	.143	.133	.125	.118	.111	.105	.100
14	.362	.308	.266	.235	.209	.189	.172	.158	.146	.135	.126	.118	.111	.105	.100	.095
15	.347	.294	.254	.223	.199	.179	.163	.150	.138	.128	.120	.112	.106	.100		
16	.334	.282	.243	.214	.190	.171	.156	.143	.132	.123	.114	.107	.101			
17	.321	.271	.233	.205	.182	.164	.149	.137	.126	.117	.110	.103	.097			
18	.310	.261	.225	.197	.175	.158	.144	.132	.122	.113	.105	.099				
19	.300	.252	.217	.190	.169	.152	.138	.127	.117	.109	.102					
20	.291	.244	.210	.184	.163	.147	.134	.123	.113	.105	.098					
21	.282	.237	.203	.178	.158	.143	.130	.119	.110	.102						
22	.275	.230	.197	.173	.154	.138	.126	.115	.106	.099						
23	.268	.224	.192	.168	.150	.135	.122	.112	.104							
24	.261	.218	.187	.164	.146	.131	.119	.109	.101							
25	.255	.213	.183	.160	.142	.128	.116	.107	.098							
26	.250	.208	.179	.156	.139	.125	.114	.104								
27	.244	.204	.175	.153	.136	.122	.111	.102								
28	.240	.200	.171	.150	.133	.120	.109	.100								
29	.235	.196	.168	.147	.131	.118	.107	.098								
30	.231	.193	.165	.145	.129	.116	.105									
31	.227	.189	.162	.142	.126	.114	.103									
32	.224	.186	.160	.140	.124	.112	.102									
33	.220	.184	.157	.138	.122	.110	.100									
34	.217	.181	.155	.136	.121	.109	.099									
35	.214	.179	.153	.134	.119	.107	.097									
36	.211	.176	.151	.132	.118	.106	.096									
37	.209	.174	.149	.131	.116	.104	.095									
38	.206	.172	.148	.129	.115	.103										
39	.204	.170	.146	.128	.113	.102										
40	.202	.168	.144	.126	.112	.101										
41	.200	.167	.143	.125	.111	.100										
42	.198	.165	.142	.124	.110	.099										
43	.196	.164	.140	.123	.109											
44	.195	.162	.139	.122	.108											
45	.193	.161	.138	.121	.107											
46	.192	.160	.137	.120	.106											
47	.190	.158	.136	.119	.106											
48	.189	.157	.135	.118	.105											
49	.187	.156	.134	.117	.104											
50	.186	.155	.133	.116	.103											

Table XX.—Factors of Safety at Speculative Rates of Interest for Valuing Equal Annual Income by Compound Interest Premises, Assuming 5% to be a Safe Rate of Interest.

Year	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%	Year
1	1.00	.991	.981	.972	.963	.955	.946	.938	.929	.921	.913	.905	.897	.890	.882	.875	1
2	1.00	.986	.972	.959	.946	.933	.921	.909	.897	.886	.874	.863	.853	.842	.832	.822	2
3	1.00	.982	.964	.946	.930	.913	.897	.882	.867	.853	.838	.825	.811	.798	.786	.774	3
4	1.00	.977	.955	.934	.914	.894	.875	.857	.839	.822	.805	.789	.774	.759	.744	.730	4
5	1.00	.973	.947	.922	.898	.876	.854	.833	.812	.793	.774	.756	.739	.722	.706	.691	5
6	1.00	.969	.939	.911	.884	.858	.833	.810	.788	.766	.746	.726	.707	.689	.672	.655	6
7	1.00	.965	.931	.900	.870	.841	.814	.789	.764	.741	.719	.698	.678	.659	.640	.623	7
8	1.00	.961	.924	.889	.856	.825	.796	.769	.742	.718	.694	.672	.651	.631	.612	.594	8
9	1.00	.957	.917	.879	.843	.810	.779	.750	.722	.696	.671	.648	.626	.605	.586	.567	9
10	1.00	.953	.910	.869	.831	.796	.763	.732	.703	.676	.650	.626	.603	.582	.562	.543	10
11	1.00	.949	.903	.859	.819	.782	.747	.715	.685	.656	.630	.606	.582	.560	.540	.521	11
12	1.00	.946	.896	.850	.808	.769	.733	.699	.668	.639	.612	.586	.563	.541	.520	.501	12
13	1.00	.942	.890	.841	.797	.756	.719	.684	.652	.622	.594	.569	.545	.523	.502	.483	13
14	1.00	.939	.884	.833	.787	.744	.705	.670	.637	.606	.578	.552	.528	.506	.485	.466	14
15	1.00	.936	.877	.825	.777	.733	.693	.656	.623	.592	.563	.537	.513	.491	.470	.450	15
16	1.00	.932	.872	.817	.767	.722	.681	.643	.609	.578	.549	.523	.499	.476	.456	.436	16
17	1.00	.929	.866	.809	.758	.712	.670	.632	.597	.565	.536	.510	.486	.463	.443	.424	17
18	1.00	.925	.861	.802	.749	.702	.659	.620	.585	.553	.524	.498	.473	.451	.431	.412	18
19	1.00	.923	.855	.795	.741	.692	.649	.609	.574	.542	.513	.486	.462	.440	.420	.401	19
20	1.00	.920	.850	.788	.733	.683	.639	.599	.564	.531	.502	.476	.452	.430	.409	.391	20
21	1.00	.918	.845	.781	.725	.675	.630	.590	.554	.522	.492	.466	.442	.420	.400	.382	21
22	1.00	.915	.840	.775	.717	.666	.621	.581	.545	.512	.483	.457	.433	.411	.391	.373	22
23	1.00	.912	.836	.769	.710	.659	.613	.572	.536	.504	.474	.448	.424	.403	.383	.365	23
24	1.00	.910	.831	.763	.703	.651	.605	.564	.528	.495	.466	.440	.416	.395	.376	.358	24
25	1.00	.907	.827	.757	.697	.644	.598	.556	.520	.488	.459	.433	.409	.388	.369	.351	25
26	1.00	.905	.823	.752	.691	.637	.590	.549	.513	.480	.452	.426	.402	.381	.362	.345	26
27	1.00	.902	.819	.747	.685	.631	.584	.542	.506	.474	.445	.419	.396	.375	.356	.339	27
28	1.00	.900	.815	.742	.679	.625	.577	.536	.499	.467	.439	.413	.390	.369	.351	.334	28
29	1.00	.898	.811	.737	.674	.619	.571	.530	.493	.461	.433	.407	.384	.364	.345	.329	29
30	1.00	.895	.807	.732	.668	.613	.566	.524	.488	.456	.427	.402	.379	.359	.340	.324	30
31	1.00	.893	.804	.728	.663	.608	.560	.519	.482	.450	.422	.397	.374	.354	.336	.320	31
32	1.00	.891	.800	.724	.659	.603	.555	.513	.477	.445	.417	.392	.370	.350	.332	.315	32
33	1.00	.889	.797	.720	.654	.598	.550	.508	.472	.440	.412	.388	.366	.346	.328	.312	33
34	1.00	.887	.794	.716	.650	.593	.545	.504	.468	.436	.408	.383	.362	.342	.324	.308	34
35	1.00	.885	.791	.712	.645	.589	.541	.499	.463	.432	.404	.380	.358	.338	.321	.305	35
36	1.00	.884	.788	.708	.641	.585	.537	.495	.459	.428	.400	.376	.354	.335	.317	.302	35
37	1.00	.882	.785	.705	.637	.581	.533	.491	.455	.424	.397	.372	.351	.332	.314	.299	37
38	1.00	.880	.782	.701	.634	.577	.529	.487	.452	.421	.393	.369	.348	.329	.312	.296	38
39	1.00	.878	.780	.698	.630	.573	.525	.484	.448	.417	.390	.366	.345	.326	.309	.294	39
40	1.00	.877	.777	.695	.627	.570	.522	.480	.445	.414	.387	.363	.342	.323	.306	.291	40
41	1.00	.875	.774	.692	.624	.567	.518	.477	.442	.411	.384	.361	.340	.321	.304	.289	41
42	1.00	.874	.772	.689	.621	.563	.515	.474	.439	.408	.382	.358	.337	.319	.302	.287	42
43	1.00	.872	.770	.686	.618	.560	.512	.471	.436	.406	.379	.356	.335	.316	.300	.285	43
44	1.00	.871	.768	.684	.615	.558	.509	.469	.434	.403	.377	.353	.333	.314	.298	.283	44
45	1.00	.870	.765	.681	.612	.555	.507	.466	.431	.401	.374	.351	.331	.312	.296	.281	45
46	1.00	.868	.763	.679	.610	.552	.504	.464	.429	.399	.372	.349	.329	.311	.294	.280	46
47	1.00	.867	.761	.677	.607	.550	.502	.461	.426	.396	.370	.347	.327	.309	.293	.278	47
48	1.00	.866	.760	.674	.605	.547	.500	.459	.424	.394	.368	.345	.325	.307	.291	.277	48
49	1.00	.865	.758	.672	.603	.545	.497	.457	.422	.392	.367	.344	.324	.306	.290	.275	49
50	1.00	.863	.756	.670	.600	.543	.495	.455	.420	.391	.365	.342	.322	.304	.288	.274	50

Table XXI.—Factors of Safety at Speculative Rates of Interest for Valuing Equal Annual Income by Compound Interest Premises, Assuming 6% to be a Safe Rate of Interest.

Year	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%	Year
1	1.00	.991	.981	.972	.964	.955	.945	.938	.930	.922	.914	.906	.898	.891	.883	1
2	1.00	.986	.973	.959	.947	.934	.922	.910	.898	.887	.876	.865	.854	.844	.833	2
3	1.00	.982	.964	.947	.930	.914	.899	.883	.869	.854	.840	.827	.813	.801	.788	3
4	1.00	.978	.956	.935	.915	.895	.877	.858	.841	.824	.808	.792	.776	.761	.747	4
5	1.00	.973	.948	.923	.900	.877	.856	.835	.815	.796	.777	.760	.742	.726	.710	5
6	1.00	.969	.940	.912	.886	.860	.836	.813	.791	.770	.749	.730	.711	.693	.676	6
7	1.00	.965	.933	.902	.872	.844	.818	.792	.768	.745	.723	.703	.683	.664	.646	7
8	1.00	.962	.925	.891	.859	.829	.800	.773	.747	.723	.699	.678	.657	.637	.618	8
9	1.00	.958	.918	.881	.847	.814	.783	.754	.727	.702	.677	.654	.633	.612	.593	9
10	1.00	.954	.912	.872	.835	.800	.768	.737	.709	.682	.657	.633	.611	.590	.570	10
11	1.00	.951	.905	.863	.824	.787	.753	.721	.691	.664	.638	.613	.590	.569	.549	11
12	1.00	.947	.899	.854	.813	.774	.739	.706	.675	.647	.620	.595	.572	.550	.529	12
13	1.00	.944	.893	.846	.802	.762	.726	.692	.660	.631	.603	.578	.555	.533	.512	13
14	1.00	.941	.887	.838	.793	.751	.713	.678	.646	.616	.588	.563	.539	.517	.496	14
15	1.00	.938	.881	.830	.783	.740	.701	.665	.632	.602	.574	.548	.524	.502	.481	15
16	1.00	.935	.876	.823	.774	.730	.690	.653	.620	.589	.561	.535	.511	.489	.468	16
17	1.00	.932	.871	.815	.766	.720	.680	.642	.608	.577	.549	.523	.498	.476	.456	17
18	1.00	.929	.866	.809	.757	.711	.670	.632	.597	.566	.537	.511	.487	.466	.444	18
19	1.00	.926	.861	.802	.750	.703	.660	.622	.587	.555	.527	.500	.476	.454	.434	19
20	1.00	.924	.856	.796	.742	.694	.651	.612	.577	.546	.517	.491	.467	.445	.425	20
21	1.00	.921	.851	.790	.735	.686	.643	.604	.568	.537	.508	.482	.458	.436	.416	21
22	1.00	.919	.847	.784	.728	.679	.635	.595	.560	.528	.499	.473	.449	.428	.408	22
23	1.00	.916	.843	.779	.722	.672	.627	.588	.552	.520	.491	.465	.442	.420	.400	23
24	1.00	.914	.839	.773	.716	.665	.620	.580	.545	.513	.484	.458	.434	.413	.393	24
25	1.00	.912	.835	.768	.710	.659	.614	.573	.538	.506	.477	.451	.428	.406	.387	25
26	1.00	.909	.831	.764	.705	.653	.607	.567	.531	.499	.471	.445	.421	.400	.381	26
27	1.00	.907	.828	.759	.699	.647	.601	.561	.525	.493	.464	439	.416	.395	.376	27
28	1.00	.905	.824	.755	.694	.642	.596	.555	.519	.487	.459	.433	.410	.390	.371	28
29	1.00	.903	.821	.750	.689	.636	.590	.550	.514	.482	.454	.428	.405	.385	.366	29
30	1.00	.901	.818	.746	.685	.632	.585	.545	.509	.477	.449	.423	.401	.380	.362	30
31	1.00	.900	.815	.743	.681	.627	.580	.540	.504	.472	.444	.419	.396	.376	.358	31
32	1.00	.898	.812	.739	.676	.623	.576	.535	.499	.468	.440	.415	.392	.372	.354	32
33	1.00	.896	.809	.735	.672	.618	.572	.531	.495	.464	.436	.411	.389	.369	.351	33
34	1.00	.895	.806	.732	.669	.615	.568	.527	.491	.460	.432	.407	.385	.365	.347	34
35	1.00	.893	.804	.729	.665	.611	.564	.523	.488	.456	.429	.404	.382	.362	.344	35
36	1.00	.892	.801	.726	.662	.607	.560	.520	.484	.453	.425	.401	.379	.359	.341	36
37	1.00	.890	.799	.723	.659	.604	.557	.516	.481	.450	.422	.398	.376	.357	.339	37
38	1.00	.889	.797	.720	.656	.601	.554	.513	.478	.447	.419	.395	.374	.354	.336	38
39	1.00	.887	.795	.717	.653	.598	.551	.510	.475	.444	.417	.393	.371	.352	.334	39
40	1.00	.886	.793	.715	.650	.595	.548	.507	.472	.441	.414	.390	.369	.349	.332	40
41	1.00	.885	.791	.713	.647	.592	.545	.505	.470	.439	.412	.388	.367	.347	.330	41
42	1.00	.884	.789	.710	.645	.590	.543	.502	.467	.437	.410	.386	.365	.345	.328	42
43	1.00	.882	.787	.708	.642	.587	.540	.500	.465	.434	.408	.384	.363	.344	.327	43
44	1.00	.881	.785	.706	.640	.585	.538	.498	.463	.432	.406	.382	.361	.342	.325	44
45	1.00	.880	.783	.704	.638	.583	.536	.496	.461	.431	.404	.380	.359	.340	.323	45
46	1.00	.879	.782	.702	.636	.581	.534	.494	.459	.429	.402	.379	.358	.339	.322	46
47	1.00	.878	.780	.700	.634	.579	.532	.492	.457	.427	.401	.377	.356	.338	.321	47
48	1.00	.877	.779	.699	.632	.577	.530	.490	.456	.425	.399	.376	.355	.336	.319	48
49	1.00	.876	.777	.697	.631	.575	.528	.488	.454	.424	.398	.374	.354	.335	.318	49
50	1.00	.876	.776	.695	.629	.574	.527	.487	.453	.423	.396	.373	.352	.334	.317	50

Table XXII.—Factors of Safety at Speculative Rates of Interest for Valuing Equal or Unequal Annual Income by Hoskold Sinking Fund Premises with 3% Sinking Fund and Safe Rate of Interest.

Year	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%	Year
1	1.00	.990	.981	.972	.963	.954	.945	.936	.928	.920	.912	.904	.896	.888	.880	.873	.866	.858	1
2	1.00	.981	.963	.946	.929	.913	.897	.882	.867	.853	.839	.826	.813	.801	.789	.777	.766	.755	2
3	1.00	.972	.946	.922	.898	.876	.855	.835	.815	.797	.780	.763	.747	.731	.716	.702	.688	.675	3
4	1.00	.964	.931	.900	.871	.843	.818	.794	.771	.749	.729	.710	.692	.674	.658	.642	.627	.613	4
5	1.00	.956	.916	.879	.845	.814	.784	.757	.732	.708	.686	.665	.645	.627	.609	.593	.577	.562	5
6	1.00	.949	.902	.860	.822	.787	.755	.725	.698	.672	.649	.627	.606	.587	.569	.552	.536	.521	6
7	1.00	.941	.889	.843	.801	.762	.728	.696	.667	.641	.616	.593	.572	.553	.534	.517	.501	.486	7
8	1.00	.934	.877	.826	.781	.740	.704	.671	.640	.613	.588	.564	.543	.523	.504	.487	.471	.456	8
9	1.00	.928	.865	.811	.765	.720	.682	.647	.616	.588	.562	.539	.517	.497	.478	.461	.445	.430	9
10	1.00	.921	.854	.796	.746	.701	.661	.626	.594	.566	.540	.516	.494	.474	.456	.439	.423	.408	10
11	1.00	.915	.844	.783	.730	.684	.643	.607	.575	.546	.519	.496	.474	.454	.436	.419	.403	.389	11
12	1.00	.909	.834	.770	.715	.668	.626	.589	.557	.527	.501	.477	.456	.436	.418	.401	.386	.371	12
13	1.00	.904	.825	.758	.702	.653	.610	.573	.540	.511	.485	.461	.439	.420	.402	.385	.370	.356	13
14	1.00	.898	.816	.747	.689	.639	.596	.558	.525	.496	.470	.446	.425	.405	.387	.371	.356	.342	14
15	1.00	.893	.807	.736	.677	.626	.583	.545	.511	.482	.456	.432	.411	.392	.374	.358	.344	.330	15
16	1.00	.888	.799	.726	.666	.614	.570	.532	.499	.469	.443	.420	.399	.380	.363	.347	.332	.319	16
17	1.00	.884	.792	.717	.655	.603	.559	.520	.487	.458	.432	.408	.388	.369	.352	.336	.322	.308	17
18	1.00	.879	.784	.708	.645	.593	.548	.509	.476	.447	.421	.398	.377	.359	.342	.326	.312	.300	18
19	1.00	.875	.777	.699	.636	.583	.538	.499	.466	.437	.411	.388	.368	.349	.333	.318	.304	.291	19
20	1.00	.870	.771	.691	.627	.573	.528	.490	.457	.428	.402	.379	.359	.341	.324	.309	.296	.283	20
21	1.00	.866	.764	.684	.619	.565	.520	.481	.448	.419	.393	.371	.351	.333	.317	.302	.288	.276	21
22	1.00	.863	.758	.677	.611	.557	.511	.473	.440	.411	.386	.363	.343	.326	.309	.295	.282	.270	22
23	1.00	.859	.753	.670	.603	.549	.503	.465	.432	.403	.378	.356	.336	.319	.303	.288	.275	.263	23
24	1.00	.855	.748	.663	.596	.541	.496	.458	.425	.396	.371	.349	.328	.312	.297	.282	.270	.258	24
25	1.00	.852	.742	.657	.589	.535	.489	.451	.415	.390	.365	.343	.324	.306	.291	.277	.264	.253	25
26	1.00	.848	.737	.651	.583	.528	.482	.444	.411	.383	.359	.337	.318	.301	.286	.272	.259	.248	26
27	1.00	.845	.732	.645	.577	.522	.476	.438	.405	.377	.353	.332	.313	.296	.280	.267	.254	.243	27
28	1.00	.842	.727	.640	.571	.516	.470	.432	.400	.372	.348	.326	.308	.291	.276	.262	.250	.239	28
29	1.00	.839	.722	.635	.566	.510	.465	.427	.394	.367	.343	.321	.303	.286	.271	.258	.246	.235	29
30	1.00	.836	.718	.630	.561	.505	.460	.422	.389	.362	.338	.317	.298	.282	.267	.254	.242	.231	30
31	1.00	.833	.714	.625	.556	.500	.455	.417	.385	.357	.333	.312	.294	.278	.263	.250	.238	.227	31
32	1.00	.831	.710	.620	.551	.495	.450	.412	.380	.353	.329	.308	.290	.273	.259	.246	.235	.224	32
33	1.00	.828	.707	.616	.546	.491	.445	.408	.376	.349	.325	.304	.286	.270	.256	.243	.231	.221	33
34	1.00	.826	.703	.612	.542	.486	.441	.403	.372	.345	.321	.301	.283	.267	.253	.240	.228	.218	34
35	1.00	.823	.699	.608	.538	.482	.437	.399	.368	.341	.318	.297	.279	.264	.249	.237	.225	.215	35
36	1.00	.821	.696	.604	.534	.478	.433	.396	.364	.337	.314	.294	.276	.261	.247	.234	.223	.212	36
37	1.00	.819	.693	.601	.530	.474	.429	.392	.361	.334	.311	.291	.273	.258	.244	.231	.220	.210	37
38	1.00	.816	.690	.597	.526	.471	.426	.388	.357	.331	.308	.288	.270	.255	.241	.229	.217	.207	38
39	1.00	.814	.687	.594	.523	.467	.422	.385	.354	.328	.305	.285	.268	.252	.238	.226	.215	.205	39
40	1.00	.812	.684	.591	.520	.464	.419	.382	.351	.325	.302	.282	.265	.250	.236	.224	.213	.203	40
41	1.00	.810	.681	.587	.516	.461	.416	.379	.348	.322	.299	.280	.263	.247	.234	.222	.211	.201	41
42	1.00	.808	.678	.584	.513	.458	.413	.376	.345	.319	.297	.277	.260	.245	.232	.220	.209	.199	42
43	1.00	.807	.676	.582	.510	.455	.410	.373	.343	.317	.294	.275	.258	.243	.229	.218	.207	.197	43
44	1.00	.805	.673	.579	.508	.452	.407	.371	.340	.314	.292	.273	.256	.241	.228	.216	.205	.195	44
45	1.00	.803	.671	.576	.505	.449	.405	.368	.338	.312	.290	.270	.254	.239	.226	.214	.203	.193	45
46	1.00	.801	.669	.574	.502	.447	.402	.366	.335	.310	.288	.268	.252	.237	.224	.212	.201	.192	46
47	1.00	.800	.666	.571	.500	.444	.400	.363	.333	.307	.286	.266	.250	.235	.222	.210	.200	.190	47
48	1.00	.798	.664	.569	.497	.442	.397	.361	.331	.305	.284	.265	.248	.233	.220	.209	.198	.189	48
49	1.00	.797	.662	.567	.495	.440	.395	.359	.329	.303	.282	.263	.246	.232	.219	.207	.197	.187	49
50	1.00	.795	.660	.564	.493	.437	.393	.357	.327	.302	.280	.261	.245	.230	.217	.206	.195	.186	50

Table XXII (Continued).—Factors of Safety at Speculative Rates of Interest for Valuing Equal or Unequal Annual Income by Hoskold Sinking Fund Premises with 3% Sinking Fund and Safe Rate of Interest.

Year	25%	30%	35%	40%	45%	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%	100%
1	.824	.792	.763	.736	.710	.687	.665	.644	.624	.606	.589	.572	.557	.542	.528	.515
2	.704	.659	.620	.585	.554	.527	.501	.478	.457	.438	.421	.404	.389	.375	.362	.350
3	.616	.567	.525	.489	.457	.429	.405	.383	.363	.345	.329	.315	.301	.289	.278	.267
4	.550	.499	.457	.421	.390	.364	.341	.321	.303	.287	.272	.259	.247	.236	.226	.217
5	.498	.447	.406	.371	.342	.317	.296	.277	.260	.246	.233	.221	.210	.201	.192	.184
6	.456	.406	.366	.333	.305	.282	.262	.245	.229	.216	.204	.193	.184	.175	.167	.160
7	.422	.373	.334	.303	.276	.255	.236	.220	.206	.193	.182	.172	.164	.156	.149	.142
8	.393	.345	.308	.278	.253	.233	.215	.200	.187	.176	.165	.156	.148	.141	.134	.128
9	.369	.322	.286	.258	.234	.215	.198	.184	.172	.161	.151	.143	.135	.129	.123	.117
10	.348	.303	.268	.241	.218	.200	.184	.171	.159	.149	.140	.132	.125	.119	.113	.108
11	.329	.286	.252	.226	.205	.187	.172	.159	.148	.139	.131	.123	.116	.110	.105	.100
12	.313	.271	.239	.214	.193	.176	.162	.150	.139	.130	.122	.115	.109	.104	.098	
13	.299	.258	.227	.203	.183	.167	.153	.142	.132	.123	.116	.109	.103	.098		
14	.287	.247	.217	.193	.174	.159	.145	.134	.125	.117	.109	.103	.097			
15	.276	.237	.207	.185	.166	.151	.139	.128	.119	.111	.104	.098				
16	.266	.228	.199	.177	.159	.145	.133	.123	.114	.106	.100					
17	.257	.220	.192	.170	.153	.139	.127	.118	.109	.102	.095					
18	.248	.212	.185	.164	.148	.134	.123	.113	.105	.098						
19	.241	.206	.179	.159	.143	.129	.118	.109	.101	.094						
20	.234	.199	.174	.154	.138	.125	.114	.105	.098							
21	.228	.194	.169	.149	.134	.121	.111	.102	.095							
22	.222	.189	.164	.145	.130	.118	.108	.099								
23	.217	.184	.160	.141	.126	.115	.105	.096								
24	.212	.179	.156	.138	.123	.112	.102	.094								
25	.207	.175	.152	.134	.120	.109	.099									
26	.203	.172	.149	.131	.118	.106	.097									
27	.199	.168	.146	.129	.115	.104	.095									
28	.195	.165	.143	.126	.113	.102	.093									
29	.192	.162	.140	.123	.110	.100										
30	.188	.159	.138	.121	.108	.098										
31	.185	.156	.135	.119	.106	.096										
32	.182	.154	.133	.117	.105	.090										
33	.180	.151	.131	.115	.103	.093										
34	.177	.149	.129	.113	.101											
35	.175	.147	.127	.112	.100											
36	.172	.145	.125	.110	.098											
37	.170	.143	.124	.109	.097											
38	.168	.141	.122	.107	.096											
39	.166	.140	.121	.106	.095											
40	.164	.138	.119	.105	.093											
41	.163	.137	.118	.103	.092											
42	.161	.135	.116	.102	.091											
43	.159	.134	.115	.101	.090											
44	.158	.132	.114	.100	.089											
45	.156	.131	.113	.099												
46	.155	.130	.112	.098												
47	.154	.129	.111	.097												
48	.152	.128	.110	.097												
49	.151	.127	.109	.096												
50	.150	.126	.108	.095												

Table XXIII.—Factors of Safety at Speculative Rates of Interest for Valuing Equal or Unequal Annual Income by Hoskold Sinking Fund Premises with 4% Sinking Fund and Safe Rate of Interest.

Year	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%	Year
1	1.000	.990	.981	.972	.963	.954	.945	.937	.929	.920	.912	.904	.897	.889	.879	.874	.867	1
2	1.000	.981	.964	.947	.930	.914	.898	.883	.869	.855	.841	.828	.815	.803	.791	.779	.768	2
3	1.000	.973	.947	.923	.900	.878	.857	.837	.818	.800	.783	.766	.750	.735	.720	.706	.692	3
4	1.000	.965	.932	.902	.873	.846	.821	.797	.775	.754	.734	.715	.697	.679	.663	.647	.633	4
5	1.000	.957	.918	.882	.849	.818	.789	.762	.737	.714	.692	.671	.652	.633	.616	.600	.584	5
6	1.000	.950	.905	.864	.827	.792	.761	.732	.705	.679	.656	.634	.614	.595	.577	.560	.544	6
7	1.000	.943	.893	.847	.806	.769	.735	.704	.675	.649	.625	.602	.581	.562	.543	.526	.510	7
8	1.000	.937	.881	.832	.788	.748	.712	.680	.650	.623	.598	.575	.553	.533	.515	.497	.481	8
9	1.000	.931	.870	.818	.771	.729	.691	.658	.627	.599	.574	.550	.528	.508	.490	.473	.457	9
10	1.000	.925	.860	.804	.755	.711	.673	.638	.606	.578	.552	.528	.507	.487	.468	.451	.435	10
11	1.000	.919	.851	.792	.741	.695	.655	.620	.588	.559	.533	.509	.488	.468	.449	.432	.416	11
12	1.000	.914	.842	.780	.727	.681	.640	.604	.571	.542	.516	.492	.470	.450	.432	.415	.400	12
13	1.000	.909	.834	.769	.715	.667	.625	.589	.556	.527	.500	.477	.455	.435	.417	.400	.385	13
14	1.000	.904	.826	.759	.703	.654	.612	.575	.542	.513	.486	.463	.441	.421	.403	.387	.371	14
15	1.000	.900	.818	.750	.692	.643	.600	.562	.529	.500	.474	.450	.428	.409	.391	.375	.360	15
16	1.000	.896	.811	.741	.682	.632	.589	.551	.518	.488	.462	.438	.417	.398	.380	.364	.349	16
17	1.000	.892	.804	.733	.673	.622	.578	.540	.507	.477	.451	.428	.407	.387	.370	.354	.339	17
18	1.000	.888	.798	.725	.664	.612	.568	.530	.497	.467	.441	.418	.397	.378	.361	.345	.331	18
19	1.000	.884	.792	.717	.656	.604	.559	.521	.488	.458	.432	.409	.388	.369	.352	.337	.322	19
20	1.000	.880	.786	.710	.648	.595	.551	.512	.479	.450	.424	.401	.380	.361	.345	.329	.315	20
21	1.000	.877	.781	.704	.641	.588	.543	.505	.471	.442	.416	.393	.373	.354	.337	.322	.308	21
22	1.000	.874	.776	.698	.634	.581	.536	.497	.464	.435	.409	.386	.366	.347	.331	.316	.302	22
23	1.000	.871	.771	.692	.627	.574	.529	.490	.457	.428	.402	.380	.359	.341	.325	.310	.296	23
24	1.000	.868	.766	.686	.621	.567	.522	.484	.450	.422	.396	.374	.353	.335	.319	.304	.291	24
25	1.000	.865	.762	.681	.615	.561	.516	.478	.445	.416	.390	.368	.348	.330	.314	.299	.286	25
26	1.000	.862	.758	.676	.610	.556	.510	.472	.439	.410	.385	.363	.343	.325	.309	.294	.281	26
27	1.000	.860	.754	.671	.605	.551	.505	.467	.434	.405	.380	.358	.338	.320	.304	.290	.277	27
28	1.000	.857	.750	.667	.600	.546	.500	.462	.429	.400	.375	.353	.333	.316	.300	.286	.273	28
29	1.000	.855	.746	.661	.595	.541	.495	.457	.424	.395	.371	.349	.329	.312	.296	.282	.269	29
30	1.000	.853	.743	.658	.591	.536	.491	.452	.420	.391	.366	.345	.325	.308	.292	.278	.265	30
31	1.000	.850	.740	.655	.587	.532	.487	.448	.415	.387	.362	.341	.321	.304	.289	.275	.262	31
32	1.000	.848	.737	.651	.583	.527	.483	.444	.411	.383	.359	.337	.318	.301	.286	.272	.259	32
33	1.000	.846	.734	.647	.579	.524	.479	.440	.408	.380	.355	.334	.315	.298	.282	.269	.256	33
34	1.000	.845	.731	.644	.576	.521	.475	.437	.404	.376	.352	.331	.312	.295	.280	.266	.253	34
35	1.000	.843	.728	.641	.573	.517	.472	.434	.401	.373	.349	.328	.309	.292	.277	.263	.251	35
36	1.000	.841	.726	.638	.569	.514	.468	.430	.398	.370	.346	.325	.306	.289	.274	.261	.248	36
37	1.000	.839	.723	.635	.566	.511	.465	.427	.395	.367	.343	.322	.303	.287	.272	.258	.246	37
38	1.000	.838	.721	.632	.563	.508	.463	.424	.392	.365	.341	.319	.301	.284	.269	.256	.244	38
39	1.000	.836	.719	.630	.561	.505	.460	.422	.390	.362	.338	.317	.298	.282	.267	.254	.242	39
40	1.000	.835	.716	.627	.558	.503	.457	.419	.387	.360	.336	.315	.296	.280	.265	.252	.240	40
41	1.000	.833	.714	.625	.556	.500	.455	.417	.385	.357	.333	.313	.294	.278	.263	.250	.238	41
42	1.000	.832	.712	.623	.553	.497	.452	.414	.382	.355	.331	.311	.292	.276	.261	.248	.236	42
43	1.000	.831	.711	.621	.551	.495	.450	.412	.380	.353	.329	.309	.290	.274	.260	.247	.235	43
44	1.000	.830	.709	.619	.549	.493	.448	.410	.378	.351	.327	.307	.289	.272	.258	.245	.233	44
45	1.000	.828	.707	.617	.547	.491	.446	.408	.376	.349	.326	.305	.287	.271	.256	.243	.232	45
46	1.000	.827	.705	.615	.545	.489	.444	.406	.374	.347	.324	.303	.285	.269	.255	.242	.230	46
47	1.000	.826	.704	.613	.543	.487	.442	.404	.373	.346	.322	.302	.284	.268	.253	.241	.229	47
48	1.000	.825	.702	.611	.541	.485	.440	.403	.371	.344	.321	.300	.282	.266	.252	.239	.228	48
49	1.000	.824	.701	.610	.539	.484	.439	.401	.369	.342	.319	.299	.281	.265	.251	.238	.227	49
50	1.000	.823	.699	.608	.538	.482	.437	.399	.368	.341	.318	.297	.280	.264	.250	.237	.225	50

Table XXIII (Continued).—Factors of Safety at Speculative Rates of Interest for Valuing Equal or Unequal Annual Income by Hoskold Sinking Fund Premises with 4% Sinking Fund and Safe Rate of Interest.

Year	25%	30%	35%	40%	45%	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%	100%
1	.832	.800	.770	.743	.717	.693	.671	.650	.630	.612	.594	.578	.562	.547	.533	.520
2	.716	.671	.631	.596	.564	.535	.510	.486	.465	.445	.428	.411	.396	.381	.368	.356
3	.632	.581	.538	.500	.468	.439	.414	.392	.371	.353	.337	.322	.308	.295	.284	.273
4	.567	.514	.471	.434	.402	.375	.351	.330	.311	.294	.280	.266	.254	.243	.232	.223
5	.517	.464	.420	.384	.354	.328	.306	.286	.269	.254	.240	.228	.217	.207	.198	.190
6	.476	.423	.381	.346	.318	.293	.272	.254	.238	.224	.212	.201	.191	.182	.173	.166
7	.442	.391	.350	.316	.289	.266	.246	.229	.215	.202	.190	.180	.171	.162	.155	.148
8	.414	.364	.324	.292	.266	.244	.226	.210	.196	.184	.173	.163	.155	.147	.140	.134
9	.390	.341	.303	.272	.247	.226	.209	.194	.181	.169	.159	.150	.142	.135	.129	.123
10	.370	.322	.285	.255	.231	.211	.195	.180	.168	.157	.148	.140	.132	.125	.119	.114
11	.352	.305	.269	.241	.218	.199	.183	.169	.158	.147	.139	.131	.124	.117	.111	.106
12	.337	.291	.256	.228	.206	.188	.173	.160	.149	.139	.130	.123	.116	.110	.105	.100
13	.323	.278	.244	.218	.196	.179	.164	.152	.141	.132	.124	.116	.110	.104	.099	
14	.311	.267	.234	.208	.188	.171	.157	.145	.134	.126	.118	.111	.105	.099		
15	.300	.257	.225	.200	.180	.164	.150	.138	.128	.120	.112	.106	.100			
16	.290	.248	.217	.192	.173	.157	.144	.133	.123	.115	.108	.101	.096			
17	.281	.240	.210	.186	.167	.152	.139	.128	.119	.111	.104	.098				
18	.273	.233	.203	.180	.162	.147	.134	.124	.115	.107	.100	.094				
19	.266	.227	.197	.175	.157	.142	.130	.120	.111	.103	.097					
20	.259	.221	.192	.170	.152	.138	.126	.116	.108	.100	.094					
21	.253	.215	.187	.165	.148	.134	.123	.113	.105	.097						
22	.248	.210	.182	.161	.144	.131	.119	.110	.102	.095						
23	.243	.206	.178	.158	.141	.128	.117	.107	.099							
24	.238	.201	.175	.154	.138	.125	.114	.105	.097							
25	.234	.198	.171	.151	.135	.122	.112	.103	.095							
26	.230	.194	.168	.148	.132	.120	.109	.100	.093							
27	.226	.191	.165	.145	.130	.117	.107	.099								
28	.222	.188	.162	.143	.128	.115	.105	.097								
29	.219	.185	.160	.141	.126	.113	.104	.095								
30	.216	.182	.157	.138	.124	.112	.102	.094								
31	.213	.179	.155	.136	.122	.110	.100	.092								
32	.210	.177	.153	.135	.120	.108	.099									
33	.208	.175	.151	.133	.118	.107	.098									
34	.205	.173	.149	.131	.117	.106	.096									
35	.203	.171	.147	.130	.116	.104	.095									
36	.201	.169	.146	.128	.114	.103	.094									
37	.199	.167	.144	.127	.113	.102	.093									
38	.197	.166	.143	.125	.112	.101	.092									
39	.196	.164	.141	.124	.111	.100										
40	.194	.163	.140	.123	.110	.099										
41	.192	.161	.139	.122	.109	.098										
42	.191	.160	.138	.121	.108	.097										
43	.189	.159	.137	.120	.107	.096										
44	.188	.158	.136	.119	.106	.096										
45	.187	.157	.135	.118	.105	.095										
46	.186	.156	.134	.117	.105	.094										
47	.185	.155	.133	.117	.104	.094										
48	.183	.154	.132	.116	.103	.093										
49	.182	.153	.131	.115	.103	.092										
50	.181	.152	.131	.114	.102	.092										

Table XXIV.—Factors of Safety at Speculative Rates of Interest for Valuing Equal or Unequal Annual Income by Hoskold Sinking Fund Premises with 5% Sinking Fund and Safe Rate of Interest.

Year	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%	Year
1	1.00	.991	.981	.972	.963	.955	.946	.937	.929	.921	.913	.905	.897	.890	.882	.875	1
2	1.00	.982	.964	.947	.931	.915	.900	.885	.871	.857	.843	.830	.818	.805	.793	.782	2
3	1.00	.973	.948	.924	.902	.880	.860	.840	.821	.803	.786	.769	.754	.739	.724	.710	3
4	1.00	.966	.934	.904	.876	.849	.825	.801	.779	.758	.738	.719	.701	.684	.668	.653	4
5	1.00	.958	.920	.885	.852	.822	.794	.767	.743	.720	.698	.677	.658	.640	.623	.606	5
6	1.00	.952	.908	.869	.831	.798	.767	.738	.711	.686	.663	.642	.621	.602	.585	.568	6
7	1.00	.945	.896	.852	.812	.776	.742	.712	.684	.658	.633	.611	.590	.571	.552	.535	7
8	1.00	.939	.886	.838	.795	.756	.721	.689	.659	.632	.607	.584	.563	.543	.525	.508	8
9	1.00	.934	.876	.824	.779	.738	.701	.668	.637	.610	.585	.561	.540	.520	.501	.484	9
10	1.00	.928	.866	.812	.764	.721	.683	.649	.618	.590	.564	.541	.519	.499	.481	.463	10
11	1.00	.923	.858	.801	.751	.707	.667	.632	.601	.572	.546	.523	.501	.481	.462	.445	11
12	1.00	.919	.849	.790	.738	.693	.653	.617	.585	.556	.530	.506	.485	.465	.446	.429	12
13	1.00	.914	.842	.780	.727	.680	.640	.603	.571	.542	.516	.492	.470	.450	.432	.415	13
14	1.00	.910	.835	.771	.716	.669	.627	.591	.558	.529	.503	.479	.457	.437	.419	.402	14
15	1.00	.906	.828	.763	.707	.658	.616	.579	.546	.517	.491	.467	.445	.426	.408	.391	15
16	1.00	.902	.822	.755	.698	.649	.606	.569	.536	.506	.480	.456	.435	.415	.397	.381	16
17	1.00	.899	.816	.747	.689	.640	.597	.559	.526	.496	.470	.446	.425	.406	.388	.372	17
18	1.00	.895	.811	.740	.681	.631	.588	.550	.517	.487	.461	.437	.416	.397	.379	.363	18
19	1.00	.892	.805	.734	.674	.623	.580	.542	.508	.479	.453	.429	.408	.389	.371	.356	19
20	1.00	.889	.800	.728	.667	.616	.572	.534	.501	.471	.445	.422	.401	.382	.364	.349	20
21	1.00	.886	.796	.722	.661	.609	.565	.527	.494	.464	.438	.415	.394	.375	.358	.342	21
22	1.00	.884	.792	.717	.655	.603	.559	.520	.487	.458	.432	.409	.388	.369	.352	.336	22
23	1.00	.881	.788	.712	.650	.597	.553	.514	.481	.452	.426	.403	.382	.363	.346	.331	23
24	1.00	.879	.784	.707	.644	.592	.547	.509	.475	.446	.420	.397	.377	.358	.341	.326	24
25	1.00	.876	.780	.703	.639	.587	.542	.503	.470	.441	.415	.392	.372	.353	.336	.321	25
26	1.00	.874	.777	.699	.635	.582	.537	.498	.465	.436	.410	.387	.367	.349	.332	.317	26
27	1.00	.872	.773	.695	.631	.577	.532	.494	.461	.431	.406	.383	.363	.344	.328	.313	27
28	1.00	.870	.770	.691	.627	.573	.528	.490	.456	.427	.402	.379	.359	.341	.324	.309	28
29	1.00	.868	.767	.688	.623	.569	.524	.485	.452	.423	.398	.375	.355	.337	.321	.306	29
30	1.00	.867	.765	.684	.619	.565	.520	.482	.448	.420	.394	.372	.352	.334	.317	.302	30
31	1.00	.865	.762	.681	.616	.562	.517	.478	.445	.416	.391	.368	.348	.330	.314	.300	31
32	1.00	.864	.760	.678	.613	.559	.513	.475	.442	.413	.388	.365	.345	.327	.311	.297	32
33	1.00	.862	.758	.676	.610	.556	.510	.472	.439	.410	.385	.362	.342	.325	.309	.294	33
34	1.00	.861	.755	.673	.607	.553	.507	.469	.436	.407	.382	.360	.340	.322	.306	.292	34
35	1.00	.859	.753	.671	.604	.550	.504	.466	.433	.404	.379	.357	.337	.320	.304	.289	35
36	1.00	.858	.751	.668	.602	.547	.502	.463	.430	.402	.377	.355	.335	.317	.302	.287	36
37	1.00	.857	.749	.666	.599	.545	.499	.461	.428	.399	.374	.352	.333	.315	.299	.285	37
38	1.00	.856	.748	.664	.597	.542	.497	.459	.426	.397	.372	.350	.331	.313	.297	.283	38
39	1.00	.855	.746	.662	.595	.540	.495	.456	.423	.395	.370	.348	.329	.311	.296	.281	39
40	1.00	.854	.744	.660	.593	.538	.493	.454	.421	.393	.368	.346	.327	.310	.294	.280	40
41	1.00	.853	.743	.658	.591	.536	.491	.452	.420	.391	.366	.345	.325	.308	.292	.278	41
42	1.00	.852	.742	.657	.589	.534	.489	.451	.418	.389	.365	.343	.324	.306	.291	.277	42
43	1.00	.851	.740	.655	.588	.533	.487	.449	.416	.388	.363	.341	.322	.305	.289	.275	43
44	1.00	.850	.739	.654	.586	.531	.485	.447	.414	.386	.361	.340	.321	.303	.288	.274	44
45	1.00	.849	.738	.652	.584	.529	.484	.446	.413	.385	.360	.338	.319	.302	.287	.273	45
46	1.00	.848	.737	.651	.583	.528	.482	.444	.411	.383	.359	.337	.318	.301	.285	.272	46
47	1.00	.848	.735	.650	.582	.527	.481	.443	.410	.382	.357	.336	.317	.300	.284	.270	47
48	1.00	.847	.734	.648	.580	.525	.480	.441	.409	.381	.356	.335	.316	.299	.283	.269	48
49	1.00	.846	.733	.647	.579	.524	.478	.440	.408	.379	.355	.333	.314	.297	.282	.268	49
50	1.00	.846	.733	.646	.578	.523	.477	.439	.406	.378	.354	.332	.313	.296	.281	.268	50

Table XXV.—Factors of Safety at Speculative Rates of Interest for Valuing Equal or Unequal Annual Income by Hoskold Sinking Fund Premises with 6% Sinking Fund and Safe Rate of Interest.

Year	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%	Year
1	1.00	.991	.981	.972	.964	.955	.946	.938	.930	.922	.914	.906	.898	.891	.883	1
2	1.00	.982	.965	.948	.932	.916	.901	.886	.872	.858	.845	.832	.820	.808	.796	2
3	1.00	.974	.949	.926	.903	.882	.862	.842	.824	.806	.789	.773	.757	.742	.728	3
4	1.00	.967	.935	.906	.878	.852	.828	.805	.783	.762	.743	.724	.706	.689	.673	4
5	1.00	.960	.922	.888	.856	.826	.798	.772	.748	.725	.704	.683	.664	.646	.629	5
6	1.00	.953	.910	.871	.836	.803	.772	.744	.718	.693	.670	.649	.629	.610	.592	6
7	1.00	.947	.900	.857	.817	.782	.749	.719	.691	.666	.642	.620	.599	.579	.561	7
8	1.00	.942	.890	.843	.801	.763	.729	.697	.668	.641	.617	.594	.573	.553	.535	8
9	1.00	.936	.880	.831	.786	.746	.710	.677	.648	.620	.595	.572	.551	.531	.512	9
10	1.00	.931	.872	.819	.773	.731	.694	.660	.629	.602	.576	.553	.531	.511	.493	10
11	1.00	.927	.864	.809	.760	.717	.679	.644	.613	.585	.559	.535	.514	.494	.475	11
12	1.00	.923	.856	.799	.749	.705	.665	.630	.599	.570	.544	.520	.498	.478	.460	12
13	1.00	.919	.850	.790	.739	.693	.653	.617	.585	.557	.530	.507	.485	.465	.447	13
14	1.00	.915	.843	.782	.729	.683	.642	.606	.574	.545	.518	.494	.473	.453	.435	14
15	1.00	.911	.837	.774	.720	.673	.632	.595	.563	.534	.507	.483	.462	.442	.424	15
16	1.00	.908	.832	.767	.712	.664	.623	.586	.553	.524	.497	.474	.452	.432	.414	16
17	1.00	.905	.827	.761	.705	.656	.614	.577	.544	.515	.488	.465	.443	.423	.405	17
18	1.00	.902	.822	.755	.698	.649	.606	.569	.536	.506	.480	.456	.435	.415	.397	18
19	1.00	.900	.818	.749	.691	.642	.599	.561	.528	.499	.473	.449	.428	.408	.390	19
20	1.00	.897	.813	.744	.685	.636	.592	.555	.521	.492	.466	.442	.421	.401	.384	20
21	1.00	.895	.810	.739	.680	.630	.586	.548	.515	.486	.459	.436	.415	.395	.378	21
22	1.00	.893	.806	.735	.675	.624	.581	.543	.509	.480	.454	.430	.409	.390	.372	22
23	1.00	.890	.803	.730	.670	.619	.575	.537	.504	.475	.448	.425	.404	.385	.367	23
24	1.00	.888	.799	.726	.666	.614	.570	.532	.499	.470	.443	.420	.399	.380	.363	24
25	1.00	.887	.796	.723	.662	.610	.566	.528	.494	.465	.439	.416	.395	.376	.358	25
26	1.00	.885	.794	.719	.658	.606	.562	.523	.490	.461	.435	.411	.391	.372	.355	26
27	1.00	.883	.791	.716	.654	.602	.558	.520	.486	.457	.431	.408	.387	.368	.351	27
28	1.00	.882	.789	.713	.651	.599	.554	.516	.483	.453	.427	.404	.383	.365	.348	28
29	1.00	.880	.786	.710	.648	.595	.551	.512	.479	.450	.424	.401	.380	.361	.345	29
30	1.00	.879	.784	.708	.645	.592	.548	.509	.476	.447	.421	.398	.377	.358	.342	30
31	1.00	.878	.782	.705	.642	.589	.545	.506	.473	.444	.418	.395	.374	.356	.339	31
32	1.00	.877	.780	.703	.640	.587	.542	.504	.470	.441	.415	.392	.372	.353	.336	32
33	1.00	.875	.778	.701	.637	.584	.539	.501	.468	.438	.413	.390	.369	.351	.334	33
34	1.00	.874	.777	.699	.635	.582	.537	.499	.465	.436	.410	.388	.367	.349	.332	34
35	1.00	.873	.775	.697	.633	.580	.535	.496	.463	.434	.408	.386	.365	.347	.330	35
36	1.00	.872	.774	.695	.631	.578	.533	.494	.461	.432	.406	.383	.363	.345	.328	36
37	1.00	.872	.772	.693	.629	.576	.531	.492	.459	.430	.404	.382	.361	.343	.326	37
38	1.00	.871	.771	.692	.627	.574	.529	.490	.457	.428	.402	.380	.360	.341	.325	38
39	1.00	.870	.770	.690	.626	.572	.527	.489	.455	.426	.401	.378	.358	.340	.323	39
40	1.00	.869	.769	.689	.624	.571	.526	.487	.454	.425	.399	.377	.356	.338	.322	40
41	1.00	.869	.768	.688	.623	.569	.524	.486	.452	.423	.398	.375	.355	.337	.321	41
42	1.00	.868	.767	.686	.622	.568	.523	.484	.451	.422	.396	.374	.354	.336	.319	42
43	1.00	.867	.766	.685	.620	.566	.521	.483	.450	.421	.395	.373	.353	.334	.318	43
44	1.00	.867	.765	.684	.619	.565	.520	.481	.448	.419	.394	.371	.351	.333	.317	44
45	1.00	.866	.764	.683	.618	.564	.519	.480	.447	.418	.393	.370	.350	.332	.316	45
46	1.00	.866	.763	.682	.617	.563	.518	.479	.446	.417	.392	.369	.349	.331	.315	46
47	1.00	.865	.762	.681	.616	.562	.517	.478	.445	.416	.391	.368	.348	.330	.314	47
48	1.00	.865	.762	.680	.615	.561	.516	.477	.444	.415	.390	.368	.347	.330	.313	48
49	1.00	.864	.761	.680	.614	.560	.515	.476	.443	.414	.389	.367	.347	.329	.313	49
50	1.00	.864	.760	.679	.613	.559	.514	.475	.442	.413	.388	.366	.346	.328	.312	50

Table XXVI.—Values of W^n for Use in Formulas for Valuing Income Decreasing in Equal Ratio. (Note: Values of W^n for income increasing in equal annual ratio or percentage are found in Table I. For income increasing 10% a year, the factor is found under 10% opposite the number of years for which the income is to be received.)

Values of (n)	.50	.55	.60	.65	.70	.75	.80	.85	.90	.95	Values of (n)
1	.5	.55	.6	.65	.7	.75	.8	.85	.9	.95	1
2	.25	.3025	.36	.4225	.49	.5625	.64	.7225	.81	.9025	2
3	.125	.166375	.216	.274625	.343	.421875	.512	.614125	.729	.857375	3
4	.0625	.091506	.1296	.178506	.2401	.316406	.4096	.522006	.6561	.814506	4
5	.03125	.050328	.07776	.116029	.16807	.237305	.32768	.443705	.59049	.773781	5
6	.015625	.027681	.046656	.075419	.117649	.177979	.262144	.377150	.531441	.735092	6
7	.007813	.015224	.027994	.049022	.082354	.133484	.209715	.320577	.478297	.698337	7
8	.003906	.008373	.016796	.031864	.057648	.100113	.167772	.272491	.430467	.663420	8
9	.001953	.004605	.010078	.020712	.040354	.075085	.134218	.231617	.387420	.630249	9
10	.000977	.002533	.006047	.013463	.028248	.056314	.107374	.196874	.348678	.598737	10
11	.000488	.001393	.003628	.008751	.019773	.042235	.085899	.167343	.313811	.568800	11
12	.000244	.000766	.002177	.005688	.013841	.031676	.068719	.142242	.282430	.540360	12
13	.000122	.000421	.001306	.003697	.009689	.023757	.054976	.120905	.254187	.513342	13
14	.000061	.000232	.000784	.002403	.006782	.017818	.043980	.102770	.228768	.487675	14
15	.000031	.000127	.000470	.001562	.004748	.013363	.035184	.087354	.205891	.463291	15
16	.000015	.000070	.000282	.001015	.003323	.010023	.028147	.074251	.185302	.440127	16
17	.000008	.000039	.000169	.000660	.002326	.007517	.022518	.063113	.166772	.418120	17
18	.000004	.000021	.000102	.000429	.001628	.005638	.018014	.053646	.150095	.397214	18
19	.000002	.000013	.000061	.000279	.001140	.004228	.014412	.045599	.135085	.377354	19
20	.000001	.000006	.000037	.000181	.000798	.003171	.011529	.038760	.121577	.358486	20
21	.000000	.000004	.000022	.000118	.000559	.002378	.009223	.032946	.109419	.340562	21
22		.000002	.000013	.000077	.000391	.001784	.007379	.028004	.098477	.323534	22
23		.000001	.000008	.000050	.000274	.001338	.005903	.023803	.088629	.307357	23
24		.000001	.000005	.000032	.000192	.001003	.004722	.020233	.079766	.291989	24
25		.000000	.000003	.000021	.000134	.000753	.003778	.017198	.071790	.277390	25
26			.000002	.000014	.000094	.000564	.003022	.014618	.064611	.263520	26
27			.000001	.000009	.000066	.000423	.002418	.012425	.058150	.250344	27
28			.000001	.000006	.000046	.000317	.001934	.010562	.052335	.237827	28
29			.000000	.000004	.000032	.000238	.001547	.008977	.047101	.225936	29
30				.000002	.000023	.000179	.001238	.007631	.042391	.214639	30
31				.000002	.000016	.000134	.000990	.006486	.038152	.203907	31
32				.000001	.000011	.000100	.000792	.005513	.034337	.193711	32
33				.000001	.000008	.000075	.000634	.004686	.030903	.184025	33
34				.000000	.000005	.000057	.000507	.003983	.027813	.174825	34
35					.000004	.000042	.000406	.003386	.025032	.166083	35
36					.000003	.000032	.000325	.002878	.022528	.157779	36
37					.000002	.000024	.000260	.002446	.020276	.149890	37
38					.000001	.000018	.000208	.002079	.018248	.142396	38
39					.000001	.000013	.000166	.001767	.016423	.135276	39
40					.000001	.000010	.000133	.001502	.014781	.128512	40
41					.000000	.000008	.000106	.001277	.013303	.122087	41
42						.000006	.000085	.001085	.011973	.115982	42
43						.000004	.000068	.000923	.010775	.110183	43
44						.000003	.000054	.000784	.009698	.104674	44
45						.000002	.000044	.000667	.008728	.099440	45
46						.000002	.000035	.000567	.007855	.094468	46
47						.000001	.000028	.000482	.007070	.089745	47
48						.000001	.000022	.000409	.006363	.085258	48
49						.000001	.000018	.000348	.005726	.080995	49
50						.000001	.000014	.000296	.005154	.076945	50

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(Roman numerals are valuation premise numbers, and capital letters refer to types of income for which valuation formulas are given under each premise.)

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